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WHEREABOUTS OF TOPICS FROM 1987 EDITION

To assist the many readers who are familiar with the 1987 edition of this book and who may have teaching notes based upon it, this fully updated 2008 edition has, as far as possible, retained a similar sequence and layout of information. The overall whereabouts of topics from the 1987 edition is as follows in the 2008 edition:

1987 Edition	**2008 Edition**
Chapter 1-3	Chapter 1-3
Chapter 4 (except parts of pages 81-83)	Chapter 4
Chapter 4 (parts of pages 81-83)	Transferred to BR 45 Vol 9
Chapter 5 (except parts of pages 94-95)	Chapter 5
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Chapter 7 (parts of pages 164,166-172)	Transferred to BR 45 Vol 4
Chapter 7 (part of page172 [Trials])	Covered in BR 45 Vol6(1)
Chapter 8	Chapter 7 and from BR 45 Vol 8
Chapter 9	Chapter 8 and from BR 45 Vol 4
Chapter 10	Chapter 9 and from BR 45 Vol 4
Chapter 11	Chapter 10 and from BR 45 Vol 4
--------------	New Chapter 11 from BR 45 Vol 4 and new data
Chapter 12	Chapters 12 / 15 and from BR 45 Vols 4 & 6(1)
Chapter 13	Chapters 12 / 15 and from BR 45 Vols 4 & 6(1)
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Chapter 19	Chapter 19 and from BR 45 Vols 4 and 8(1)
Appendices 1-6	Appendices 1-6
Appendix 7	Appendices 7 and 10 (Appendices 8 / 9 are spare)

FORMULA NUMBERING: 1987 AND 2008 EDITIONS

Existing formula numbers from the 1987 edition have been retained, although their order of presentation may now be out of numerical sequence. Where formulae have been transferred to an Appendix or were in chapters whose numbers have altered (ie new Chapters 8 & 10 and new Appendices 5, 7 & 10) the new Chapter / Appendix number prefix is used but the 1987 number is shown as well (eg "**. . . 8.1 (1987 Ed . . . 9.1)**"). <u>No formulae have been re-allocated to a different previously-used number; where additional formulae have been introduced into a sequence, they have been inserted with an alphabetical suffix (eg "**. . . 5.24a**")</u>.

BIBLIOGRAPHY

The following references have been consulted in the production of this book:

Admiralty Manual of Navigation (BR 45) - Volume 1 [1987 Edition].
Admiralty Manual of Navigation (BR 45) - Volumes 3, 4, 6 and 8.
Norrie's Nautical Tables (Imray Laurie Norie and Wilson Ltd, 1977).
UKHO Publications and Charts.
Bowditch, N (American Practical Navigator Vol I ,1977) Defence Hydrographic Mapping Centre.

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Mr Christopher Peacock, MA (Cantab), C Eng, M I Mech E

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Lieutenant Commander A S Peacock, MSc, FNI, AFRIN, RN

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General

Fig 0-1.   T23 Frigate Entering Portsmouth Harbour (UK)

CHAPTER 1

POSITION AND DIRECTION ON THE EARTH'S SURFACE

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CHAPTER 1

POSITION AND DIRECTION ON THE EARTH'S SURFACE

0101. Scope of Chapter

Chapter 1 introduces the basic terms dealing with position and direction on the Earth's surface. More detailed geodetic information about the Earth's shape and definition of position on the Earth's surface in relation to *Spheroids* and *Datums* is at Chapter 3.

0102. Choice of Units for Measuring Angles

From ancient Greek times, 'degrees' (˚) have been used to measure angles, with 360˚ in one complete revolution. Conveniently, 360 is exactly divisible by all single-digit prime numbers except 7 (ie 2, 3 & 5) and by their multiples (ie 4, 6, 8 & 9). See also Appendix 1.

0103-0109. Spare

SECTION 1 - POSITION ON THE EARTH'S SURFACE

0110. The Shape of the Earth

a. **The Earth's Dimensions.** The Earth is not a perfect *Sphere*; it is slightly flattened at the top and bottom, the smaller diameter being about 23.1 *n.miles* less than the larger. The Earth's 'flattened' shape is known as an *Oblate Spheroid* (see Fig 1-1 below) with an *Equatorial* radius '*a*' of approximately 3443.9 *n.miles* and a *Polar* radius '*b*' of 3432.4 *n.miles* (*International Nautical Miles* of 1852 m based on *WGS 84 Datum*).

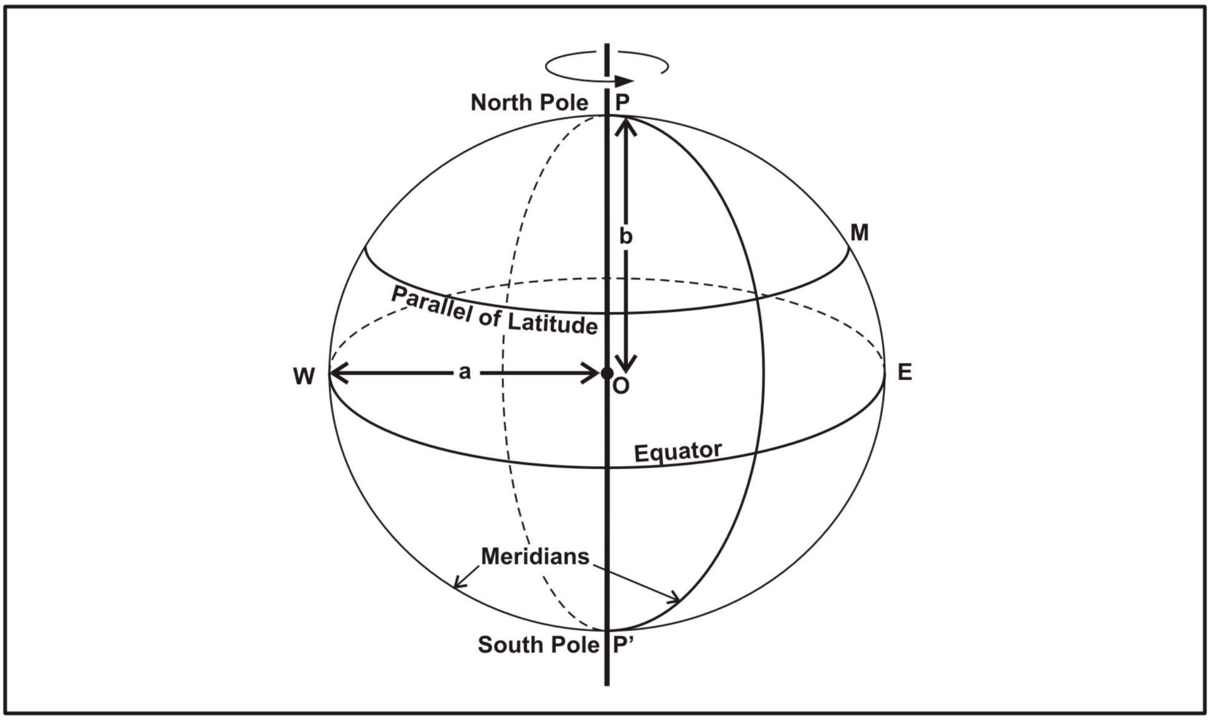


Fig 1-1. The Shape of the Earth - an Oblate Spheroid

b. **The Earth's Rotation.** The Earth turns about its shortest diameter (*PP*' in Fig 1-1 above). An *Oblate Spheroid* is a figure traced out by the revolution of a semi-ellipse such as *PWP*', about its minor axis *PP*'. The Earth revolves about its *Axis PP'* in the direction shown; this direction of revolution is East, the opposite direction is West. The North *Pole* is on the left and the South *Pole* on the right of an observer facing East.

(0110) c. **Terminology.** The following definitions have been established to describe key aspects and measurements of the Earth's shape (see Fig 1-1 previous page).

- **Axis (of the Earth).** The Earth's *Axis* is its shortest diameter (*PP*'), about which it rotates in space.
- **Poles (of the Earth).** The *Poles* are the extremities of the *Axis* of the Earth.
- **Great Circle.** A *Great Circle* is the intersection of a *Spherical* surface and a plane which passes through the centre of the *Sphere*. It is the shortest distance between two points on the surface of a *Sphere*. See *Geodesic* and Note 1-1.
- **Geodesic.** See Note 1-1 (below) and full definition at Para 0540a.
- **Small Circle.** A *Small Circle* is the intersection of a *Spherical* surface and a plane which does NOT pass through the centre of the *Sphere* (see also Para 0115 / Fig 1-9). See Note 1-1 below.
- **Meridian.** A *Meridian* is a semi-*Great Circle* on the Earth's surface whose ends lie at opposite *Poles*. *Meridians* are shown by the successive positions of *PWP*' in Fig 1-1. **See Para 0202a for explanation of *Rhumb Line*.**
- **Prime Meridian.** The *Prime Meridian* (also known as the *Greenwich Meridian)* passes through the Greenwich Observatory (London, UK). The *Prime Meridian* is the starting point (0˚) for the measurement of *Longitude*, East and West from this *Meridian*.
- **Equator.** The *Equator* is the line traced out on the Earth's surface by the mid points of the *Meridians*. The *Equator* is shown by the successive positions of *W* in Fig 1-1 and its plane is perpendicular to the Earth's *Axis*.

Note 1-1. *The Geodesic is the equivalent of a Great Circle on a Spheroid. In everyday use, the terms 'Great Circle' (in lieu of 'Geodesic') and 'Small Circle' are applied to the Earth's Oblate Spheroidal shape. See details at Paras 0115 and 0540.*

0111. Latitude and Longitude

a. **Latitude.** A position on the Earth's surface may be expressed by reference to the planes of the *Equator* and the *Prime Meridian* (see Note 1-2). Fig 1-2 (opposite) shows a *Meridional* section of the *Spheroid*; the *Latitude* of point *M* is the angle *MLE* (ϕ), where *L* is the point of intersection of the perpendicular to the Earth's surface at *M* and the plane of the *Equator* OE. It should be noted that in a *Spheroid*, point *L* may not coincide with point *O* (centre of the Earth). Two definitions arise from this:

- **Definition of Latitude.** The *Latitude* of a place on the Earth's surface (also called the *Geodetic, Geographical* or *True Latitude*) is the angle that the perpendicular at that place makes with the plane of the *Equator* and is measured from 0˚ to 90˚ North or South of the *Equator*.
- **Parallels of Latitude.** Planes parallel to the plane of the *Equator* are known as *Parallels of Latitude*. Except for the *Equator* itself, which is a *Great Circle*, they also comprise *Small Circles* (see Para 0115 / Fig 1-9).

Note 1-2. *A technical definition of 'Absolute Position' for use in Naval Command Systems is at BR 45 Volume 9.*

(0111a continued)

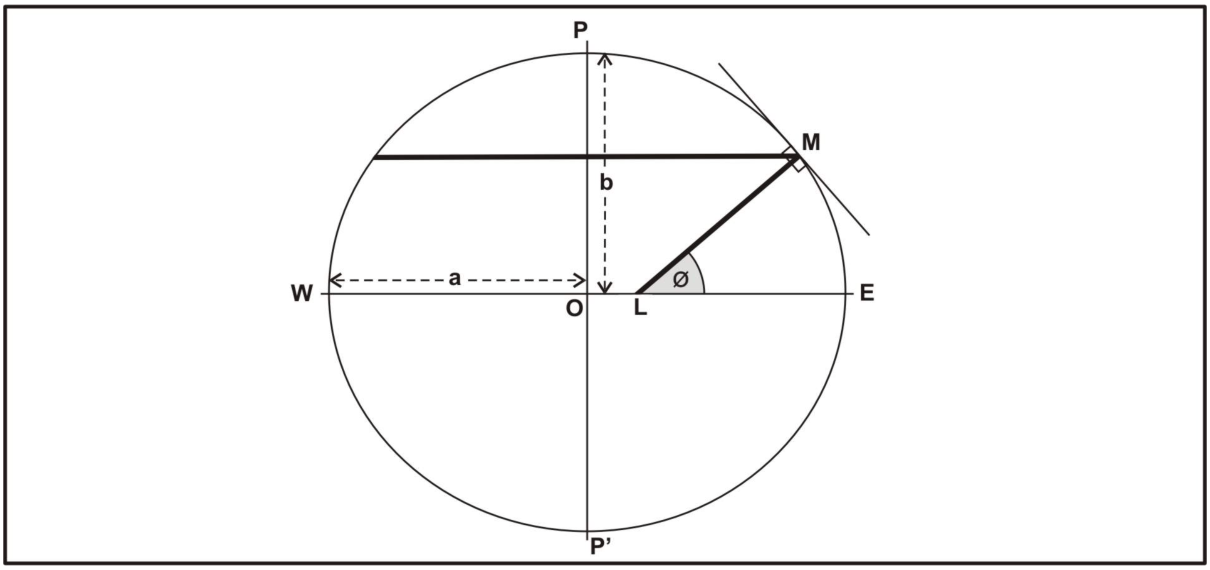


Fig 1-2. The *Latitude of Point M on the Earth's Surface
(also called Geodetic Latitude, Geographical Latitude or True Latitude*)**

(0111) b. **Longitude.** The *Longitude* of a place on the Earth's surface is the angle between the *Prime* (*Greenwich*) *Meridian* and the *Meridian* of that place, measured from 0˚ to 180˚ East or West of Greenwich (see Fig 1-3 below). In Fig 1-3, the *Longitude* of *F* is the arc *AB* = angle *AOB* (East of Greenwich).

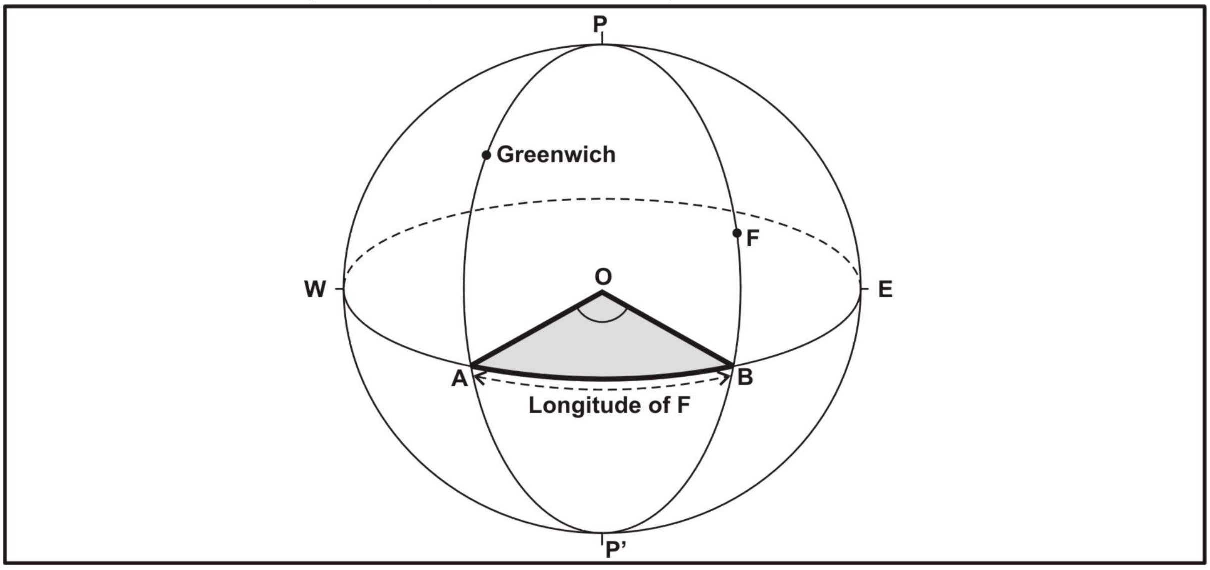


Fig 1-3. The Longitude of Point F on the Earth's Surface

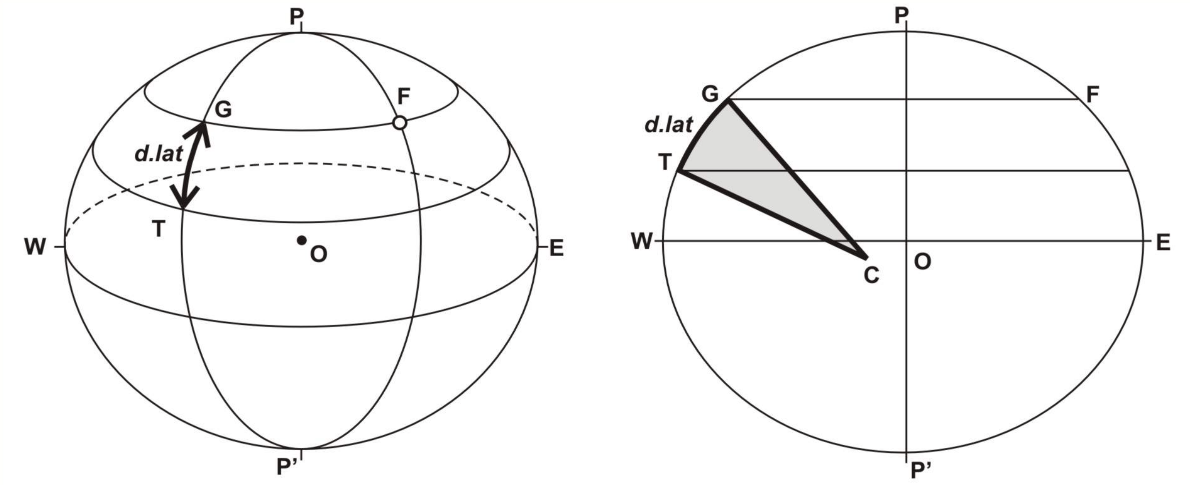
c. **Notation.** From the chart, Semaphore Tower in Portsmouth Naval Base, is in *Latitude* 50 degrees 47 minutes 59 seconds North of the *Equator* and in *Longitude* 1 degree 6 minutes 37 seconds west of Greenwich (*WGS 84*). This may be expressed as:

- 50˚ 47' 59"N   1˚ 06' 37"W   (for traditional use)
- 50˚ 47'.98N   1˚ 06'.62W   (*UKHO* accepted notation for Admiralty charts)
- 50˚ 47.98'N   1˚ 06.62'W   (alternative notation - common usage)
- + 50˚.79972   - 1˚.11028   (for calculator use, +ve for N and E)

The alternative notation (eg 50˚ 47.98'N   1˚ 06.62'W) is used in this book.

0112. Difference of Latitude and Difference of Longitude

a. **Difference of Latitude (d.lat).** The *Difference of Latitude* (*d.lat*) between two places is the arc of the *Meridian* between the two *Parallels of Latitude*. When proceeding from one place to another, *d.lat* is named North or South according to whether the *Latitude* of the destination is North or South of the *Latitude* of the place of departure. In Fig 1-4 (below) the *d.lat* between *F* and *T* is the same as the *d.lat* between *G* and *T*, where *GF* is the *Parallel of Latitude* through *F*.

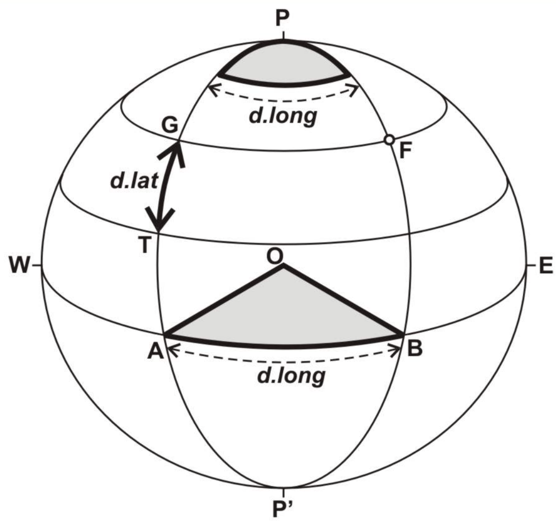


Explanation: d.lat from F to T = angle GCT (south) = Lat F - Lat T

Note: 'C' is the centre of a circle and arc 'GT' is part of its circumference. Thus on a Spheroid (Ellipsoid), the position of 'C' will move relative to 'O' as arc 'GT' moves over the surface of the Spheroid.

Fig 1-4. Explanation of the "d.lat" Calculation

b. **Difference of Longitude (d.long).** The *Difference of Longitude* (*d.long*) between two places is the smaller arc of the *Equator* between their *Meridians*. When proceeding from one place to another, *d.long* is named East or West according to whether the *Meridian* of the destination is East or West of the *Meridian* of the place of departure. In Fig 1-5 (below), the *d.long* between *F* and *T* is the same as the *d.long* between *B* and *A*, where *FB* is the *Meridian* through *F*.



Explanation: d.long from F to T = arc BA = angle BOA (West) = angle FPT.
Angle FPT is the angle at the Pole between the Meridians of F and T.

Fig 1-5. Explanation of the "d.long" Calculation

(0112) c. **Calculation of "d.lat" and "d.long".** The rule for finding the *d.lat* and the d.*long* is as follows:

<u>Same Names</u>: SUBTRACT **<u>Opposite Names</u>: ADD**

If, when using this rule, the sum of the *Longitudes* exceeds 180˚, this sum is subtracted from 360˚ to find the smaller angle and the <u>name is reversed</u>.

Examples 1-1 to 1-3.

Find the d.lat and d.long between:

1-1. Portsmouth (F): (50˚48'N, 1˚07'W) and New York (T): (40˚40'N, 74˚00'W).
1-2. Malta (F): (35˚53'N, 14˚31'E) and Gibraltar (T): (36˚07'N, 5˚21'W).
1-3. Sydney (F): (33˚52'S, 151˚13'E) and Honolulu (T): (21˚18'N, 157˚52'W).

Example 1-1.	Lat F 50˚48'N	Long F	1˚07'W
	Lat T <u>40˚40'N</u>	Long T	<u>74˚00'W</u>
	d.lat 10˚08'S	d.long	72˚53'W
Example 1-2.	Lat F 35˚53'N	Long F	14˚31'E
	Lat T <u>36˚07'N</u>	Long T	<u>5˚21'W</u>
	d.lat 0˚14'N	d.long	19˚52'W
Example 1-3.	Lat F 33˚52'S	Long F	151˚13'E
	Lat T <u>21˚18'N</u>	Long T	<u>157˚52'W</u>
	d.lat 55˚10'N	d.long	309˚05'W
		subtract from	<u>360˚</u>
		d.long	50˚55'E

0113. Linear Measurement of Distance and Speed

a. **The Statute Mile**. The *Statute Mile* (also known as the *Land Mile*) is a standard fixed length of 1760 yards or 5280 feet (1609.36 m).

b. **The Geographical Mile.** A *Geographical Mile* is the length of 1' of arc measured along the *Equator* (ie 1' of *Longitude*); its value is 1855.3 m (*WGS 84*). As the *Equator* is a circle (*Great Circle*), the *Geographical Mile* is the same length at all parts of the *Equator* and is equal to "*a* sin 1' of arc", where "*a*" is the radius of the *Equator*.

c. **The International Nautical Mile.** The *International Nautical Mile* is a standard fixed length of 1852 m. Its abbreviation is the term "*n mile*" (or "*n.mile*" or "*nm*").

d. **The Knot.** In navigation, it is convenient to have a fixed or standard unit for measuring speed. This unit is called a *Knot*, and is one *International Nautical Mile* (1852 m) per hour. Its abbreviation is the term "*kn*" (NOT "kt"). The name '*Knot*' originates from running out a log line with distances marked by knots tied in the line. In normal practice, the errors arising from using *Sea Miles* (see Paras 0113e/f overleaf) instead of *International Nautical Miles* (*n.miles*) are very small (less than 0.5%); see details at Para 0113g (overleaf).

(0113) e. **The Sea Mile.** The *Sea Mile* is the length of one minute of arc (1') measured along the *Meridian* in the *Latitude* of the position (see explanation at Fig 1-6 below). On Admiralty charts on the *Mercator Projection* (see Chapter 4), the *Latitude* graduations form a *Scale* of *Sea Mile* (see Note 1-3 below). Except on charts (where the symbol 'M' is used) the *Sea Mile* is denoted by ', which is also the symbol for a minute of arc. Thus, 10'.8 (or 10.8') means 10.8 *Sea Miles*. Traditionally, the symbol was always placed before the decimal point, but increasingly, this convention has altered and placing the symbol at the end of the expression (eg 10.8') is now considered equally acceptable.

Note 1-3. *It is a common but mistaken practice for mariners to refer to a 'Sea Mile' as a 'Nautical Mile'. The British Standard Nautical Mile was discarded in 1970.*

f. **Length of the Sea Mile.** The *Radius of Curvature* in the *Meridian* (see Fig 1-6 below) increases as *M* moves from the *Equator* to the *Pole*; thus, the distance subtended by 1' of arc also increases. The length of the *Sea Mile* is shortest at the *Equator* (1842.9 m) and longest at the *Poles* (1861.6 m), with a mean value of 1852.2 m at 45˚ *Latitude* (all *WGS 84*).

g. **Practical Differences: International Nautical Miles and Sea Miles.** In normal practice, the errors arising from using *Sea Miles* instead of *International Nautical Mile* are very small (effectively zero error at 45˚ *Latitude,* 0.5% [short] at the *Equator* and 0.5% [long] at the *Poles*). However, it is sometimes necessary to determine the error and the procedure for this is at Para 0314. Formulae (3.8 to 3.10) [see Para 0314] give the length of 1' of arc of *Latitude*; their derivation is at Appendix 5.

h. **Length of a Cable.** One-tenth of a *Sea Mile* is known as a *Cable*, which varies between 184.3 m (201.55 yds) and 186.2 m (203.63 yds) according to *Latitude*. A *Cable* thus approximates to 200 yards, and this nominal distance is a convenient measure normally used at sea for short-range navigational purposes.

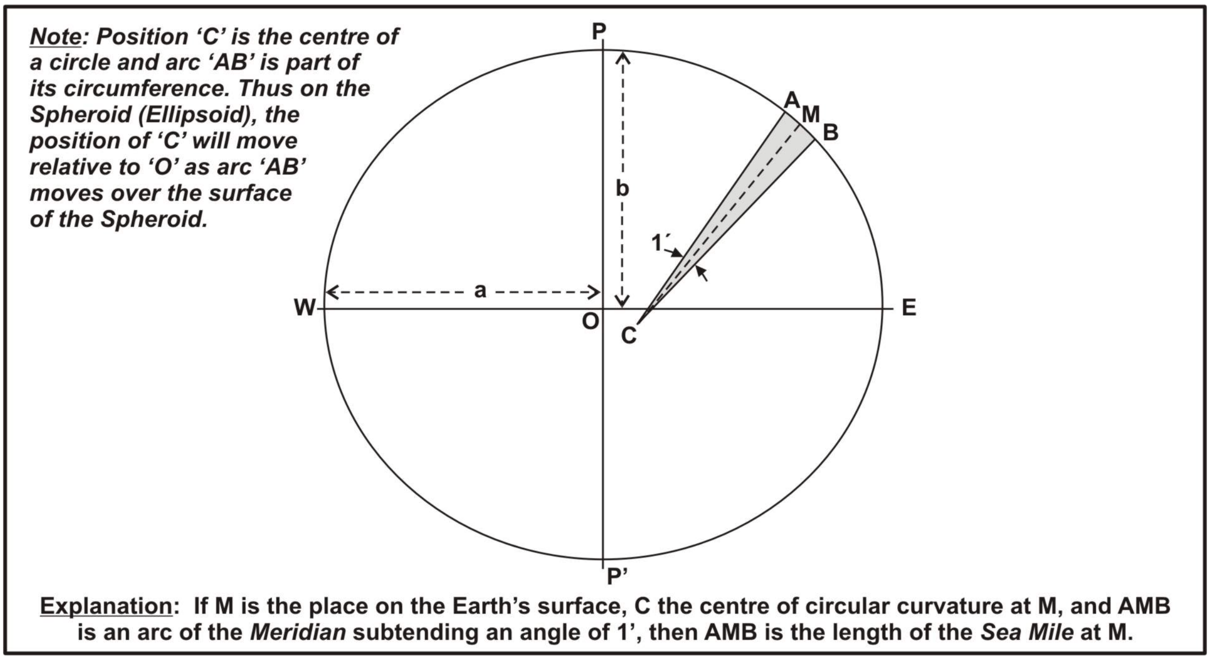
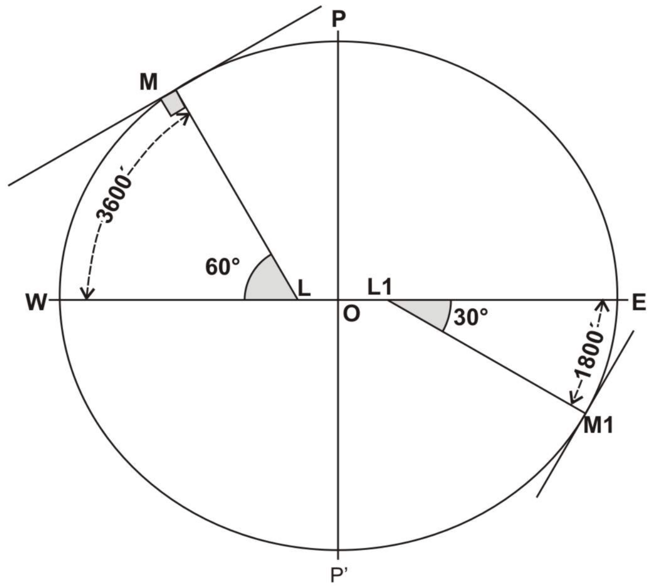


Fig 1-6. The Sea Mile

0114. Linear Measurement of Latitude and Longitude

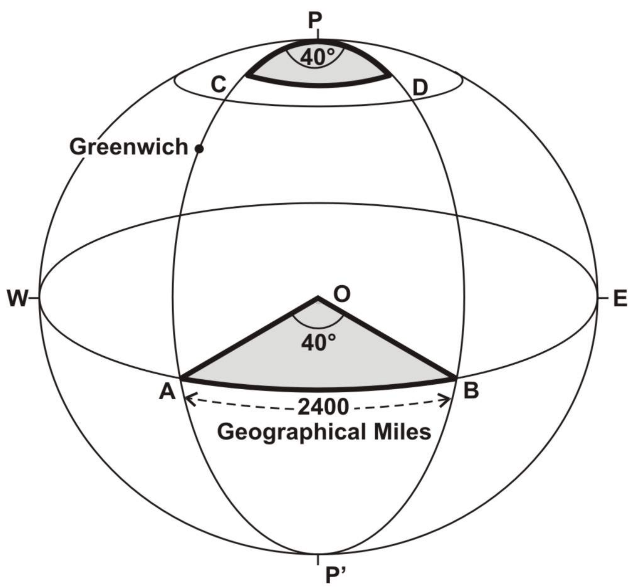
a. **Linear Measurement of Latitude.** The *Linear Latitude* of a place is the length of the arc of the *Meridian* between the *Equator* and that place. It is measured in *Sea Miles* North or South of the *Equator* (see explanation at Fig 1-7 below).



Explanation: If M is in *Latitude* 60°N, then angle MLW = 60° = 60 x 60 minutes of arc = 3600'.
If place M_1 is 1800' south of the *Equator*, its *Latitude* is 1800 / 60 = 30°S.

Fig 1-7. Linear Measurement of Latitude

b. **Linear Measurement of Longitude.** The *Linear Longitude* of a place is the smaller arc of the *Equator* between the *Prime Meridian* and the *Meridian* of the place. Along the *Equator*, it is measured in *Geographical Miles*, (see Para 0113b) East or West of the *Prime Meridian* (see explanation at Fig 1-8 below).



Explanation: If point B is 40°E of the *Prime Meridian* PAP', the angle AOB is 40°, the arc AB of the *Equator* is 40° = 40 x 60 = 2400 minutes of arc along the *Equator* = 2400 *Geographical Miles*

Fig 1-8. Linear Measurement of Longitude

(0114) c. **Variation in Linear Distance of Longitude with Change in Latitude.** *Meridians* converge to meet at a point at the *Poles*, and thus the distance on the Earth's surface between any two *Meridians* is greatest at the *Equator* and diminishes until it is zero at the *Poles* (see Fig 1-8, previous page). The linear distance of a degree of *Longitude* on the surface of a *Spherical* Earth varies with the cosine of the *Latitude* [see formula (1.1) below], but in practice with the *Spheroidal* Earth's shape this is only an approximation.

With a *Spherical Earth* at *Latitude* ϕ :

$$1^{\circ}\,\text{Longitude} = \text{Linear distance at Equator} \times \text{cosine Latitude}\ \phi \qquad \ldots\ \mathbf{1.1}$$

d. **Linear Distance of Longitude - Errors.** In practice (*WGS 84 Spheroid*), the percentage errors in assuming formula (1.1) increase towards the *Poles*, but as linear distance between *Meridians* decreases towards the *Poles,* these two variables work against each other [see formula (3.12)]. Formulae (1.1) and (3.12) produce a maximum linear error at about *Latitude* 60 ˚, as follows:

- At the *Equator*: 0.0% error (0.0 metres)
- At *Latitude* 10˚: 0.01% error (0.6 metres)
- At *Latitude* 30˚: 0.08% error (81 metres)
- At *Latitude* 45˚: 0.17% error (132 metres)
- At *Latitude* 60˚: 0.25% error (140 metres)
- At *Latitude* 80˚: 0.33% error (63 metres)
- At *Latitude* 89˚: 0.34% error (0.6 metres)

In all cases the approximate distance is greater than the precise calculation. Formulae (3.8 to 3.12) [see Para 0314] give the precise *Spheroidal* distances.

0115. The Earth as a Sphere

a. **Properties of a Sphere.** A *Sphere* is the figure formed by rotating a semi-circle about its diameter. The following definitions are repeated from Para 0110c for the convenience of readers (see *Great* & *Small Circles* at Para 0115c / Fig 1-9 opposite).

(Extracts from Para 0110c):

- **Great Circles.** Any plane through the centre of the *Sphere* cuts the surface in a *Great Circle.*
- **Small Circles.** Any plane which cuts the surface of the *Sphere*, but does not pass through the centre, is called a *Small Circle.*

b. **Effect of Assuming the Earth as a Sphere.** Although the shape of the Earth is that of an *Oblate Spheroid*, for most purposes of navigation it may be assumed to be a *Sphere*, with radius equal to the mean of the greatest and least radii and measuring approximately 3438 *n.miles* (*International Nautical Miles* with *WGS 84* - see Note 1-4 below). Thus, when the Earth is regarded as a *Sphere*, *Meridians* of *Longitude* become *Great Circle*s which cut the *Equator* at right angles and join the *Poles*. The *Equator* is a *Great Circle* but all other *Parallels of Latitude* are *Small Circles*.

Note 1-4. *Where 'a' is the Equatorial radius and 'b' the Polar radius, the Earth's arithmetical mean radius is:*

$$\frac{a+b}{2} = 6{,}367{,}444.657 \text{ metres} = 3438.14506 \text{ n miles} \quad (WGS\ 84)$$

(0115) c. **Practical Use of the Great Circle.** The *Great Circle* gives the shortest distance between two points on the surface of a *Sphere*; for practical purposes, this term may also be applied to the surface of the Earth's *Oblate Spheroid* shape (ie as a *Geodesic* - see Para 0540a) without appreciable error. A *Great Circle* may thus be regarded as the shortest distance between two points on the Earth's surface and is also the path taken by an electro-magnetic radiation near the Earth's surface (radio, radar, light, etc).

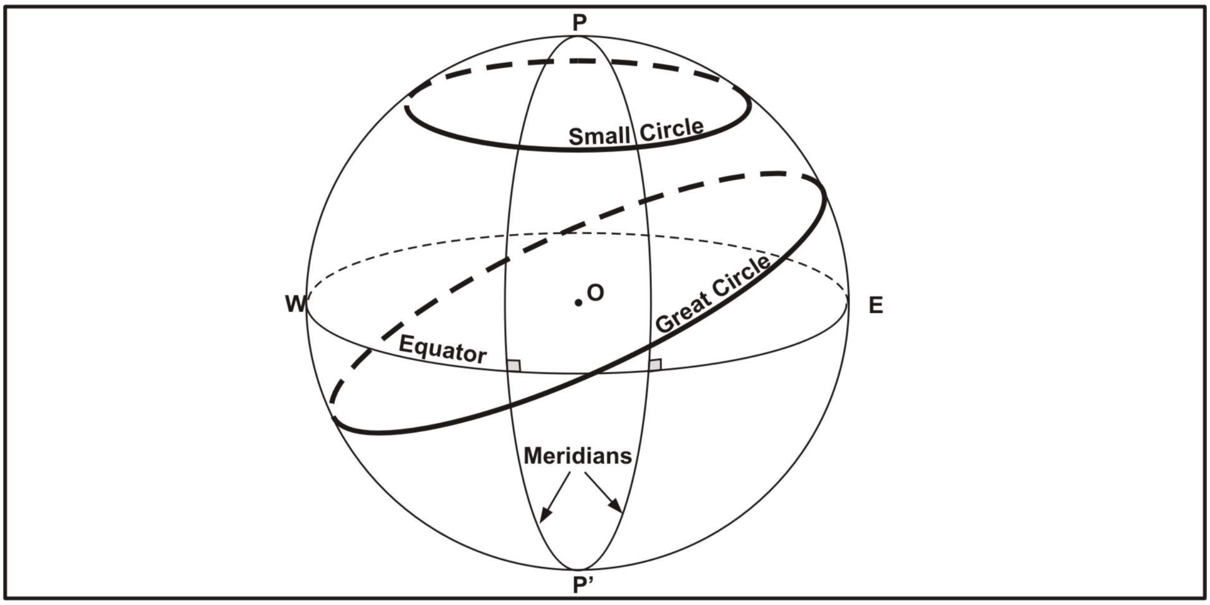


Fig 1-9. The Earth as a Sphere

d. **Approximation of Distances - Assuming the Earth as a Sphere.** Using the mean radius for the *Sphere* derived from *WGS 84* (see Para 0115b opposite), the length of 1' of *Latitude* on the *Meridian* (or 1' of *Longitude* on the *Equator*) equals 1852.2 m. This distance approximates very closely to the length of the *International Nautical Mile* of 1852 m. Thus without appreciable error (see Note 1-5 below), the Earth may be treated as a *Sphere* where:

- 1' of *Latitude* equates to 1 *n.mile* (*International Nautical Mile*) anywhere.
- On the *Equator*, 1' of arc of *Longitude* equates to one *n.mile*.
- *Linear Latitude* and *Longitude* may be measured in the same units (*n.miles*).

Note 1-5. *The errors introduced by assuming a Spherical Earth based on the International Nautical Mile are not more than 0.5% for Latitude, 0.2% for Longitude.*

0116. Astronomical Positions Used with a Spheroidal Earth

Astronomical observations measure angles from the horizon, which is itself referenced to the local vertical (*Zenith*). If the Earth was *Spherical*, the *Zenith* line produced would pass through the centre of the Earth (which is also the centre of the *Celestial Sphere*) and positions from astronomical observations could be plotted without error. With a *Spheroidal Earth*, the *Zenith* line produced does NOT pass through the centre of the Earth except at the *Equator* and *Poles*; elsewhere, a small error occurs which is maximum at *Latitude* 45°. This error is NOT significant for distant celestial bodies (ie the sun, planets and stars), but can reach 0.2' of sextant altitude for the Moon; a correction for this is applied automatically in HM Nautical Almanac Office's *NAVPAC* software (available from *UKHO* as DP330 - see details at Para 0210a).

0117-0119. Spare.

SECTION 2 - DIRECTION ON THE EARTH'S SURFACE

0120. Direction

a. **True Direction.** The true direction between two points on the Earth's surface is given by the *Great Circle* between them; it is expressed as the angle between the *Meridian* and *Great Circle* (angle *PFT* in Figs 1-10a/b below). A technical definition of '*Direction*' for use in Naval Command Systems is at BR 45 Volume 9.

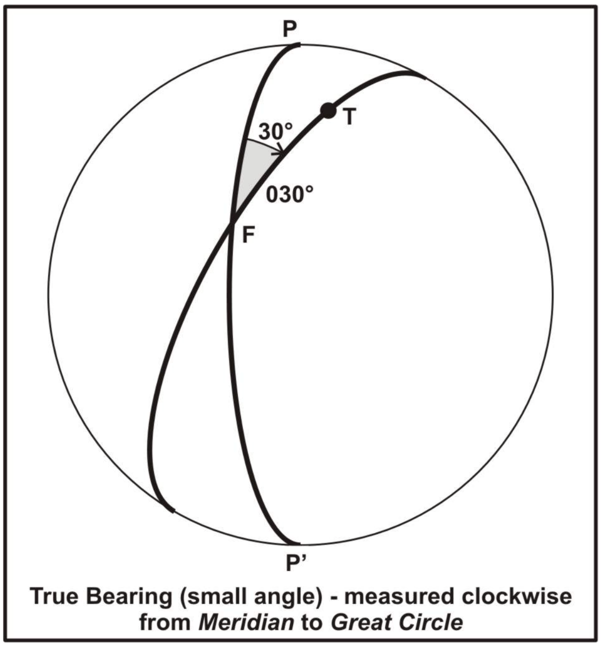


Fig 1-10a. True Bearing (Small Angle)

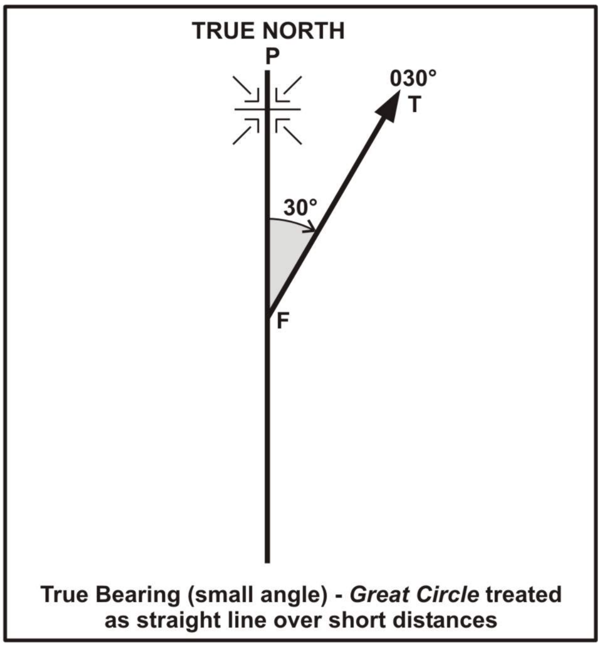


Fig 1-10b. True Bearing (Small Angle)

b. **True North.** *True North* is the northerly direction of the *Meridian* and is the reference from which true bearings and courses are measured.

c. **The Navigational Compass.** The navigational compass provides the reference direction from which courses and bearings may be measured. There are two principal types of compass: the *Gyro Compass* and the *Magnetic Compass*, both of which are described in detail in BR 45 Volume 3. The general principles of the two principal types of compass are described at Paras 0121 and 0122 respectively, with an explanation as to how true courses and true bearings may be obtained from them.

d. **True Bearing.** The true bearing of an object is the angle between the *Meridian* and the direction of the object.

- The true bearing of *T* from *F* is given by the angle *PFT* in Fig 1-10a (above) and Fig 1-11a (opposite), where *PF* is the *Meridian* through F and *FT* is the *Great Circle* joining *F* to *T.*
- Angle *PFT* is measured clockwise from 000˚ to 360˚. In Fig 1-10a (above), *T* bears 030˚ from F; in Fig 1-11a (opposite), T bears 330˚ from F.
- Over short distances the *Great Circle* may be drawn as a straight line without appreciable error, as in Fig 1-10b (above) and Fig 1-11b (opposite). The error varies with the *Latitude* and the bearing.

(0120d continued)

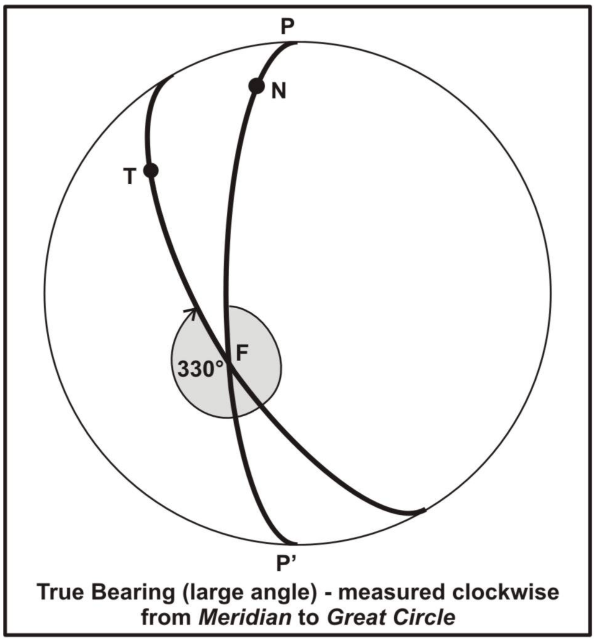


Fig 1-11a. True Bearing (Large Angle)

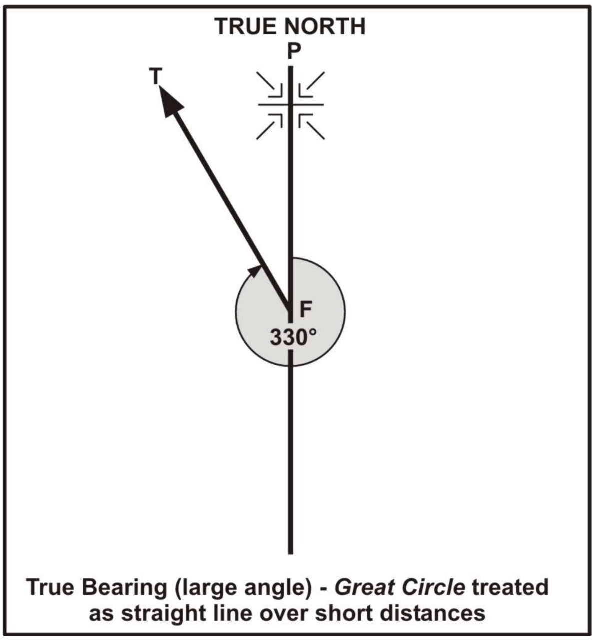


Fig 1-11b. True Bearing (Large Angle)

(0120) e. **Position by Bearing and Distance - Notation.** Rather than specify a position by *Latitude* and *Longitude*, it is often convenient to indicate <u>either</u> a position of an object from an observer by bearing and distance, <u>or</u> a position by bearing and distance from a known object. There are two notations for this: bearings centred on the observer and bearings centred on the object. It is essential to establish clearly the difference between these notations as otherwise an error of 180˚ may occur:

- **Position of an Object From an Observer.** When the position of an object is indicated <u>from an observer</u> by bearing and distance (as when taking a *Fix*), the convention is the state the object first, then the bearing and distance (eg Start Point Light 270˚ 4.0 *n.miles*). This indicates that the object is to the West of the observer.

- **Position from an Object.** When a position <u>from a known object</u> is indicated by bearing and distance (as in specifying a rendezvous position), the convention is to state the bearing first, then the object and then distance (eg 090˚ Start Point Light 4.0 *n.miles*). This indicates that the position is to the East of the object.

f. **True Course.** True course is the direction along the Earth's surface in which the vessel is being steered (or is intended to be steered). It is measured by the angle between the *Meridian* through the vessel's position and the fore-and-aft line, clockwise from 000˚ to 360˚. A technical definition of '*Course*' for use in Naval Command Systems is at BR 45 Volume 9.

g. **True Heading.** True course is not to be confused with *Heading* (or *Ship's Head*), which is the instantaneous direction of the ship and is thus a constantly changing value if the ship *Yaws* across the course due to the effect of wind, sea and steering errors. A technical definitions of '*Yaw*' and '*Heading*' for use in Naval Command Systems is at BR 45 Volume 9.

0121. The Gyro Compass

The *Gyro Compass* first made its appearance in the early part of the 20th century; in its simplest form it comprised a rapidly rotating wheel (*Gyroscope*), the *Spin Axis* of which was made to point along the *Meridian* to *True North*. By the late 20th / early 21st centuries, *Gyro Compasses* became extremely accurate, reliable instruments; some now use *Fibre Optic Gyro* (*FOG*) technology with few, if any, moving parts. See Para 0920 for *Gyro Compass* theory.

a. **Gyro Compass Output.** Provided there is no *Gyro* error, the *Gyro Compass* provides true courses and bearings, measured clockwise from 000˚ to 360˚.

b. **Gyro Compass Errors.** For a number of reasons the *Gyro Compass* may not always point exactly towards *True North*. Any *Gyro* error must be established before the *Gyro Compass* may be used as an accurate reference (see details at Para 0811).

c. **Gyro Repeater Error.** The alignment of the *Lubbers Line* of *Gyro* repeaters to *Ship's Head* (see Fig 8-1 at Para 0802) should be checked frequently and adjusted if necessary. On offset repeaters (eg Bridge Wings) an incorrect alignment may not be easily noticed but will cause significant errors (see procedure at Para 1230f).

d. **Gyro Error Magnitudes.** The maximum error in Royal Navy *Gyro Compasses* in adverse conditions is normally better than ¼˚ at the *Equator* and ½˚ at *Latitude* 60˚, and much better in good conditions (see details at BR 45 Volume 3). However, in some commercial *Gyro Compasses* the error may exceed this by one or two degrees.

e. **Calculating the Gyro Error.** If the true bearing of an object is known to be 075˚ and the *Gyro* bearing is 077˚, then the *Gyro* is reading 2˚ high (see Fig 1-12a below); similarly, if the *Gyro* bearing is 073˚, the *Gyro* is reading 2˚ low (see Fig 1-12b below).

f. **Correcting the Gyro Error.** In order to calculate the true bearing:

- **Gyro Error High.** If the *Gyro* error is **<u>high</u>**, it must be **<u>subtracted</u>** from the *Gyro* bearing **<u>observed</u>** (or added to bearings from the chart).
- **Gyro Error Low.** If the *Gyro* error is **<u>low</u>**, it must be **<u>added</u>** to the *Gyro* bearing **<u>observed</u>** (or subtracted from bearings from the chart).
- **Suffixes 'G' and 'T'.** The suffixes 'G' or 'T' should be used where appropriate to denote *Gyro* or 'True' courses / bearings respectively.

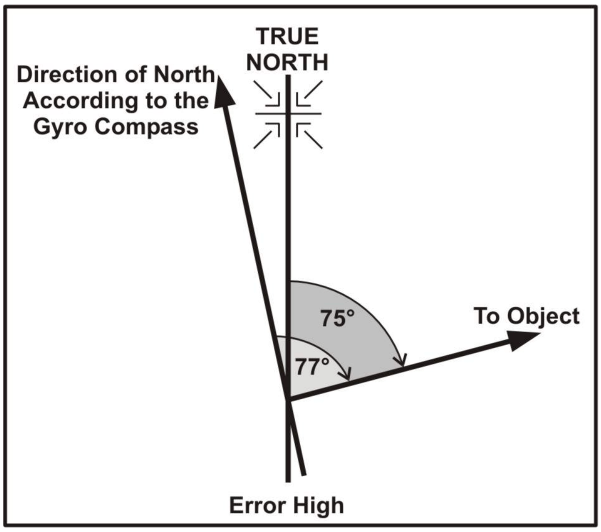


Fig 1-12a. Gyro Error High

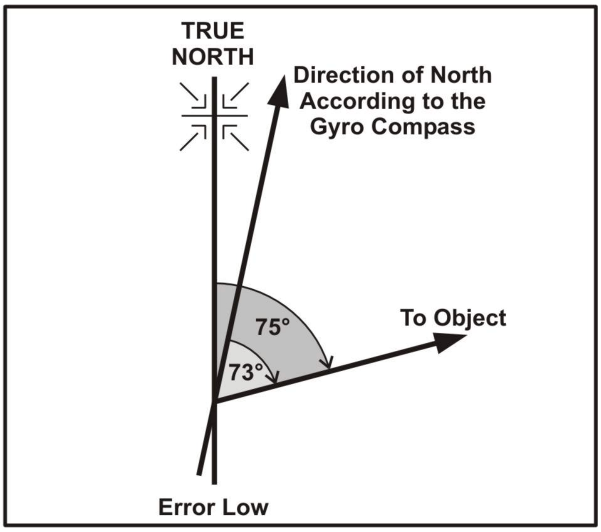


Fig 1-12b. Gyro Error Low

0122. The Magnetic Compass

The *Magnetic Compass* is a bar magnet freely suspended in the horizontal plane and acted upon by both the Earth's magnetic field and the magnetic properties of the ship.

a. **The Earth's Magnetic Field.** The Earth may be considered as a gigantic magnet. Magnetic lines of force emanate from the South *Magnetic Pole* (located in Antarctica). These lines of force follow approximate semi-*Great Circle* paths to the North *Magnetic Pole* (located in the Canadian Arctic). These *Magnetic Pole*s are not stationary but are continually moving over a largely unknown path in a cycle of some hundreds of years.

b. **The Magnetic Meridian.** A freely suspended *Magnetic Compass* needle acted on only by the Earth's magnetic field will lie in the vertical plane along the Earth's magnetic field line of force. This vertical plane is known as the *Magnetic Meridian* (*Magnetic North*). However, as the Earth's magnetic field is irregular, *Magnetic Meridian*s do not always point towards the *Magnetic Pole*s. In addition, the *Magnetic Pole*s are not 180˚ apart; thus, it is rare for the magnetic needle to point towards the *Magnetic Pole*.

c. **Magnetic North.** *Magnetic North* is the name given to the direction in which the 'North' end of a magnetic needle, suspended so as to remain horizontal, would point when subject only to the influence of the Earth's magnetism. It is the northerly direction of the *Magnetic Meridian*.

d. **Magnetic Variation.** *Magnetic Variation* (normally abbreviated to '*Variation*') is the angle between the geographic (true) *Meridian* and the *Magnetic Meridian*. It is measured East or West from *True North*; in Fig 1-13 (below) the *Variation* at point *F* is shown as 20˚ West. *Variation* has different values at different places and is gradually changing. Its value at any place may be found from navigational charts which give the *Variation* for a certain year, together with a note of its annual value of secular change <u>for which allowance must be made</u>. *Variation* may also be taken from special *Isogonic* charts on which all places of equal *Variation* are joined by *Isogonic* lines known as *Isogonals* (not to be confused with *Magnetic Meridian*s, which are lines of force).

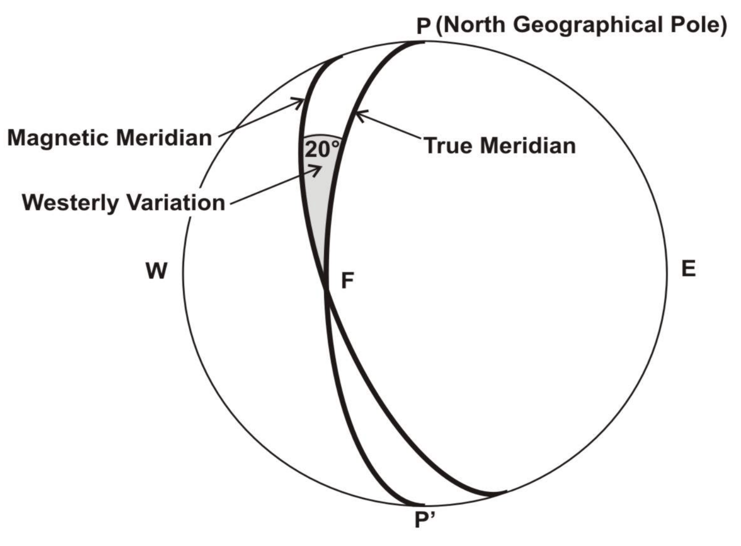


Fig 1-13. Magnetic Variation (Example)

(0122) e. **Magnetic Deviation and Compass North.** If a *Magnetic Compass* is put in a ship, the presence of iron, steel or electrical equipment will cause the *Magnetic Compass* to deviate from the *Magnetic Meridian*. The angle between the *Magnetic Meridian* (*Magnetic North*) and the direction in which the needle points (*Compass North*) is called *Magnetic Deviation* (normally abbreviated to '*Deviation*'). It is measured East or West from *Magnetic North*.

f **Reducing Deviation.** As a ship alters course in a particular *Magnetic Latitude*, the magnetic field of the ship changes, both in direction and magnitude. Consequently, *Deviation* is different for different compass courses. In practice, the *Deviation* of a ship's *Magnetic Compass* is reduced to a minimum (usually less than 3˚) by the fitting and adjustment of permanent magnets and soft-iron *Spheres* at the compass binnacle.

g. **'Swinging the Ship' to Establish Residual Deviation.** After adjustment of the permanent magnets and soft-iron *Sphere*s at the compass binnacle, the ship's residual *Deviation* is found by slowly 'swinging the ship' through 360˚ and noting the *Deviation* for the various compass *Headings* (see Note 1-6 below); this may easily be done at sea as a 'Comparison Swing' against the *Gyro Compass* (see Para 0125c). The residual *Deviation* may be tabulated (see Table 1-1 below) or drawn as a graph (see Fig 1-14 opposite); intermediate values may be obtained by interpolation. If there is any significant change of *Magnetic Latitude*, the swinging (and sometimes adjustment) procedures should be repeated. Detailed procedures are at BR 45 Volume 3.

Table 1-1. Table of (Example) Residual Deviations

COMPASS HEADING		BEARING OF DISTANT OBJECT		DEVIATION
		MAGNETIC (FROM CHART)	COMPASS (OBSERVED)	
N	(000˚)	236˚ M	237½ ˚C	1½ ˚W
NNE	(022½˚)	236˚ M	237¾ ˚C	1¾ ˚W
NE	(045˚)	236˚ M	237¾ ˚C	1¾ ˚W
ENE	(067½˚)	236˚ M	237½ ˚C	1½ ˚W
E	(090˚)	236˚ M	237 ˚C	1 ˚W
ESE	(112½˚)	236˚ M	236½ ˚C	½ ˚W
SE	(135˚)	236˚ M	235½ ˚C	½ ˚E
SSE	(157½˚)	236˚ M	235 ˚C	1 ˚E
S	(180˚)	236˚ M	234½ ˚C	1½ ˚E
SSW	(202½˚)	236˚ M	234 ˚C	2 ˚E
SW	(225˚)	236˚ M	234 ˚C	2 ˚E
WSW	(247½˚)	236˚ M	234¼ ˚C	1¾ ˚E
W	(270˚)	236˚ M	234¾ ˚C	1¼ ˚E
WNW	(292½˚)	236˚ M	235¼ ˚C	¾ ˚E
NW	(315˚)	236˚ M	236 ˚C	NIL
NNW	(337½˚)	236˚ M	237 ˚C	1 ˚W

Note 1-6. *In the Royal Navy, Forms RNS 374A (Record of Observations for Deviation) and RNS 387 (Table of Deviations) are used to record Deviations. They are tabulated every 22½ ˚ to facilitate the calculation of the various compass coefficients (see BR 45 Volume 3).*

(0122g continued)

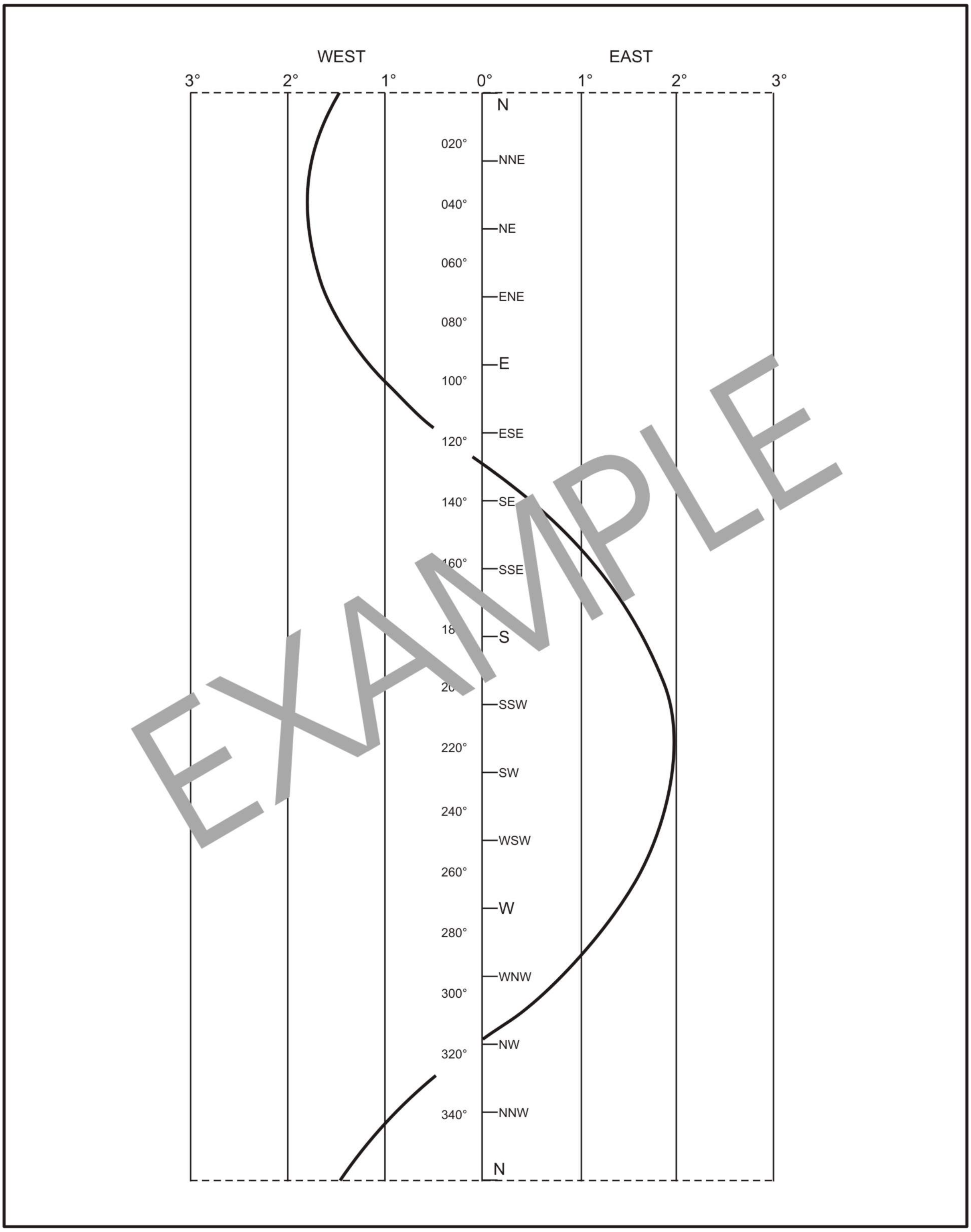


Fig 1-14. Graph of (Example) Residual Deviations

0123. Magnetic Courses / Bearings and Compass Courses / Bearings

a. **Magnetic Courses and Bearings.** Magnetic courses and bearings are measured clockwise from 000˚ to 360˚ from *Magnetic North* (the *Magnetic Meridian*) and are given the suffix 'M', eg 075˚M. They differ from true courses and bearings by the *Variation* (see Fig 1-15). See also Para 0123c opposite.

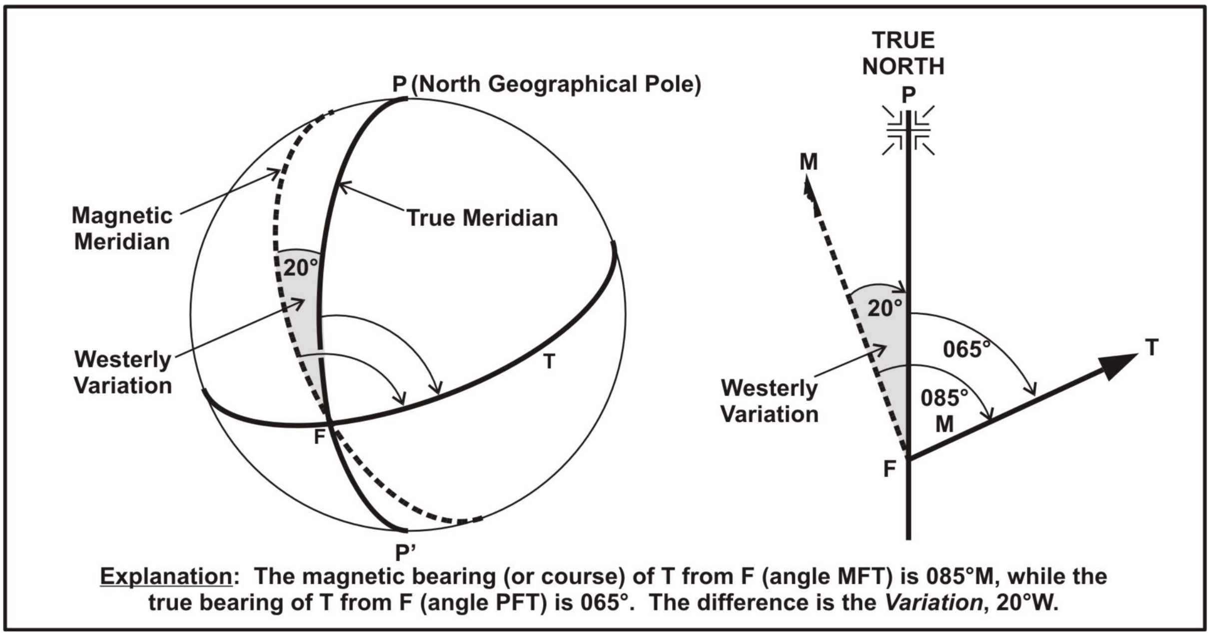


Fig 1-15. Magnetic Courses and Bearings

b. **Compass Courses and Bearings.** Compass courses and bearings are measured clockwise from 000˚ to 360˚ from *Compass North*, and are given the suffix 'C', eg 195˚C. They differ from true courses and bearings by the *Variation* for the geographical location and the *Deviation* for the compass *Headings* (see Fig 1-16). See also Para 0123c opposite.

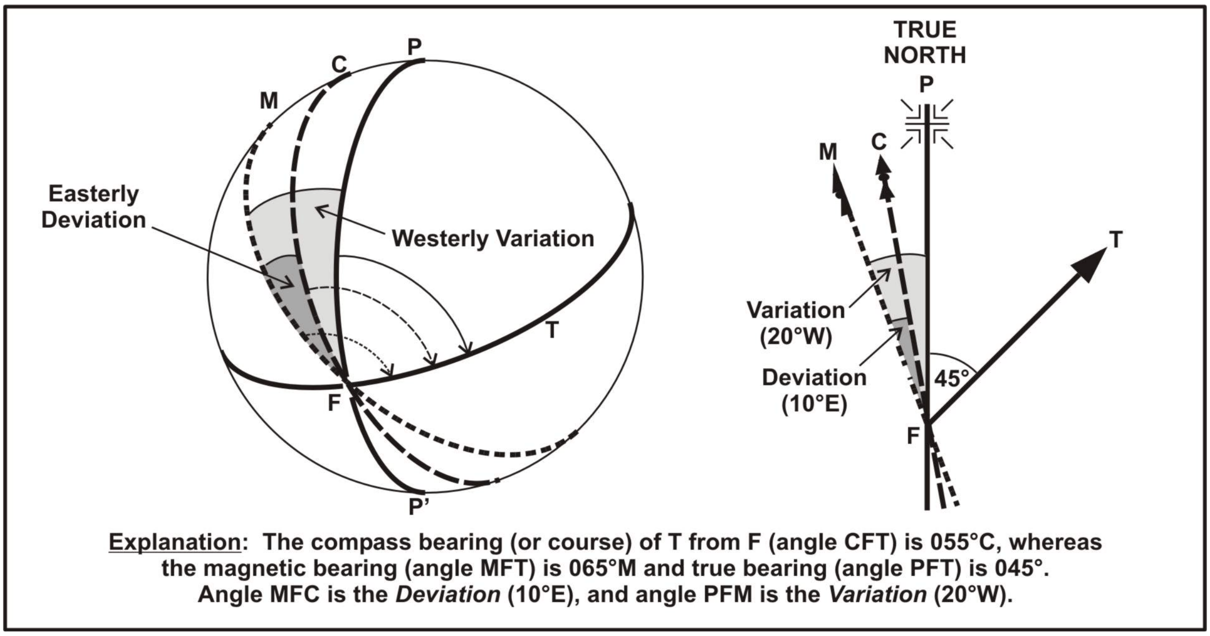


Fig 1-16. Magnetic Courses / Bearings and Compass Courses and Bearings

(0123) c. **'Traditional' Graduation of Older Magnetic Compass Cards.** Some older (mostly historical) *Magnetic Compass* cards may still be found which do not use 000˚ to 360˚ graduations, but instead, are graduated with 'traditional' markings:

- **Older Compass Cards.** Older *Magnetic Compass* cards were divided into four quadrants of 90˚, the angles being measured from North and South to East and West. For example, the bearing 137˚M would be shown as S43˚E.

- **Even Older Compass Cards.** Even older *Magnetic Compass* cards were divided into four quadrants by the cardinal points, North, East, South, West. Each quadrant is divided into eight equal parts, the division marks being called *Points*; each point has a distinctive name - North, North by East, North North East and so on. There were 32 *Points* in the whole card.

0124. Practical Application of Magnetic Compass Errors

a. **Compass Rose Magnetic Information.** All Admiralty (*UKHO*) charts have *Compass Roses* printed on them, containing the following information.

- **True and Magnetic Roses.** When there are two concentric *Roses*, the outer *Rose* represents the true compass and the inner the *Magnetic Compass* (see Fig 1-17 below). Some small-*Scale* charts have only the true *Compass Rose*.

- **Variation, Variation Change (and Deviation).** The *Variation,* the year for which it is correct and its annual rate of secular change are normally shown in the *Compass Rose* on its *Magnetic Meridian* (see Fig 1-17 below). Before using the magnetic *Rose* to lay off compass bearings or the compass courses, correction for the change in *Variation* (and *Deviation*) must be applied.

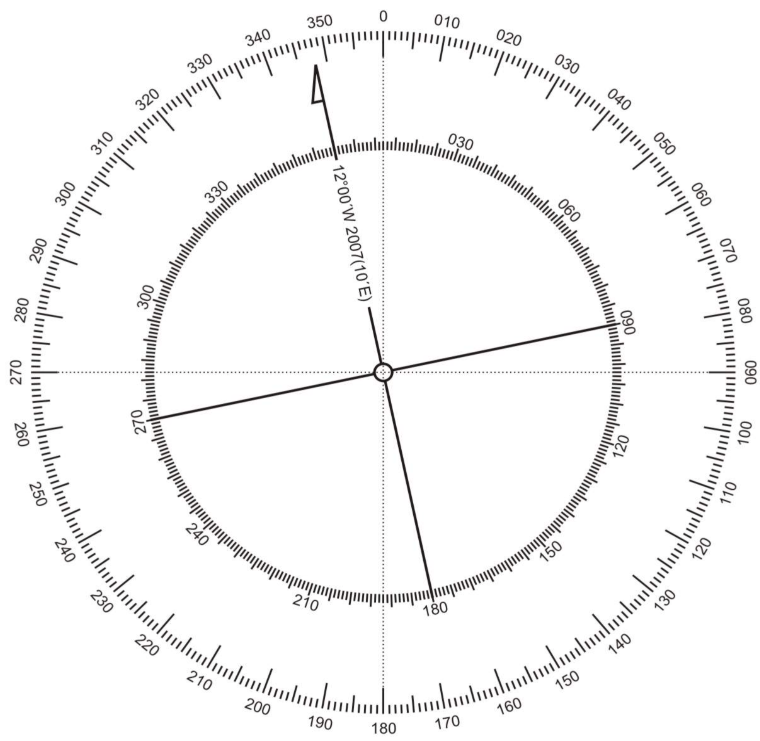


Fig 1-17. Compass Rose of the Type Printed on Admiralty Charts

(0124) b. **Difference Between 'Magnetic' and 'Compass'.** As defined at Para 0122, *Variation* is the difference between True and Magnetic, while *Deviation* is the difference between Magnetic and Compass, ie:

Magnetic = True ± *Variation*
Compass = Magnetic ± *Deviation* = True ± *Variation* ± *Deviation*

c. **Conversion of Magnetic and Compass Courses and Bearings to True.** The following rules should be applied for the conversion of magnetic or compass courses and bearings to true:

- Easterly *Variation* and *Deviation* are added or applied clockwise.
- Westerly *Variation* and *Deviation* are subtracted or applied anti-clockwise.
- The order of conversions (Compass-to-Magnetic-to-True) is:
  "Compass ± *Deviation* = Magnetic. Magnetic ± *Variation* = True"
  For True-to-Magnetic-to-Compass conversions, see Para 0124f overleaf.

d. **Mnemonics.** The conversion rules (Para 0124c above) may be easily memorised by the use of either of the two mnemonics shown below:

- '**CADET**'. When converting from compass to true, add East (ie **'C**ompass to True, **Ad**d **E**as**T'** ie ***"CADET"***). It follows that West *Variation* and *Deviation* are subtracted (see Para 0124f overleaf).

- **Alternative Rhyme.** An alternative (rhyming) mnemonic which may be used is: **"Error West, Compass Best; Error East, Compass Least.**"

- **'CDMVT'**. The 'Compass: *Deviation*: Magnetic: *Variation*: True' sequence may be summarised as '*CDMVT*' (ie **"*Cadbury's Dairy Milk Very Tasty*"**).

e. **Methods for Laying off the Compass Course or Compass Bearing.** There are two methods available for laying off the compass course or bearing:

- **Method 1.** *Deviation* (for the compass course steered) and *Variation* (corrected to date) are applied in one step to the compass course or bearing in accordance with the above rule to obtain the true course or bearing. The parallel ruler is then placed at the true reading on the true *Compass Rose*.

Note 1-7. *The application of compass error in one step in Method 1 avoids a very common mistake, that of taking out the Deviation for the compass bearing of the object instead of the compass course of the ship.*

- **Method 2.** The parallel ruler is placed on the given compass bearing or course on the *Magnetic Compass Rose*. It is then slewed through a small angle (sometimes known as the *'Rose Correction'*) which is the algebraic sum of the *Deviation* and the change in *Variation* (taking +ve for East, -ve for West in accordance with mnemonics), to allow for:
  - The change in *Variation* to bring it up to date.
  - The *Deviation* for the compass course being steered.

Methods 1 and 2 are demonstrated overleaf at Examples 1-4 and 1-5 respectively.

(0124e) **Example 1-4 (Demonstrating Method 1 from Para 0124e opposite)**

A ship is steering 260˚C. *Variation* from the chart was 12˚W in 2007, decreasing 10' annually. The compass bearing of an object is 043˚C. Using the *Deviation* from Fig 1-14, what is the true course and how would the bearing be plotted using the above two methods? The year is 2010.

- **Calculation of the Deviation value that applies for Heading 260˚C**

Variation in 2007	12	˚W
Change in *Variation* 2007 to 2010: 3 x 10'E	½	˚E
Variation in 2010	11½	˚W
Thus *Deviation* applicable for 260˚C *Headings* =	1½	˚E

- **Application of CADET / CDMVT**

Compass *Headings*	260	˚C
Deviation	+ 1½	˚E
Magnetic *Headings*	261½	˚M
Variation	-11½	˚W
True course	250	˚T

Solution by Method 1 - Plotting the True Bearing

Compass bearing	043	˚C
Deviation	+ 1½	˚E
Magnetic bearing	044½	˚M
Variation	-11½	˚W
True bearing to be plotted	033	˚T

Note 1-8. *The compass error may be applied in one step (see Pare 0124e) to avoid the common mistake of taking out the Deviation for the compass bearing of the object instead of the compass course of the ship. In this case, the total error correction = +1½˚E - 11½˚W = -10˚W. Thus to convert to true while on compass Headings 260˚C, all compass bearings should be reduced by 10˚.*

(0124e) **Example 1-5 (Demonstrating Method 2 from Para 0124e opposite)**

Take the scenario from Example 1-4 (above). Place the parallel rule on the *Magnetic Compass Rose* in the direction 043˚M. Slew through a total *Rose Correction* of +2˚ clockwise (½˚ clockwise to allow for the easterly change of *Variation* and 1½˚ clockwise to allow for the easterly *Deviation*). Plot the bearing on the *Magnetic Compass Rose*, 045˚M. As *Magnetic North* on the *Magnetic Compass Rose* is already offset 12˚ to the West (see Fig 1-17), it will be immediately apparent that 045˚M is the same as 033˚T, the true bearing.

(0124) f. **Conversion of True Course to Compass Course.** To find the compass course from the true course, the mnemonic '*CADET*' (see Para 0124d) is used in the reverse direction, although it no longer forms a mnemonic:

True to Compass, add West (subtract East)

There is, however, a small complication. Before the Compass course may be found, the *Deviation* must be known, but that cannot be established until the Compass course in known. An iterative process follows, where the *Deviation* table is entered with the Magnetic course in lieu of Compass course. If the *Deviation* is large, a second calculation is required to establish the exact *Deviation* for the Compass course. This process is shown at Example 1-6 below.

Example 1-6. First and Second Approximations of Deviation for Compass Course

Using the *Deviations* at Fig 1-14 and a true *Headings* of 260˚ with 10˚W *Variation*, the following calculations are made:

1st Approximation

True course	260	˚T
Variation	+10	˚W
Magnetic course	270	˚M
Deviation (for 270˚M)	- 1¼	˚E
Approximate Compass course	268¾	˚C

The graph at Fig 1-14 indicates with this approximate course of 268¾˚C, the correct *Deviation* to use is nearer 1½˚E than 1 ¼˚E, thus giving the following 2nd approximation:

2nd Approximation

True course	260	˚T
Variation	+10	˚W
Magnetic course	270	˚M
Deviation (for 268¾˚M)	- 1½	˚E
Refined Compass course	268½	˚C

Note 1-9. *For practical purposes, providing that the Deviation is small, a second approximation is rarely necessary.*

0125. Checking Magnetic Deviation

a. **Principles.** If an accurate and reliable *Gyro Heading* is known, or a compass bearing is taken of a terrestrial or astronomical object whose true bearing is known, then provided the *Variation* is also known, the *Deviation* may be calculated and used to establish a new *Deviation* table. Various detailed procedures to establish *Deviation* exist; either by 'Comparison Swing' at Para 0125c (below) or by other methods at Para 0811, but the calculation required for all of them is at Example 1-7 (below).

Example 1-7. Calculating Deviation

By calculation, the sun's true bearing is 230˚T, the compass bearing is 235˚C and *Variation* 12˚W. What is the *Deviation*? (The same calculation is used if a Gyro 'Comparison Swing' is made, with Gyro *Heading* 230˚T and compass *Heading* 235˚C)

True bearing / *Headings*	230	˚T
Variation	+12	˚W
Magnetic bearing / *Headings*	242	˚M
Deviation (by subtraction 242˚-235˚ = 7˚)	**-7**	
Compass bearing / *Headings*	235	˚C

As 7˚ has to be subtracted from 242˚M to reach 235˚C, and since when calculating from 'True to Compass', East is subtracted (reverse of *CADET*), the *Deviation* is 7˚**E.**

b. **Acceptable Deviation Limits.** In practice, it should be possible to adjust a well placed *Magnetic Compass* to limit the residual *Deviation*s to less than 3˚. However, if a large change in *Magnetic Latitude* has taken place, any structural alterations have taken place near the compass, ferrous cargo loading / unloading has occurred etc, or if more than one year has elapsed since the previous compass swing (see full list of factors at BR 45 Volume 3), then a check 'Comparison Swing' (see Para 0125c below) should be carried out to establish a new *Deviation* table, as the residual *Deviations* may have changed substantially. Adjustment of the compass (ie moving magnets / *Spheres*) should only be carried out by a suitably qualified 'Compass Swinging Officer'.

c. **Establishing Magnetic Deviation by Gyro Compass 'Comparison Swing'.** As stated at Para 0122g, the ship's residual *Deviation* is found by slowly 'swinging the ship' through 360˚ and noting the *Deviation* for the various compass *Headings*, as compared to *Gyro Headings* (after making allowance for the correct *Variation* - see Para 0125a above); this may easily be done at sea as a 'Comparison Swing' without the need for a qualified 'Compass Swinging Officer'(as no compass adjustments are carried out).

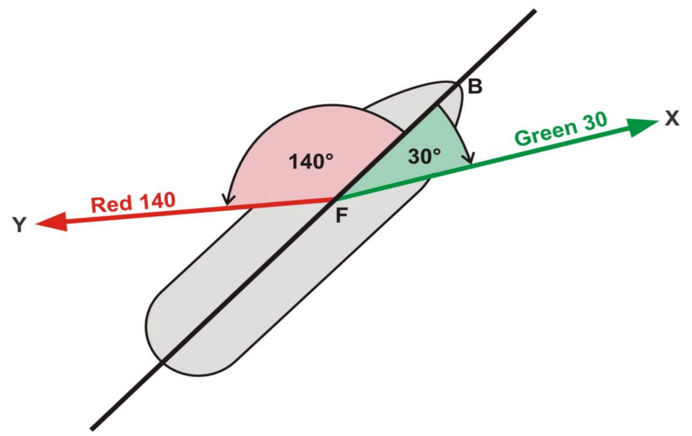
- **Rate of Turn.** It is essential that the swing is carried out slowly, with a rate of turn no greater than 8˚ per minute (ie 360˚ in not less than 45 minutes); if necessary the ship may be steadied on each *Heading* for a period. Failure to observe this very slow rate of turn will not allow the ship's magnetism to adjust to each new *Heading*, and will result in a flawed set of readings.

0126. Relative Bearings

a. **"Red" or " Green".** *Relative Bearings* are normally stated relative to the ship's fore-and-aft line looking forward (ie from *Ship's Head*) and are measured from the bow from 0˚ to 180˚ on each side. Starboard bearings are prefixed "Green" and port bearings prefixed "Red". See Fig 1-18 below.

b. **"... Relative".** Occasionally, *Relative Bearings* are measured clockwise from 000˚ to 360˚ from *Ship's Head* and, if so, are given the suffix "Relative" (eg "135˚ Relative"). See Fig 1-18 below.

c. **"On the Bow" etc.** The expressions "On the bow", "On the beam" and "On the quarter" may be used with or without any specified number of degrees (or *Points* if 'traditional' language is used). When used without any specified number of degrees (or *Points* if 'traditional' language is used), they mean respectively 45˚ (4 *Points*), 90˚ (*8 Points*), and 135˚ (*12 Points*) from *Ship's Head*. When used with a specified angle, the meaning is as stated (see Fig 1-18 below).



Explanation: The bearing of X is "Green 30" (or 030° Relative) and that of Y is "Red 140" (or 220° Relative).
Alternatively, X could be said to be "30° on the starboard bow" and Y to be "40° on the port quarter".
If the ship is steering 045°, the true bearings of X and Y are 075° and 265° respectively.

Fig 1-18. Relative Bearings

0127. The Radian Rule

The 'Radian' is defined as the angle subtended at the centre of a circle by a length of arc equal to the radius. 'π' is defined as the ratio of the circumference of a circle to its diameter; this ratio is constant in all cases and π is approximately equal to 3.1415927. It can thus be shown (see Appendix 1 Para 2b) that 360˚ equates to 2π radians. From these definitions, it follows that:

$$1^{\circ} = \frac{2\pi}{360} \text{ radians}$$

and, at 1 *n.mile* (1852 metres):

$$1^{\circ} \text{ subtends: } \frac{2\pi}{360} \times 1852 \text{ metres} = 32.32 \text{ metres} = 35.35 \text{ yards} = 106.05 \text{ feet}$$

With small angles at reasonably short ranges, 1˚ may be taken to subtend 35 yards (approximately 100 feet) at 1 *n.mile* without serious error; this is known colloquially as the "*Radian Rule*".

CHAPTER 2

THE SAILINGS (1) - BASIC CALCULATIONS

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CHAPTER 2

THE SAILINGS (1) - BASIC CALCULATIONS

0201. Scope of Chapter

a. **'Sailings'.** '*Sailings*' are terms used to describe the various mathematical methods of finding course and *Distance* from one place on the Earth's surface to another. Chapter 2 covers the following *Sailings*:

- *Parallel Sailing*.
- *Plane Sailing*.
- *Mean Latitude Sailing* and *Corrected Mean Latitude Sailing*.
- *Traverse Sailing*.
- *Mercator Sailing* (brief overview only).
- *Spherical Sailing* - *Great Circle* tracks (introduction only).
- *Spherical Sailing* - *Composite Tracks* (introduction only).

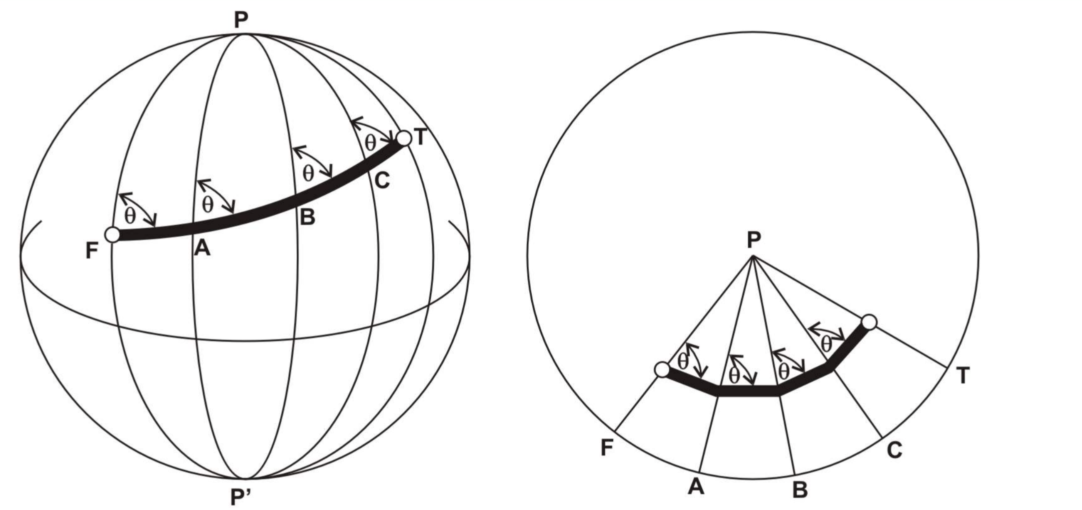
b. **Further Details.** *Mercator Sailing* is only introduced very briefly in Chapter 2, and further details are as follows:

- **Chapter 4.** *Mercator Projections* are covered in detail at Chapter 4.
- **Chapter 5.** The calculations involved in both *Spherical* and *Spheroidal Mercator Sailing* are set out Chapter 5, together with *Vertex* and the *Composite Track* calculations.

0202. Rhumb Lines and Departure

a. **Rhumb Lines.** A *Rhumb Line* is a curve drawn on the Earth's surface which cuts all *Meridians* at the same angle (see Fig 2-1 below).

- **Special Cases - Equator, Parallels of Latitude and Meridians.** As special cases, the *Equator* and *Parallels of Latitude* are *Rhumb Lines* of either 090˚ or 270˚, while *Meridians* are *Rhumb Lines* of either 000˚ or 180˚.
- **Other Rhumb Lines - Loxodromes.** Other *Rhumb Lines* (cutting *Meridians* at the same angle) spiral towards the *Pole* and are also called *Loxodromes*.
- **Use.** *Rhumb Lines* appear as straight lines on *Mercator Projection* charts and thus represent a ship's steady course.



Explanation: ***Rhumb Line*** **curve *FT* cuts *Meridians* F, A, B, C and T at the same angle(θ) .**

Fig 2-1. The Rhumb Line

b. **Departure.** *'Departure'* is the *Distance* made good in an East-West direction in sailing from one place to another along a *Rhumb Line*. The units of *Distance* used in *Departure* are the same as in *d.lat* (ie *Sea Miles*).

0203. Parallel Sailing

Parallel Sailing is a method of converting *Departure* along a *Parallel of Latitude* into *Longitude,* and assumes the Earth is a *Sphere*.

a. **The Arc of a Parallel of Latitude.** In Fig 2-2a (below), for a ship travelling along the *Equator* from *A* to *B*, the *Departure* and the *d.long* (*Difference of Longitude*) are equal. However, if the ship is travelling from *F* to *T* along any other *Parallel of Latitude* ϕ , the *d.long* λ is still *AB*, but the *Departure FT* (ie *Distance*) is numerically less than the *d.long*. The nearer the *Parallel of Latitude* is to the *Pole* (ie the higher the *Latitude*) the shorter *Departure FT* becomes, but the *d.long AB* does not alter.

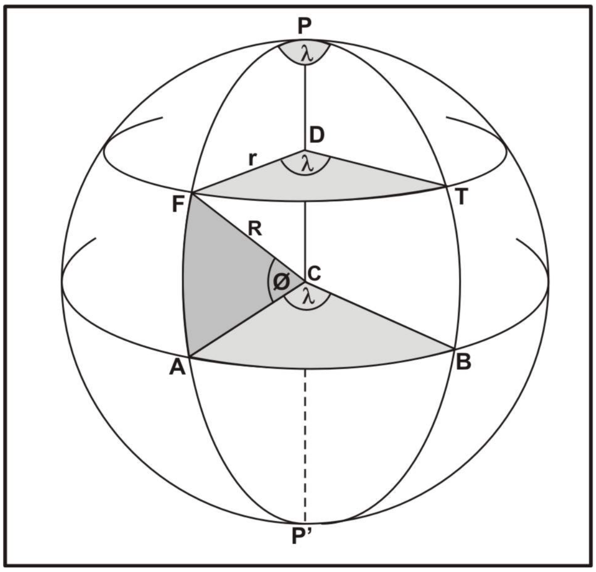


Fig 2-2a. The Arc of a Parallel of Latitude

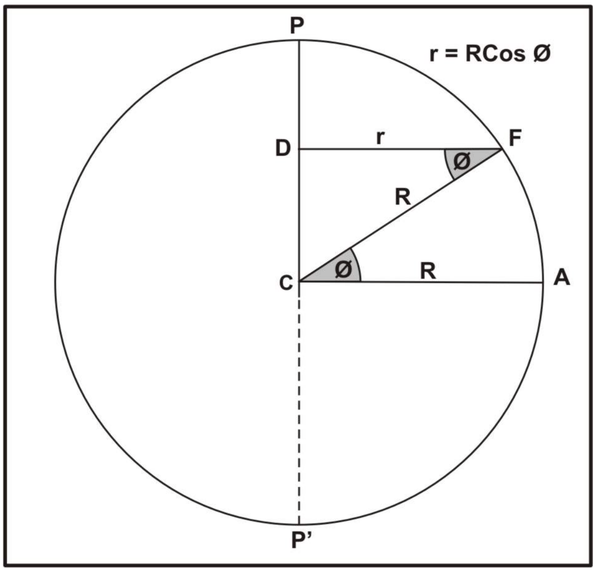


Fig 2-2b. Alteration of the Arc with a Change of Latitude

b. **Alteration of the Arc with a Change of Latitude.** The relationship between *Departure* and d.*long* may be found as follows (see Figs 2-2a/b above):

- The radius *r* of the circle of *Latitude* ϕ is *R cos* ϕ , where *R* is the radius of the *Sphere* (see Fig 2-2b above).
- The *Departure* (ie *Distance*) *FT* along the *Parallel of Latitude* is:

$= \lambda r$, where λ is in radians (see Fig 2-2a above)
$= \lambda R \cos \phi$ (see Fig 2-2b above)
$\therefore = AB \cos \phi$ (see Fig 2-2a above)
$\therefore = \lambda \cos \phi$ (where λ is in minutes, see Fig 2-2a above)
Departure = *d.long* (in minutes) cos *Latitude* . . . **2.1**

Thus for a *Sphere*, the *Departure* (*Distance*) along a *Parallel of Latitude* (in minutes of *Latitude*) equals *d.long* (in minutes of arc), times the cosine of the *Latitude*.

Examples 2-1 and 2-2.

2-1. At *Latitude* 40˚N with *Longitudes* of F (15˚E) and T (60˚E), the *d.long* is 45˚, or 2700' minutes of arc along the *Equator* (ie FT = 2700' cos 40˚ = 2068'.3).

2-2. Had the *Latitude* been 60˚N instead of 40˚N, the *Distance* along this new *Parallel of Latitude* would have been 2700' cos 60˚, ie 1350'.

0204. Plane Sailing

Plane Sailing is a method of solving the relationship between *d.lat* (*Difference of Latitude*), *Departure* (see Para 0202b), *Distance* and *Course*, when NOT *Parallel Sailing*; in this case, *Departure* does NOT equal *Distance*. *Plane Sailing* assumes the Earth is a *Sphere.* It does not involve *d.long* (*Difference of Longitude*) except indirectly (see Para 0205).

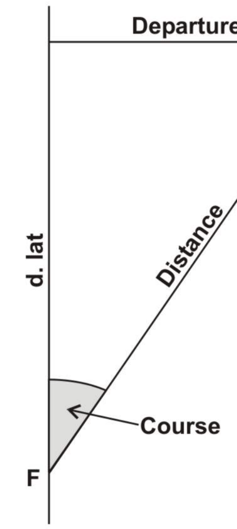


Fig 2-3. Departure, d.lat and Distance when Plane Sailing.

a. **Plane Sailing Formulae.** When a ship travels along any *Rhumb Line* other than a *Parallel of Latitude* or a *Meridian of Longitude* (ie not *Parallel Sailing*), its *d.lat*, *Departure*, *Distance* and *Course* may be considered as forming a plane right-angled triangle (see Fig 2-3 above). From this triangle it may be shown (and proved opposite at Para 0204b) that:

$$\text{Departure} = \text{Distance} \sin \text{Course} \qquad \ldots \mathbf{2.2}$$

$$\text{d.lat} = \text{Distance} \cos \text{Course} \text{ (but see Note 2-1)} \qquad \ldots \mathbf{2.3}$$

Dividing (2.2) by (2.3):

$$\tan \text{course} = \frac{\text{Departure}}{\text{d.lat}} \qquad \ldots \mathbf{2.4}$$

Note 2-1. *If using formula (2.3) to find the Distance, it has a fundamental weakness as the Course approaches 90°, because small errors in the Course introduce large errors in the Distance. Formula (2.2) should be used instead to find Distance in such cases.*

(0204) b. **Proof of the Plane Sailing Formulae.** The *Plane Sailing* formulae may be proved as follows. Consider the *Rhumb Line FT* in Fig 2-4 (below):

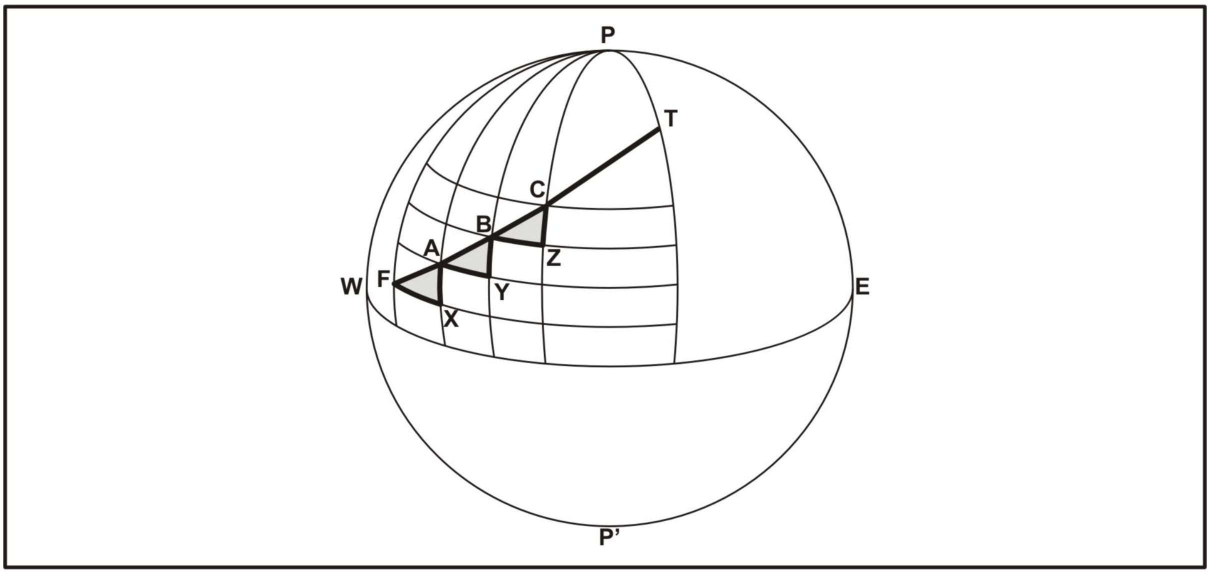


Fig 2-4. Division of the Rhumb Line (to Prove the Plane Sailing Formulae)

- **Division of the Rhumb Line**. In Fig 2-4 (above), let the *Rhumb Line FT* be divided into a large number '*n'* of equidistant *Parallels of Latitude* cutting the *Rhumb Line* at *F*, *A*, *B*, *C*, etc. Let the *Meridians* through the points cut the *Parallels of Latitude* at *X*, *Y*, *Z*, etc.

- **Small Triangles.** In the small triangles *FAX*, *ABY*, *BCZ* etc, the angles *FXA*, *AYB*, *BZC* are right angles. The angles *FAX*, *ABY*, *BCZ* are all equal, being equal to the *Course*. The sides *AX*, *BY*, *CZ* are all equal.

- **Consider as Plane Right-Angled Triangles.** These triangles are therefore equal in all respects and, as they are very small, may be considered as plane right-angled triangles. Thus, in the triangle *FAX* (et al):

$$AX = FA \cos Course$$
$$nAX = nFA \cos Course$$
$$d.lat = Distance \cos Course \qquad \text{. . . (formula 2.3)}$$
$$FX = FA \sin Course$$
$$nFX = nFA \sin Course$$
$$Departure = Distance \sin Course \qquad \text{. . . (formula 2.2)}$$

Dividing (2.2) by (2.3):

$$\tan Course = \frac{Departure}{d.lat} \qquad \text{. . . (formula 2.4)}$$

0205. Mean and Corrected Mean Latitude Sailing

There are two calculation methods by which a ship may determine its *Latitude* and *Longitude* after travelling along a *Rhumb Line* other than in a North-South or East-West direction. One of these methods uses the *Mean Latitude* or *Corrected Mean Latitude*, the other uses *Mercator Sailing* (introduced at Para 0207 and described at Chapter 5).

a. **Mean Latitude Sailing.** Consider the *Rhumb Line FT* in Fig 2-5 (below).

- **Relative Sizes of Departures.** The *Departure* of *FT* is greater than that of *HT* (ie *Departure* along the *Parallel of Latitude* through *T*), but less than that of *FG* (ie *Departure* along the *Parallel of Latitude* through *F)*. Therefore, the *Departure* from *F* to *T* must equal the *Departure* along a *Parallel of Latitude* lying somewhere between *FG* and *HT*. Let this *Parallel of Latitude* be *UV*.

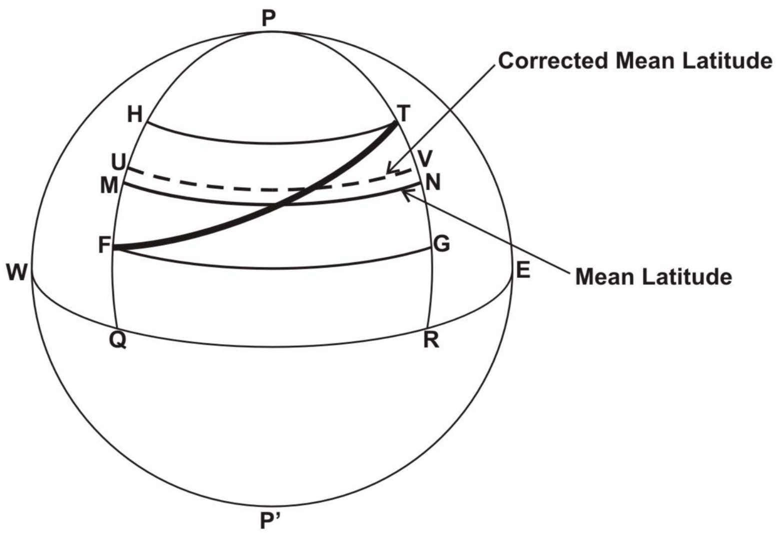


Fig 2-5. Mean and Corrected Mean Latitudes

- **Mean Latitude.** Provided that the *d.lat* between *F* and *T* is fairly small and the *Latitudes* of *F* and *T* are not too high, this *Departure* is approximately equal to the arc of the *Parallel of Latitude MN*, which has as its *Latitude* the mathematical mean between *F* and *T*. This *Latitude* is referred to as the *'Mean Latitude'* (or *'Mean Lat'*). In these particular circumstances *MN* and *UV* are almost identical.

- **Mean Latitude Formula.** If *QR* is the *d.long* between *F* and *T*:

$$MN = QR \cos QM \qquad \text{. . . (formula 2.1)}$$

$$Departure = d.long \cos Mean\ Latitude \ \text{(for the } Sphere\text{)} \qquad \text{. . . } \mathbf{2.5}$$

- **Errors.** However, formula (2.5) is not mathematically accurate except when *F* and *T* are on the same *Parallel of Latitude*.
  - **Distance.** In practice, the accuracy of this formula depends on how close *T* is to *F*, and it should NOT be used for *Distance*s exceeding 600'.
  - **Proximity to Equator.** If the *Latitudes* of *F* and *T* are on each side of the *Equator* and also within 10˚ of *Latitude* of the *Equator*, the *Departure* may be taken as the d.*long* without appreciable error (the maximum error in *Departure* cannot exceed 0.4%.)

(0205) b. **Corrected Mean Latitude Sailing - Formula.** The true or '*Corrected Mean Latitude*' between *F* and *T* is given by *UV* (see Fig 2-5 opposite). For the <u>*Sphere*</u> (see Note 2-2 below for the *Spheroid*), it may be shown (see Appendix 3) that the *Latitude* *L* of *UV* may be found from the following formulae:

<u>Either</u>:

$$sec\ L = \frac{7915.7045}{d.lat(mins\ of\ arc)}\left[log_{10}tan\left(45°+\frac{T°}{2}\right) - log_{10}tan\left(45°+\frac{F°}{2}\right)\right] \quad \ldots \mathbf{2.6}$$

<u>Or</u> (see Para 0422):

$$sec\ L = \frac{DMP}{d.lat\ (mins\ of\ arc)} \quad \ldots \mathbf{2.7}$$

c. **Corrected Mean Latitude Sailing - Terminology.** '*Corrected Mean Latitude*' is frequently referred to in nautical tables and other navigational publications as the '*Middle Latitude*', but in the BR 45 series the term '*Corrected Mean Latitude*' is used.

Note 2-2. *Rhumb Line position calculations for the <u>Spheroid</u> are at Paras 0530-0531.*

Example 2-3: Low Latitudes

A ship transits from position F (30°N, 40°W) to a point T (34°N, 36°W). Find the Departure, Course and *Distance* by both *Mean Latitude* and *Corrected Mean Latitude* methods.

- **Mean Latitude Sailing:**

$$\begin{aligned} d.lat &= 4°N = 240'N \\ d.long &= 4°E = 240'E \\ Mean\ Lat\ Sailing &= ½\ (30° + 34°)N = 32°N \\ Departure &= 240'\cos 32°E = 203'.53E \end{aligned}$$

... (formula 2.5)

$$tan\ Course = \frac{Departure}{d.lat}$$

... (formula 2.4)

$$Course = 040.3°$$

$$\mathbf{Distance} = d.lat\ sec\ Course = 240'\ sec\ 040.3° = \mathbf{314'.68}$$

... (formula 2.3)

- **Corrected Mean Latitude Sailing:**

$$sec\ L = \frac{7915.7045}{240'}\left(log_{10}\ tan\left(45° + \frac{34°}{2}\right) - log_{10}\ tan\left(45° + \frac{30°}{2}\right)\right)$$

... (formula 2.6)

$$\begin{aligned} L &= 32°.033158\ (32°02') \\ Departure &= 203.46' \\ Course &= 040.3° \\ \mathbf{Distance} &= \mathbf{314.64'} \end{aligned}$$

- **Difference between Mean Latitude / Corrected Mean Latitude Sailing Results:** The difference in *Distances* (0.013%) between 314.68' for *Mean Latitude Sailing* and 314.64' for *Corrected Mean Latitude* is small, so *Mean Latitude Sailing* may be used here without appreciable error.

Example 2-4: High Latitudes

A ship in a high *Latitude* position F (70˚N 20˚W), steers a *Course* of 020˚ for a *Distance* of 600 miles. What is its *Latitude* and *Longitude* at the end of the run, by both *Mean Latitude* and *Corrected Mean Latitude Sailing* methods?

- **Mean Latitude Sailing:**

$$d.lat = 600' \cos 20^\circ N \qquad \text{. . . (formula 2.3)}$$
$$= 563.81557'N$$
$$= 9.3969262^\circ \quad = 9^\circ 23.8'N$$
$$\textbf{Latitude T} = \mathbf{79^\circ 23'.8N}$$

$$Departure = 600' \sin 20^\circ E = 205.21209'E \qquad \text{. . . (formula 2.2)}$$
$$Mean\ Lat = 74.6984631^\circ N = 74^\circ 41.9'N$$

$$d.long = 205.21209' \sec 74.6984631^\circ E \qquad \text{. . . (formula 2.5)}$$
$$= 777.61623'E = 12^\circ 57.6'E$$
$$\textbf{Longitude T} = \mathbf{7^\circ 02.4'W}$$

- **Corrected Mean Latitude Sailing:**

$$\sec L = \frac{7915.7045}{563.81557'}\left(\log_{10} \tan 84.698463^\circ - \log_{10} \tan 80^\circ\right) \qquad \text{. . . (formula 2.6)}$$

$$L = 75.197922^\circ \ \left(75^\circ 11.9'\ N\right)$$

$$d.long = 205.21209' \sec 75.197922^\circ \qquad \text{. . . (formula 2.5)}$$
$$= 803.23871'E$$
$$= 13.387312^\circ E = 13^\circ 23.2'E$$
$$\textbf{Longitude T} = \mathbf{6^\circ 36.8'W}$$

- **Difference between Mean Latitude / Corrected Mean Latitude Sailing Results:** The difference in *Longitude* between *Mean Latitude* and *Corrected Mean Latitude Sailing* results is 25.6'E. From formula (1.1) [at *Latitude* 79˚ 23.8'N] this equates to 4.7 *n. miles*; over 600 *n. miles* this is 0.8% of the *Distance*. The discrepancy in position at the end of the run illustrates the danger of using the *Mean Latitude* method in high *Latitudes*, even though the *Distance* is only 600 *n. miles*.

0206. Traverse Sailing

Traverse Sailing is the term given to *Traverse* solutions for single or multiple *Plane Sailing*s (see Para 0204). The various leg(s) of the ship's track are the hypotenuses of a series of *Plane Sailing* triangles (see Fig 2-4 at Para 0204b). The individual *d.lats* and *Departures* may be found using formulae (2.2 to 2.4) and *d.long* using formula (2.5).

a. **Traverse Table.** The 'Traverse Table' in Norie's Nautical Tables solves the *d.lat* / *Departure* / *Distance* / *Course* plane triangles for any *Distance* up to 600'. Instructions for the use of these tables are given in the explanation within Norie's. The tables may also be used to find *d.long* by formula (2.5) by treating the *Course* as *Mean Latitude*, *d.lat* as *Departure* and *Distance* as *d.long*.

b. **Calculator or Spreadsheet.** A calculator or spreadsheet with trigonometric functions is quicker and more accurate to use than the Traverse Table. If a calculator or spreadsheet with co-ordinate conversion is available, it should be possible to read off *d.lat* and *Departure* directly using *Cartesian Coordinates* (x, y) (see Note 2-3 below). Using a calculator or spreadsheet avoids the need to interpolate between sets of figures in the Traverse Table.

c. **Polar and Cartesian Coordinates.** The position of point T (see Fig 2-6 below) may be defined in *Polar Coordinates* (r, θ) or *Cartesian Coordinates* (x, y) where:

r = *Distance*
θ = *Course*
x = *Departure*
y = *d.lat* (see Note 2-3 below)

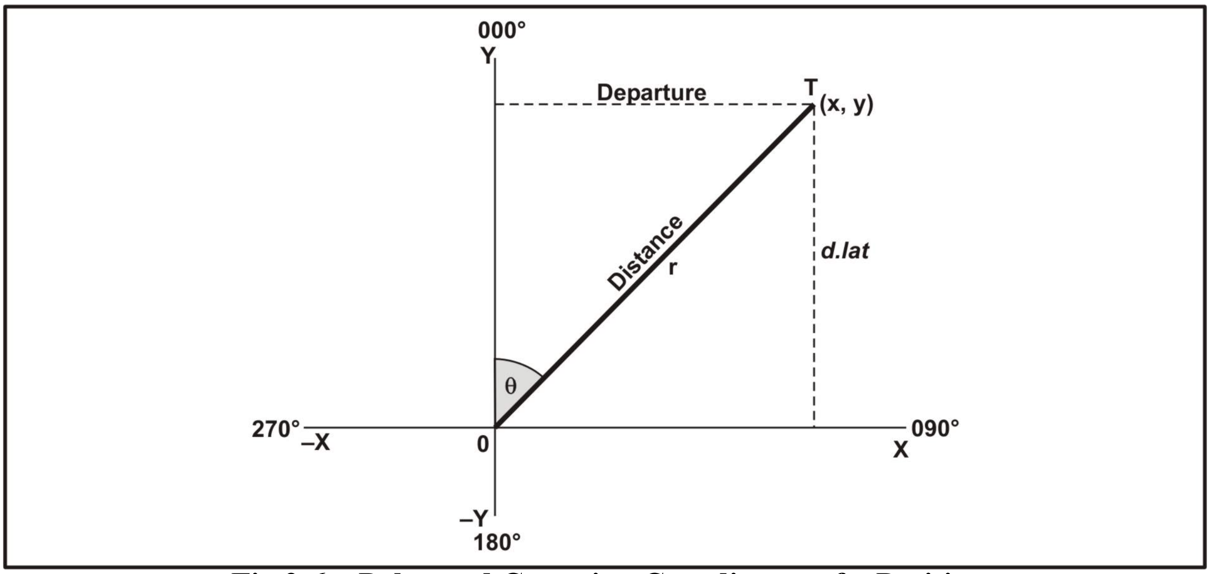


Fig 2-6. Polar and Cartesian Coordinates of a Position

Note 2-3. *If converting between Polar and Cartesian Coordinates using a calculator, d.lat appears as 'x' and Departure as 'y' because of the difference between mathematical and navigational conventions on the initial line from which angles are measured. In navigational notation, Course is measured clockwise from the north-south line, while in mathematical notation, it is measured anti-clockwise from the east-west line.*

Example 2-5: Traverse Sailing by Traverse Table and Calculator Methods

A ship in position 45˚25'N, 15˚05'W at 0900 steers the following *Courses* and speeds. What is its position at 1315?

TIME	COURSE	SPEED	
0900-0946	045˚	15	knots
0946-1015	312˚	16½	knots
1015-1122	217˚	14¾	knots
1122-1247	103˚	17	knots
1247-1315	190˚	15	knots

The ships track and the overall *d.lat / d.long* is shown at Fig 2-7 (below). Details for each leg by Traverse Table and calculator methods are at Table 2-1 (below):

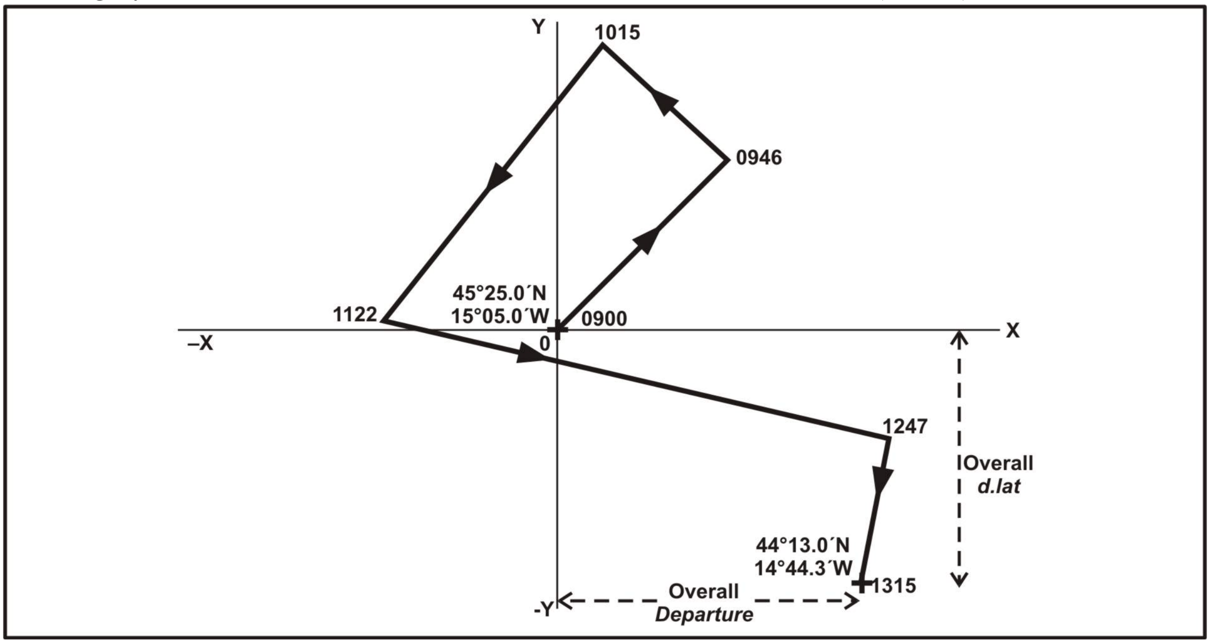


Fig 2-7. Ship's Track and Overall d.lat / d.long from Example 2-5

Table 2-1. Details for each leg (Example 2-5) by Traverse Table and Calculator Methods

Time	Course θ	Speed	Distance *r*	Traverse Table Method			Calculator Method	
				Course	*Departure*	*d.lat*	*Dep* (*x*)	*d.lat* (*y*)
0900-0946	045˚	15 kn	11.5'	N45˚E	08.13'E	08.13'N	+08.132'	+08.132'
0946-1015	312˚	16½ kn	7.975	N48˚W	05.97'W	05.33'N	-05.927'	+05.336'
1015-1122	217˚	14¾ kn	16.471'	S37˚W	09.91'W	13.16'S	-09.912'	-13.154'
1122-1247	103˚	17 kn	24.083'	S77˚E	23.46'E	05.42'S	+23.466'	-05.417'
1247-1315	190˚	15 kn	7.0'	S10˚W	01.22'W	06.89'S	-01.216'	-06.894'
TOTALS (0900-1315)					**14.5'E**	**12.01'S**	**+14.543(E)**	**-11.997'(S)**

Mean Latitude (0900-1315):	45˚ 19.0'N
Departure:	00˚ 14.5'E (both methods - see Table 2-1 above)
d.long (from formula 2-5):	00˚ 20.7'E
d.lat:	00˚ 12.0' S (both methods - see Table 2-1 above)
Position at 1315:	**45˚ 13.0'N 14˚ 44.3'W**

0207. Mercator Sailing Overview

a. **Concept.** *Mercator Sailing* provides a method of determining position after travelling along a *Rhumb Line*, other than in a North-South or East-West direction. As stated at Para 0205, *Mercator Sailing* is an alternative to *Mean Latitude* / *Corrected Mean Latitude Sailing*. It uses *Difference of Meridional Parts* (*DMP*) instead of *d.lat*, and *d.long* instead of *Departure*.

b. **Meridional Parts.** *Meridional Parts* are a feature of the *Mercator* projection on which the great majority of small-*Scale* Admiralty (*UKHO*) navigational charts are based. *Meridional Parts* are discussed at length in Chapter 4.

c. **Further Details.** *Mercator Projection* charts are covered in detail at Chapter 4. The calculations involved in both *Spherical* and *Spheroidal Mercator Sailing* are set out Chapter 5, together with *Vertex* and the *Composite Track* calculations.

0208. Spherical Sailing - Great Circle Tracks

The definition of *Great Circle* is repeated for the convenience of readers, as follows:

> **(Extract from Para 0110c): Great Circle.** A *Great Circle* is the intersection of a *Spherical* surface and a plane which passes through the centre of the *Sphere*. It is the shortest distance between two points on the surface of a *Sphere*.

a. **The Great Circle - Explanation.** A straight line is the shortest distance between two points. However, when two points lie on the surface of a *Sphere*, the arc of the *Great Circle* joining them is the curve that most nearly approaches the straight line, because it has the greatest radius and therefore the least curvature.

- The smaller arc of the *Great Circle* joining two places on the Earth's surface (eg arc *FT* at Fig 2-8 below) is the shortest route between *F* and *T* (or *T* ').
- PF and *PT* are arcs of the *Meridians* passing through *F* and *T* (or *T* ') and are also arcs of *Great Circles*.
- The triangle *PFT* (or *PF T* ') is therefore a *Spherical* triangle, and the *Distance FT* (or *F T* ') is the length of the side opposite the *Pole* in this triangle.

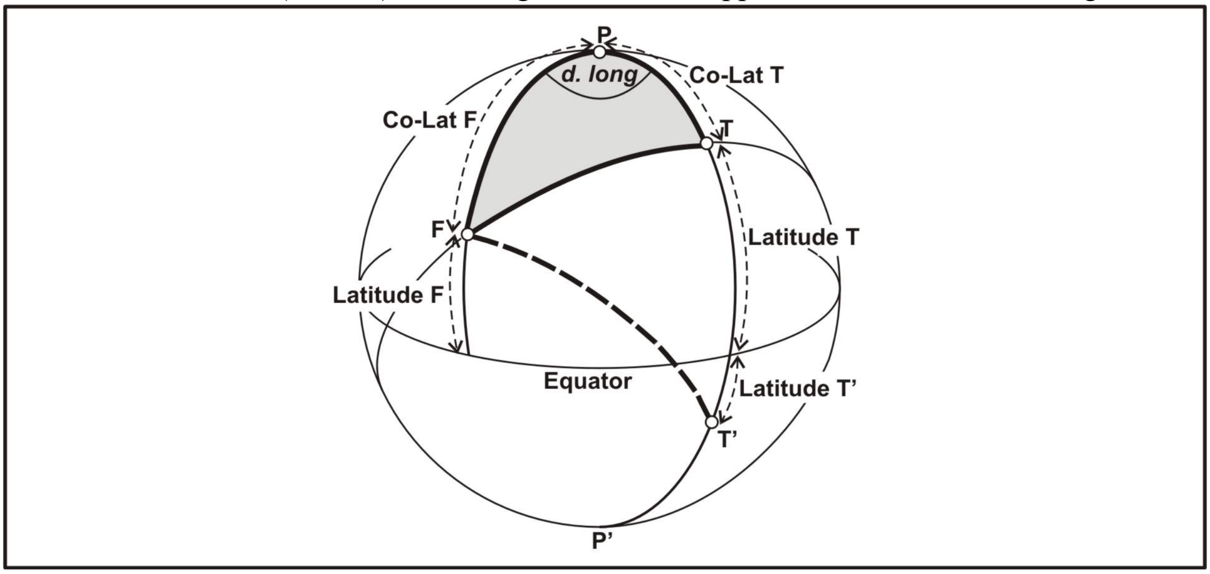


Fig 2-8. The Great Circle (shown in Northen Hemisphere)

(0208) b. **True Bearings on a Great Circle.** The true bearing of *T* from *F* (see Fig 2-9a/b below) is the angle between the *Meridian* through *F* and the *Great Circle* joining *F* and *T*, measured clockwise from the *Meridian* (ie the angle *PFT)*. This angle is the initial *Course* to be steered by a ship sailing on a *Great Circle* from *F* to *T*. Radio waves also travel along *Great Circles* near the Earth's surface (see Note 2-4 below) and the angle *PFT* is thus the bearing of *T* from *F* as given by a radio-direction finder.

Note 2-4. *The 'Geodesic' is the equivalent of a Great Circle on a Spheroid. However, in everyday use, the terms 'Great Circle' (in lieu of 'Geodesic') and 'Small Circle' are applied to the Earth's Oblate Spheroidal shape (see Note 1-1 at Para 0110c).*

c. **True Bearings at Intermediate Points on a Great Circle.** At any intermediate point *G* between *F* and *T* on a *Great Circle* (see Fig 2-9a below), the true bearing of *T* is the angle *PGT*, and this is <u>not</u> equal to the angle *PFT* (see Fig 2-9b below). To an observer moving along the *Great Circle* from *F* to *T*, the true bearing of *T* changes continuously. Only when *T* is close to *F* may this change be neglected (as the area of the Earth's surface traversed by *FT* is then sufficiently small to be considered as a plane or flat surface, on which *Great Circles* appear as straight lines).

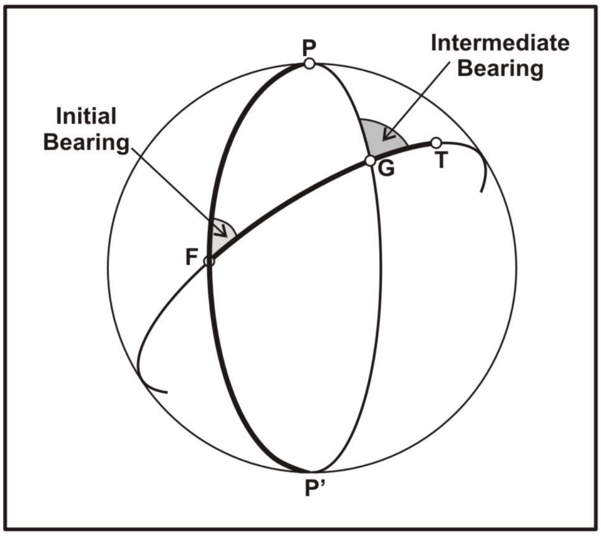


Fig 2-9a. Great Circle True Bearings

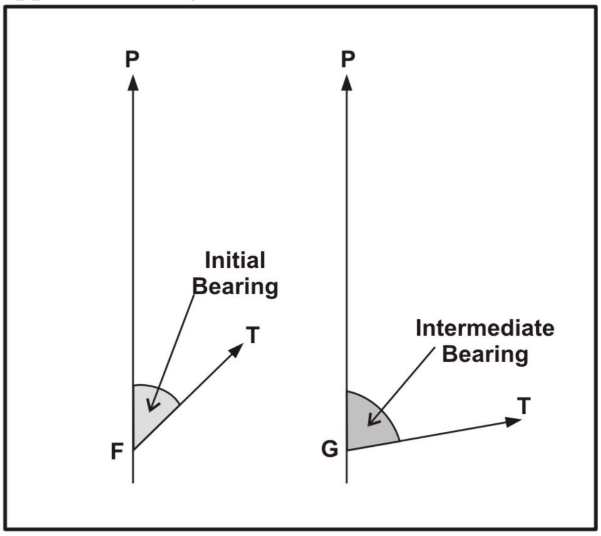


Fig 2-9b. Initial and Intermediate Bearings

d. **Great Circle Distance and Bearing.** In *Spherical* triangle *FPT* (see Fig 2-8, previous page), the length of the side *FT* and its true bearing (angle *PFT*) may be calculated (see calculation at Para 0211), as the angle *FPT* and sides *PF* and *PT* (or PT_1) are known:

- **Angle *FPT*.** The angle *FPT* is the *d.long* between *F* and *T*.
- **Sides PF and PT - 'SAME' Names.** When the *Latitudes* of the sides *PF* and *PT* are in the same hemisphere (ie have *'SAME'* names, as in Fig 2-8 where *F* and *T* are both North), *PF* = (90˚ - *Latitude F*) and *PT* = (90˚ - *Latitude T*). The distance (90˚ - *Latitude*) is known as the *Co-Latitude*.
- **Sides PF and PT_1 - 'CONTRARY' Names.** When the *Latitudes* of the sides *PF* and *PT* are in different hemispheres (ie have *'CONTRARY'* names, as in Fig 2-8 where *F* is North and T_1 is South), PT_1 = (90˚ + *Latitude* T_1).

(0208d continued)

- **For Pole P (Northern Hemisphere).** From Fig 2-8, it can be seen that:

  For *Latitudes* of '*SAME*' name:

  $PF = 90° - Latitude\ F = Co\text{-}Latitude\ F$

  $PT = 90° - Latitude\ T = Co\text{-}Latitude\ T$

  angle $FPT = d.long$

  For *Latitudes* of '*CONTRARY*' names:

  $PF = 90° - Latitude\ F = Co\text{-}Latitude\ F$

  $PT_1 = 90° + Latitude\ T_1$

  angle $FPT_1 = d.long$

- **For Pole P'(Southern Hemisphere).** When *F* is also in southern *Latitudes* (see Fig 2-10), *P'* may substituted for *P* in the above expressions.

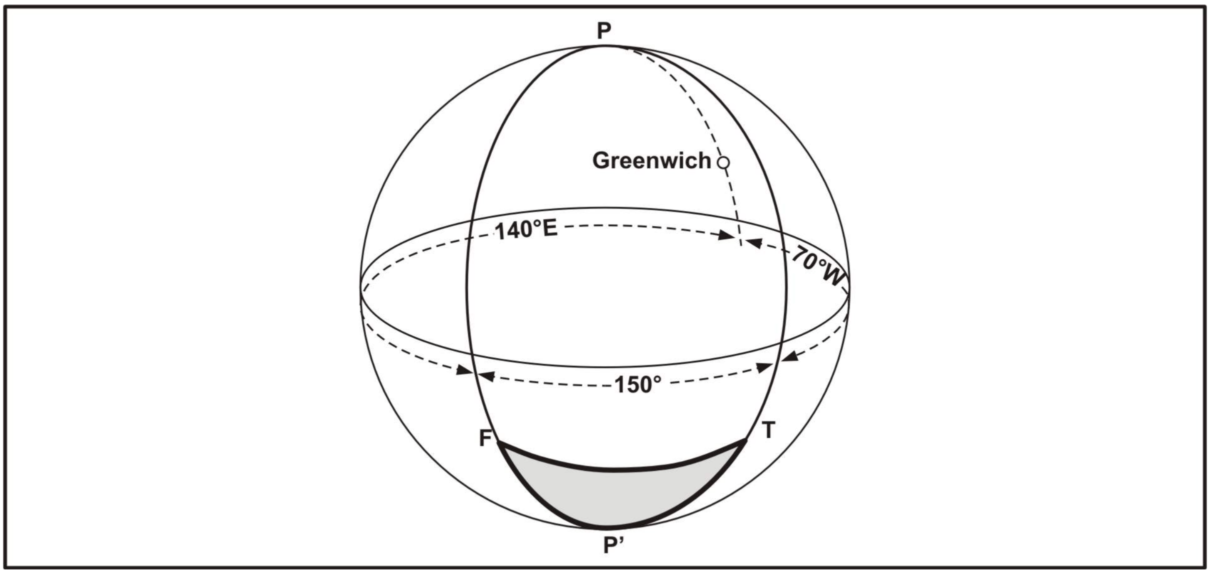


Fig 2-10. The Spherical Triangle (Southern Hemisphere)

- **Summary For Either Hemisphere.** So, for either hemisphere:

  $PF = 90° \pm Latitude\ F$

  $PT = 90° \pm Latitude\ T$

  The sign of the expression is determined by the NAME of the *Pole* and NAME of the *Latitude* of the places, summarised as:

SAME Names:	SUBTRACT
CONTRARY Names:	ADD

e. **Great Circle Sailing.** To follow a *Great Circle* track exactly would involve continuous *Course* changes. In practice, the *Great Circle* track is divided into a series of *Rhumb Lines*, approximating the *Great Circle* (see Fig 2-11 below, and Para 0441a).

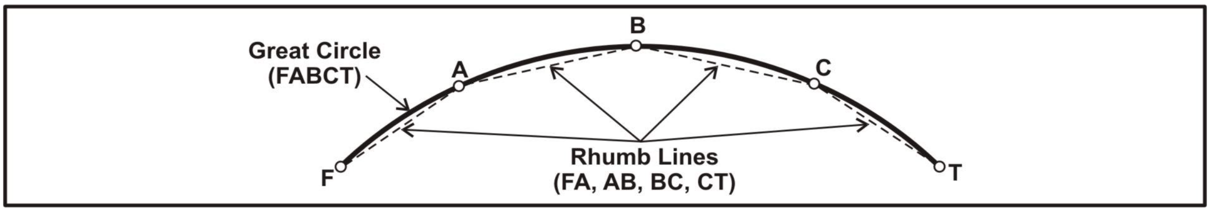


Fig 2-11. Rhumb Line tracks approximating the Great Circle

0209. Spherical Sailing - The Vertex and Composite Tracks

a. **The Vertex**. The point at which a *Great Circle* most nearly approaches the *Pole* is called its *Vertex* - shown as '*V*' at Fig 2-12 (below). At the *Vertex*, the *Great Circle* ceases to approach the *Pole* and begins to curve away. Therefore, at the *Vertex*, the *Great Circle* must cut the *Meridian* at right angles. The method of finding this position involves the use of right-angled *Spherical* triangles, and is described in Chapter 5.

b. **The Composite Track.** A *Great Circle* track between any two points passes nearer to the *Pole* than does the *Rhumb Line* track (unless both points are on the *Equator*); the *Great Circle* track can easily reach high *Latitudes* (eg route from USA / Canada to UK). If ice is likely to be encountered, the *Great Circle* track should be modified to clear any ice danger by avoiding high *Latitudes*, while remaining the shortest possible safe track. This modified track is known as the *Composite Track*, and is formed by two *Great Circle* arcs joined at their *Vertices* by the '*Safe Parallel*' *of Latitude*. In Fig 2-12 below:

- **Great Circle FLVMT.** Track *FLVMT* is the *Great Circle* joining *F* and *T*.

- **'Safe Parallel' LM.** *Latitudes* higher than the *Safe Parallel LM* are assumed to be dangerous and *Great Circle* arc *LVM* cannot be used.

- **Shortest Track.** The track *FLMT* is not the shortest safe route. The shortest safe route is *FABT*, where *FA* and *BT* are *Great Circle* arcs tangential at *A* and *B* to the *Safe Parallel*. *FABT* is thus the *Composite Track* in this example.

- **Proof of FABT as the Shortest Track.** By inspection:
    - If *L* and *M* are taken as any points on the *Safe Parallel LM* outside the arc *AB*, (*FL* + *LA*) is greater than *FA* itself
    - Similarly, (*BM* + *MT*) is greater than *BT*.
    - By addition, *FL* + *LA* + *AB* + *BM* + *MT* is greater than *FA* + *AB* + *BT*.

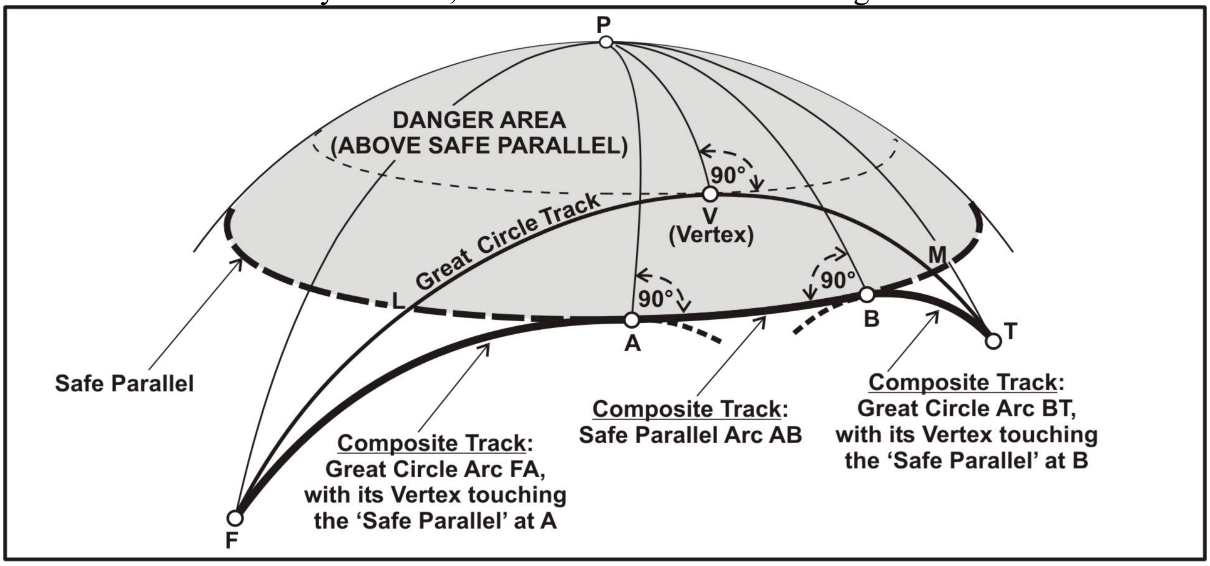


Fig 2-12. The Great Circle, Vertex and Composite Track

c. **Calculation.** The calculation of the *Composite Track* is at Para 0522.

0210. Summary of Methods for Spherical Great Circle Calculations

Table 2-2 (below) lists 7 methods for solving *Spherical Great Circle* calculations, with their applicability to finding the *Distance* and *Course* / bearing. The calculation of *Great Circle* (*Geodesic*) *Course*s and *Distance*s taking into account the *Spheroidal* shape of the Earth are at Paras 0541-0542.

Table 2-2. Methods for Spherical Great Circle Calculations

Method	Distance	Course / Bearing
Software - 'NAVPAC 'program (UKHO: DP 330)	YES	YES
WECDIS and ECDIS equipments	YES	YES
Cosine	YES	YES
Sine	NO	YES
Haversine	YES	NO
Sight Reduction Tables (NP 401)	YES	YES
Half Log Haversine	NO	YES
ABC Tables (NP 320 - Norie's Nautical Tables)	NO	YES

a. **'NAVPAC' Program (UKHO - DP 330).** The *NAVPAC* software program is produced by HM Nautical Almanac Office (available to the public from *UKHO,* Taunton as DP 330); *NAVPAC* provides astronomical data and is a fast, reliable, authoritative method of solving *Spherical Great Circle*, *Spheroidal Rhumb Line* and astro-navigation calculations (see *NAVPAC* details at Para 0551). Its operation is fully described in BR 45 Volume 2 (available to the public and published by the Nautical Institute, London). ***NAVPAC* is in Royal Navy service and is the Royal Navy's preferred method for obtaining astronomical data and solving the above calculations.**

b. **WECDIS / ECDIS.** *WECDIS* also provides a fast, reliable and authoritative method of solving *Spherical Great Circle* and *Spheroidal Rhumb Line* calculations. *ECDIS* equipments are also capable of these calculations.

c. **Cosine and Sine Methods.** The '*Cosine Method*' is very suitable for use with a pocket calculator and is explained at Para 0211. The *'Sine Method'* may be used to cross-check the *Cosine* solution and may also be used to determine the *Course* or bearing. Both the *Cosine* and the *Sine Formulae* are set out in Appendix 2. Although the *Sine Formula* is ambiguous, this ambiguity is easily resolved in most cases, and the calculation is simpler than the *Cosine Method*. An example is given at Para 0211.

d. **Haversine and Half Log Haversine Methods.** The '*Haversine Method*' and *'Half Log Haversine Method*' are set out in Appendix 2.

e. **Sight Reduction and ABC Table Methods.** A full explanation of the *'Sight Reduction Method*' and '*ABC Table Method*' are set out in BR 45 Volume 2 (see details of Volume 2's availability to the public at Para 0210a above).

0211. Spherical Great Circle Calculations - Cosine Method

Although <u>*Spherical*</u> *Great Circle Distance* and *Course* / bearing calculations are most easily solved by using *NAVPAC* (see Para 0210a), they may also be solved effectively by the '*Cosine Method*', which is very suitable for use with a pocket calculator (see Fig 2-13 below).

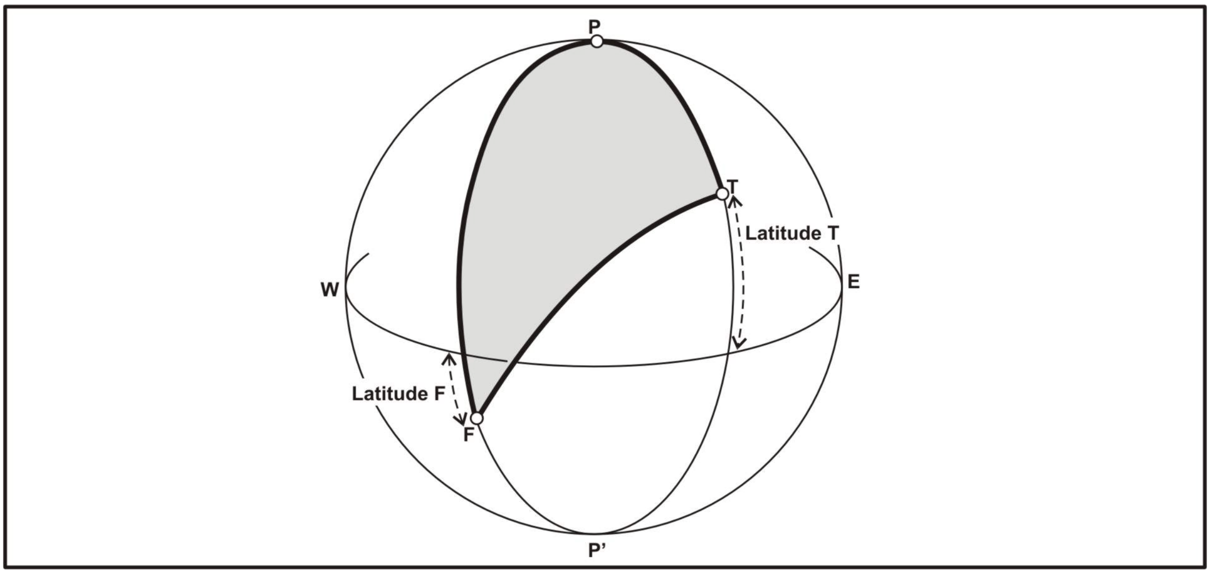


Fig 2-13. Spherical Great Circle Calculations

a. **Great Circle Distance.** The equations for *Great Circle Distance* are:

$$\cos FT = \cos FP \cos PT + \sin FP \sin PT \cos FPT$$

$$\cos Distance = \cos (90^\circ \pm Lat\ F) \cos (90^\circ \pm Lat\ T) + \sin (90^\circ \pm Lat\ F) \sin (90^\circ \pm Lat\ T) \cos d.\ long \quad \ldots \mathbf{2.8}$$

- **Signs in Formula (2.8).** Signs are determined in formula (2.8) by the name of the elevated *Pole* and the *Latitude* of the place, as follows:

SAME Names:	SUBTRACT
CONTRARY Names:	ADD

  In Fig 2-13 (above), *F* and *T* are on opposite sides of the *Equator*; thus in formula (2-8), the *Latitude* of *F* would be <u>added</u> and that of *T* <u>subtracted</u>.

- **Same and Opposite Hemispheres - Formula (2.9).** Formula (2.8) may be resolved into formula (2.9) below. In formula (2.9), when *F* and *T* are on the same side of the *Equator*, *F* and *T* are both positive; when *F* and *T* are different sides of the *Equator*, the *Latitude* of the point opposite the elevated *Pole* is negative (eg At point *F* in Fig 2-13, sin *Lat* (-*F*) and cos *Lat* (-*F*) would be used). Example 5-6 at Para 0541 provides an illustration of this concept.

$$\cos Distance = \sin Lat\ F \sin Lat\ T + \cos Lat\ F \cos Lat\ T \cos d.long \quad \ldots \mathbf{2.9}$$

- **Modified Formula (2.10).** Formula (2.9) may also be modified as follows:

$$\cos Distance = (\tan Lat\ F \tan Lat\ T + \cos d.long) \cos Lat\ F \cos Lat\ T \quad \ldots \mathbf{2.10}$$

(0211) b. **Great Circle Course / Bearing**. The equations for *Great Circle Course* / bearing are:

$$\cos PT = \cos FP \cos FT + \sin FP \sin FT \cos PFT$$

$$\cos PFT = \frac{\cos PT - \cos FP \cos FT}{\sin FP \sin FT}$$

$$\cos PFT = \frac{\cos (90° \pm lat\ T) - \cos (90° \pm lat\ F) \cos FT}{\sin (90° \pm lat\ F) \sin FT} \qquad \ldots \mathbf{2.11}$$

- **Opposite Hemispheres.** In Fig 2-13, *F* and *T* are on opposite sides of the *Equator*; thus the *Latitude* of *T* would be <u>subtracted</u> and that of *F* <u>added</u>.

- **Same Hemispheres.** When *F* and *T* are both on the same side of the *Equator*, formula (2.11) resolves into:

$$\cos initial\ course = \frac{\sin lat\ T - \sin lat\ F \cos distance}{\cos lat\ F \sin distance} \qquad \ldots \mathbf{2.12}$$

$$\sin initial\ course = \frac{\sin (90° \pm Lat\ T) \sin d.long}{\sin distance} \qquad \ldots \mathbf{2.13}$$

- **Initial Courses.** In the Northern hemisphere, the initial *Course* is defined from North (and as either East or West). In the Southern hemisphere, it is defined from South unless Southern *Latitudes* are entered as negative values.

Example 2-6: Great Circle Calculations by the Cosine and Sine Methods

A ship transits from position F (45˚N, 140˚E) to T (65˚N, 110˚W). Find the *Great Circle Distance* and the initial *Course* by the *Cosine Method*, and the initial *Course* by the *Sine Method*.

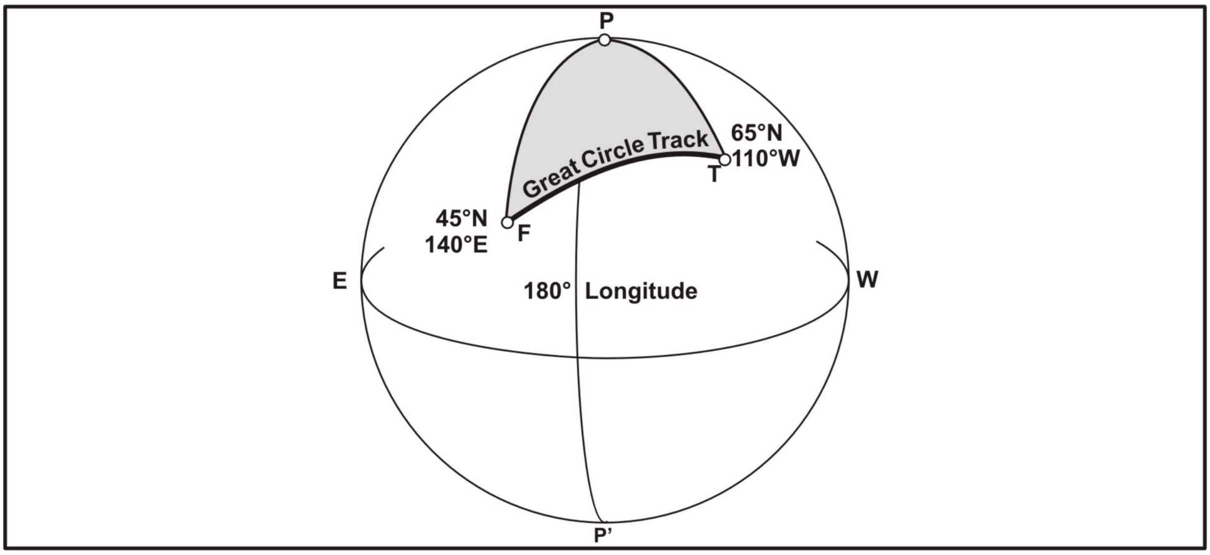


Fig 2-14. Ship's Great Circle Track from Example 2-6

- **Great Circle Distance - Cosine Method:**

$$\cos Distance = \sin Lat\ F \sin Lat\ T + \cos Lat\ F \cos lat\ T \cos d.long \qquad \ldots \text{(formula 2.9)}$$

$$= \sin Lat\ 45° \sin Lat\ 65° + \cos Lat\ 45° \cos Lat\ 65° \cos 110°$$

$$= 0.53864837$$

$$Great\ Circle\ Distance = 57.408325° = 3444.5'$$

(0211b Example 2-6 continued)

- **Great Circle Initial Course - Cosine Method:**

$$cos\ initial\ course = \frac{sin\ lat\ T - sin\ lat\ F\ cos\ distance}{cos\ lat\ F\ sin\ distance} \quad \text{. . . (formula 2.12)}$$

$$= \frac{sin\ lat\ 65° - sin\ lat\ 45°\ cos\ 57.408325°}{cos\ lat\ 45°\ sin\ 57.408325°}$$

$$= \frac{0.52542587}{0.59575915} = 0.88194343$$

$$Initial\ Course = N28.122305°E = 028.1°$$

- **Sine Rule Check**:

$$\frac{sin\ FT}{sin\ FPT} = \frac{sin\ PT}{sin\ PFT} \quad \text{. . . (Appendix 2, formula A2.5)}$$

$$\frac{sin\ 57.408325°}{sin\ 110°} = 0.8966024 = \frac{sin\ 25°}{sin\ 28.122305°}$$

- **Great Circle Initial Course - Sine Method:**

$$\frac{sin\ FT}{sin\ FPT} = \frac{sin\ PT}{sin\ PFT} \quad \text{. . . (Appendix 2, formula A2.5)}$$

$$sin\ PFT = \frac{sin\ PT\ sin\ FPT}{sin\ FT}$$

$$sin\ initial\ course = \frac{sin\ (90° \pm Lat\ T)\ sin\ d.long}{sin\ distance} \quad \text{. . . (formula 2.13)}$$

$$= \frac{cos\ lat\ T\ sin\ d.long}{sin\ distance}$$

$$= \frac{cos\ 65°\ sin\ 110°}{sin\ 57.408325°} = 0.47135526$$

$$PFT = N28.122305°E\ or\ N151.87769°E$$

In this case the ambiguity is easily resolvable, as the *Great Circle Course* from F to T must lie to the North of East. Thus:

$$Initial\ Course = 028.1°$$

CHAPTER 3

AN INTRODUCTION TO GEODESY

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CHAPTER 3

AN INTRODUCTION TO GEODESY

0301. Scope of Chapter

Geodesy is the branch of mathematics concerned with large areas of the Earth's surface in which allowance must be made for Earth's shape and the curvature of its surface. Chapter 3 introduces the basic concepts of this subject.

0302-0309. Spare

SECTION 1 - SPHEROIDS

0310. The Shape of the Earth - Oblate Spheroid

As stated at Para 0110a / Fig 1-1, and repeated below for the convenience of readers:

> **(Extract from Para 0110a):** The Earth is not a perfect *Sphere*; it is slightly flattened at the top and bottom, the smaller diameter being about 23.1 *n.miles* less than the larger. The Earth's 'flattened' shape is known as an *Oblate Spheroid* (see Fig 3-1 below) with an *Equatorial* radius '*a*' of approximately 3443.9 *n. miles* and a *Polar* radius '*b*' of 3432.4 *n. miles* (*International Nautical Miles* of 1852 m based on *WGS 84 Datum*).

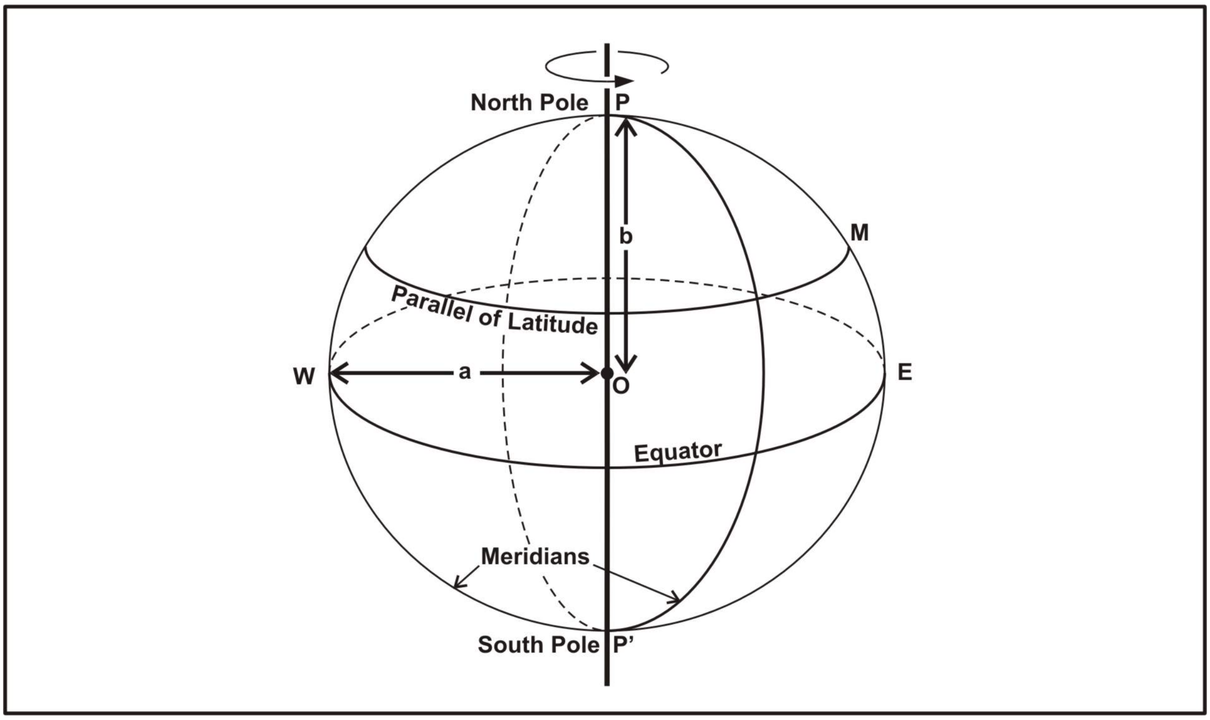

Fig 3-1. The Shape of the Earth - an Oblate Spheroid (*Copy of Fig 1-1*)

0311. The Surfaces of the Earth (Physical Surface, Geoid and Spheroid)

Within the overall concept of the Earth being an *Oblate Spheroid*, the Earth has 3 distinct surfaces (see Fig 3-2 below): the *Physical Surface*, the *Geoid*, and the *Spheroid* (also called the *Ellipsoid* - see Note 3-1 below).

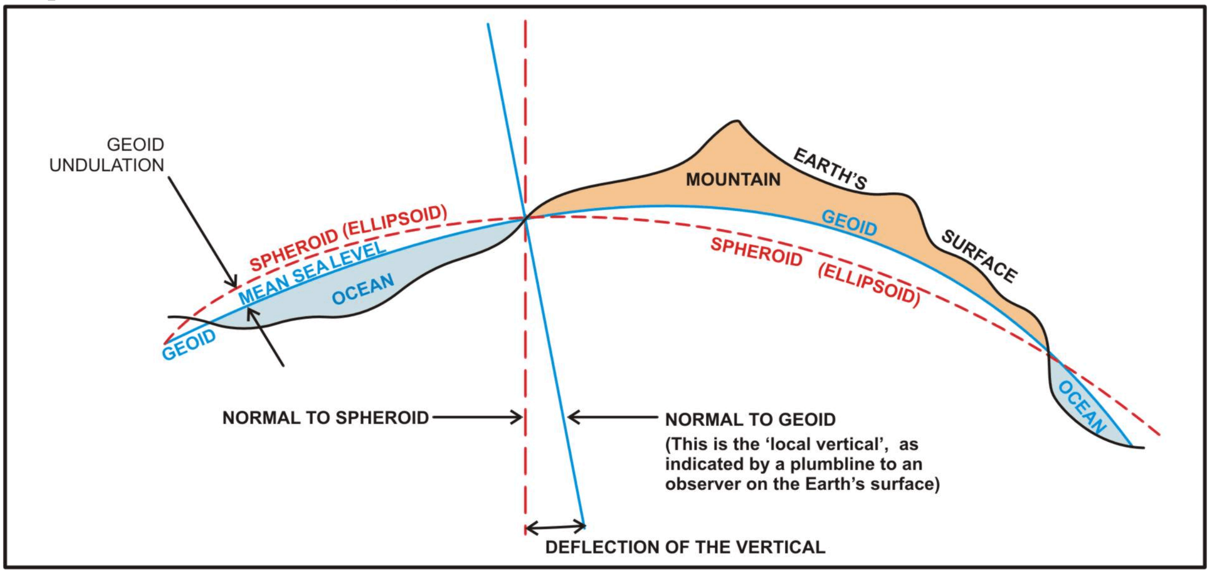

Fig 3-2. The Surfaces of the Earth (Physical Surface, Geoid and Spheroid)

Note 3-1. *The term 'Ellipsoid' is also widely used for 'Spheroid'. For the purposes of simplicity, the term 'Spheroid' is used in BR 45 Volume 1.*

a. **Defining the Surfaces of the Earth.** The 3 surfaces of the Earth may be defined as follows:

- **Physical Surface.** The Earth's *Physical Surface* does not have a perfectly *Spherical* shape, but is slightly flattened in the *Polar* regions (see Para 0310a / Para 0110a previous page). In addition, although the Earth's shape approximates to an *Oblate Spheroid*, it is highly irregular with local departures of several kilometres from a 'pure' *Spheroida*l shape. Thus, it is NOT a suitable surface on which to base calculations.

- **The Geoid.** The *Geoid* is a more regular shape and is defined as:

  "An 'Equipotential Surface' (see Note 3-2 below) which is equivalent to Mean Sea Level" .

  However, due to topography and variations in the earth's density, the *Geoid* is NOT a sufficiently regular surface on which to base calculations.

Note 3-2. *An 'Equipotential Surface' is one on which the gravity potential is always uniform and to which the direction of gravity is always perpendicular.*

- **The Spheroid (Ellipsoid).** The *Spheroid (Ellipsoid)* may be considered as an ellipse which has been rotated about its semi-minor axis. Calculations based on the semi-major and semi-minor axes provide a figure for the *Spheroid 's 'Flattening'* and *'Eccentricity'* (see Figs 3-3 and 3-4 opposite). The *Spheroid* forms a convenient surface on which to base positions and positional calculations. A large number of *Spheroids* have been developed for a number of *Local*, *Regional* or worldwide *Geodetic Datums* (see details at Para 0321).

(0311) b. **Flattening.** The *'Flattening'* (or ellipticity) of the *Spheroid* is an important factor in *Geodetic Datum* conversion calculations. The reciprocal of *Flattening* (ie '1/*f*') and the Earth's semi-major *Equatorial* axis '*a*' are defined values; others values are derived from these. The relationship between '*f*,' '*a*' and '*b*' (semi-major *Polar* axis) is:

$$f = \frac{a - b}{a} \qquad \ldots 3.1$$

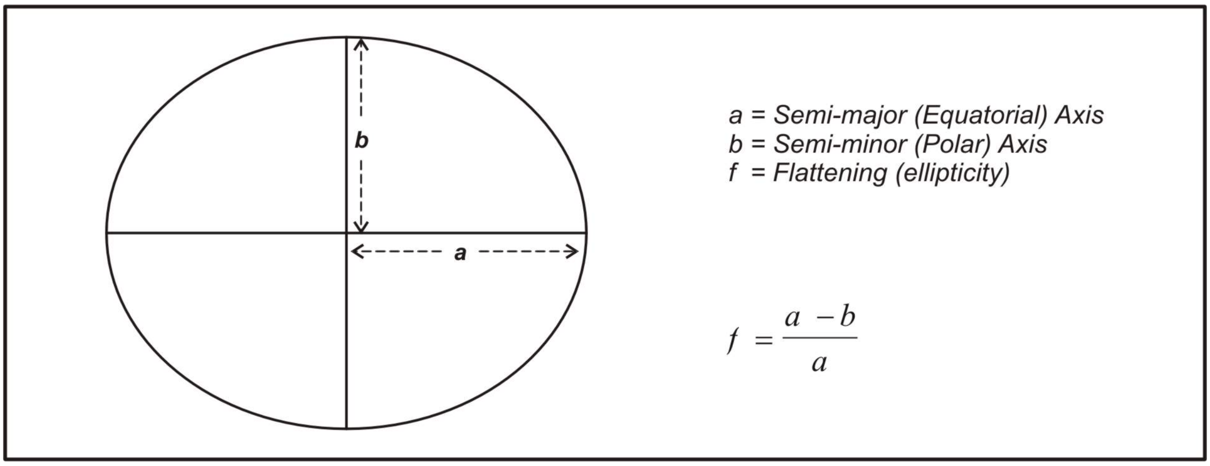

Fig 3-3. The Spheroid - Flattening 'f' (*ellipse exaggerated for clarity*)

c. **Eccentricity.** The '*Eccentricity'* of the *Spheroid* is another important factor in *Geodetic Datum* conversion calculations. If point *'M'* moves so that its distance from a fixed point *'S'* (the focus of the ellipse) is always in a constant ratio *'e'* (less than unity) to its perpendicular distance from a fixed straight line *AB* (the directrix), then the locus of a point '*M'* is called an ellipse of *Eccentricity* '*e*'(see Fig 3-4 below). *Eccentricity* '*e*' may be calculated from the Earth's semi-major (*Equatorial*) axis '*a*' and semi-minor (*Polar*) axis '*b*' (see Fig 3-3 above), as follows:

$$e = \left(\frac{a^2 - b^2}{a^2}\right)^{1/2} \quad \text{and} \quad e^2 = \frac{a^2 - b^2}{a^2} \qquad \ldots 3.2$$

and from . . . 3.1

$$e = \left(2f - f^2\right)^{1/2} \qquad \ldots 3.3$$

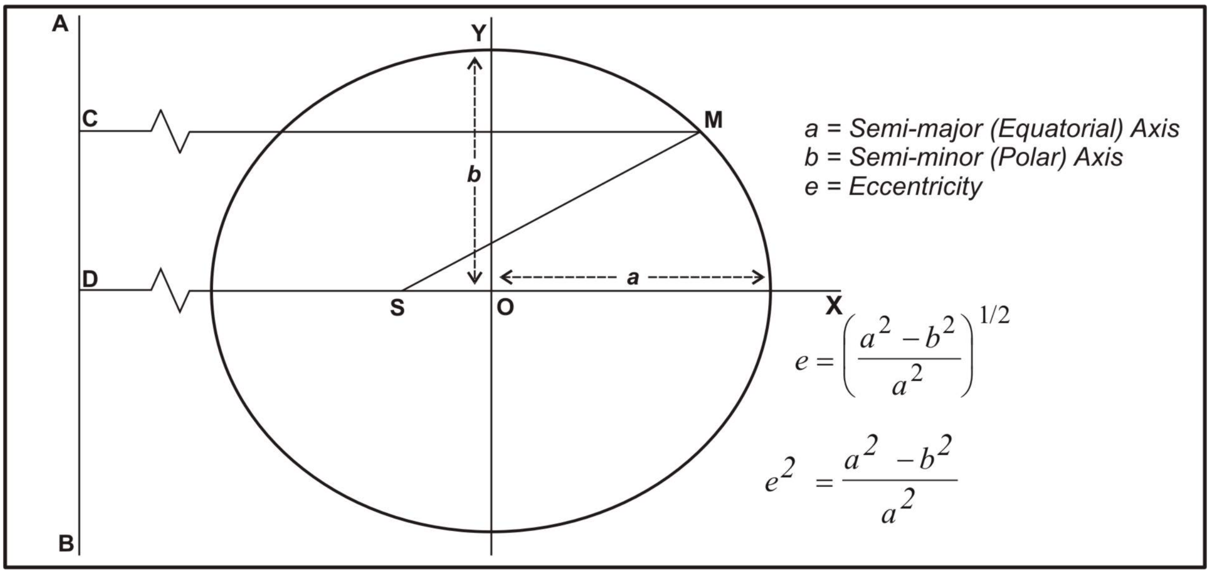

Fig 3-4. The Spheroid - Eccentricity 'e' (*ellipse exaggerated for clarity*)

0312. Spheroid - Geodetic and Geocentric Latitudes

Fig 3-5 (below) shows a *Meridional* section of the *Spheroid*. *'M'* is a point on the *Meridian PAP'*, and *MK* is the tangent to the *Meridian* at *M*.

a. **Geodetic Latitude.** If the normal *LM* to the tangent *MK* cuts *OA* at *L*, the angle *MLA* is called the *Geodetic Latitude* of *M*, and denoted by the symbol ϕ . *ML* does NOT pass through the centre *O* of the *Spheroid,* except when *M* is on the *Equator* or at the *Poles*.

b. **Geocentric Latitude.** The angle *MOA* (which by definition, does pass through the centre *O* of the *Spheroid*) is called the *Geocentric Latitude* of *M* and is denoted by the symbol θ .

c. **Connection between Geodetic Latitude and Geocentric Latitude.** The *Geodetic Latitude* ϕ and *Geocentric Latitude* θ are connected by the following formula (see Appendix 5):

$$\tan\,\theta = \frac{b^2}{a^2}\tan\,\phi \qquad \ldots\ \mathbf{3.4}$$

$$= (1-f)^2 \tan\,\phi \qquad \ldots\ \mathbf{3.5}$$

$$= (1-e^2)\tan\,\phi \qquad \ldots\ \mathbf{3.6}$$

d. **Difference between Geodetic Latitude and Geocentric Latitude.** The difference between *Geodetic Latitude* and *Geocentric Latitude* is zero at the *Equator* and the *Poles*, and has a greatest value when ϕ = 45˚. For *WGS 84 Datum,* the greatest value is 11.54 minutes of arc.

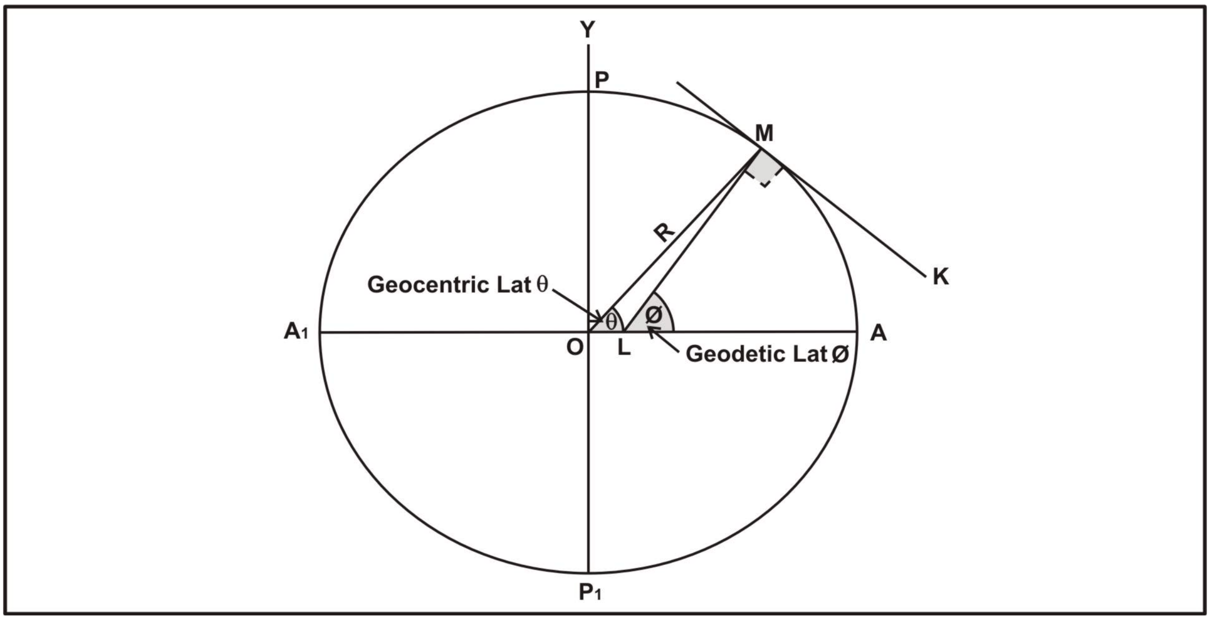

Fig 3-5. Geodetic Latitude and Geocentric Latitude

0313. Spheroid - Parametric Latitude

Fig 3-6 shows a *Meridional* section of a *Spheroid WPE* and *Sphere WBE*:

- The *Polar* axis of *Spheroid WPE* is *OP* and its shape and size are defined by the radii $OE = a$, and $OP = b$.
- *WBE* is the *Meridional* section of a *Sphere* with centre *O*, *Polar* axis *OB* and radii $OE = OB = a$.
- *M* is a point on the *Spheroid* with *Geodetic Latitude* ϕ .
- *HM* is parallel to *OP* and produced to cut the circle *WBE* at U. The radius *OU* makes an angle β with the X axis.

a. **Parametric Latitude.** Angle β is the *Parametric Latitude* (or *Reduced Latitude*) of point *M*. The *Parametric Latitude* is often used for *Geodesic* calculations on the *Spheroid* (see Paras 0541-0542).

b. **Connection between Geodetic Latitude and Parametric Latitude.** The *Geodetic Latitude* ϕ and *Parametric Latitude* β are connected by the following formula (see Appendix 5):

$$\tan\beta = \frac{b}{a}\tan\phi \qquad \text{. . . } \mathbf{3.7}$$

The difference between the *Geodetic Latitude* and *Parametric Latitude* is zero at the *Equator* and at the *Poles* and has a greatest value when $\phi = 45^\circ$. For the *WGS 84 Spheroid*, where $f = 1/298.257223563$, the greatest value is approximately 5.85 minutes of arc.

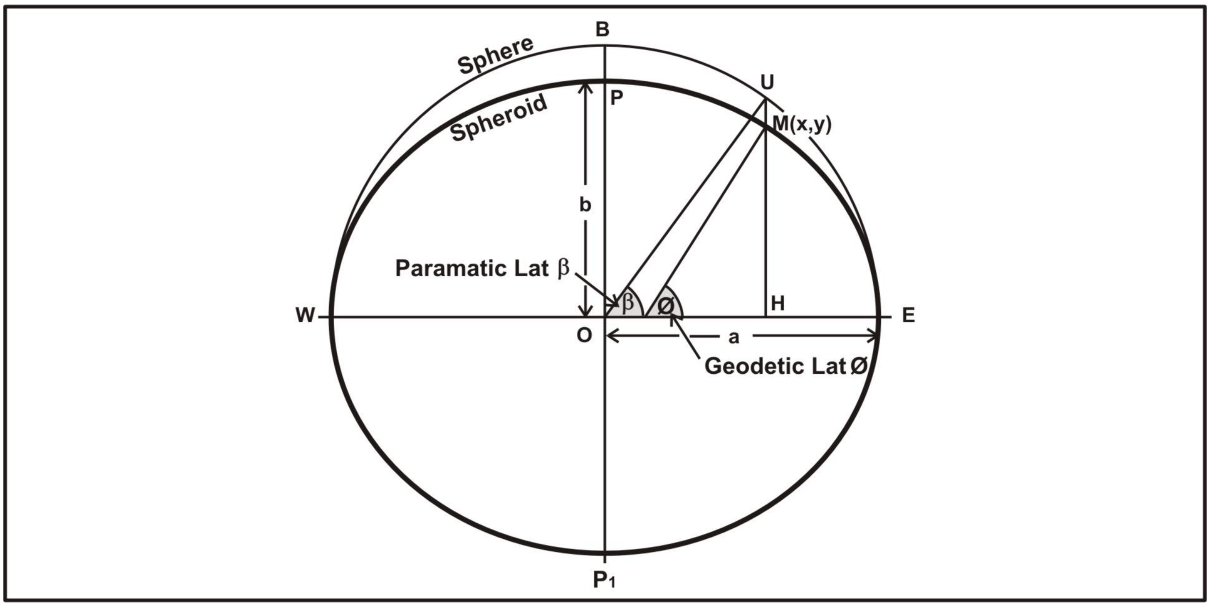

Fig 3-6. Geodetic Latitude and Parametric Latitude (*ellipse exaggerated for clarity*)

0314. Spheroid - Length of 1' in Latitude and Longitude

As explained at Para 0113, the length of the *Sea Mile* varies between the *Equator* and the *Poles* because of the Earth's changing *Radius of Curvature*.

a. **Latitude Formula.** The length of 1 minute of *Latitude* may be found from the formula $\rho\delta\phi$, where ρ is the circular *Radius of Curvature* in the *Meridian* and $\delta\phi$ is a small increase (measured in radians) in the *Geodetic Latitude* ϕ (see Fig 3-7 below). It may be shown (Appendix 5) that:

$$\rho = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}} \qquad \ldots \mathbf{3.8}$$

When $\delta\phi$ = 1 minute of arc:

$$1' \text{ of Latitude } = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}} \sin 1' \qquad \ldots \mathbf{3.9}$$

When ϕ = zero

$$1' \text{ of Latitude at the Equator} = a\left(1-e^2\right)\sin 1' \qquad \ldots \mathbf{3.10}$$

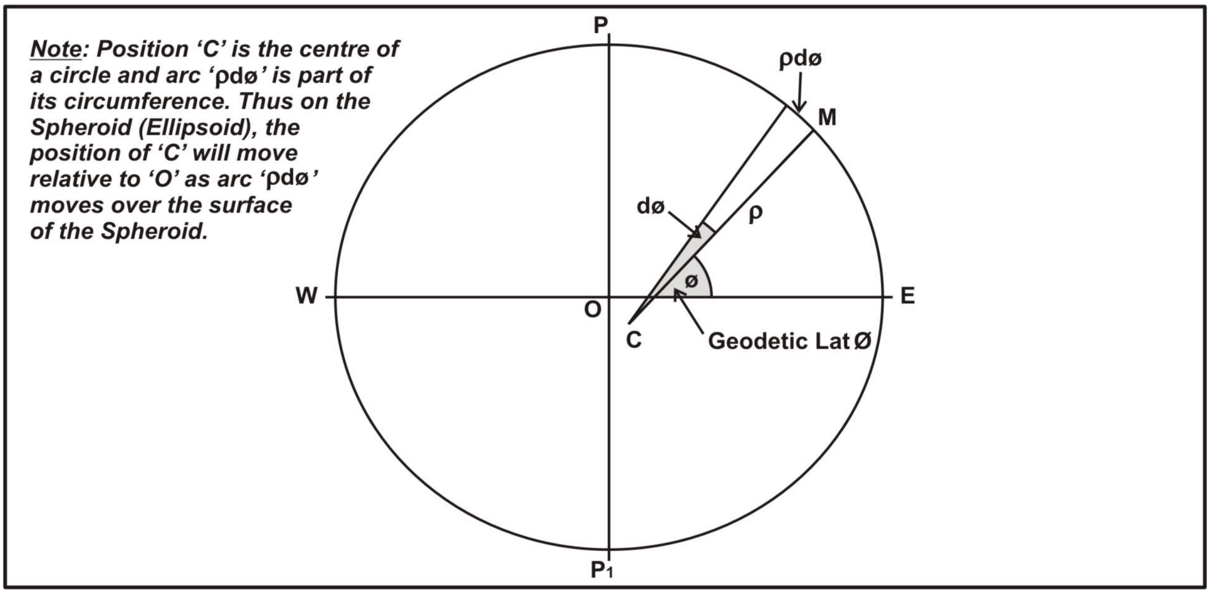

Fig 3-7. Length of One Minute of Latitude

b. **Longitude Formulae**

$$1' \text{ of Longitude at the Equator } = a \sin 1' \quad \text{(minutes of arc)} \qquad \ldots \mathbf{3.11}$$

At *Latitude* ϕ,

$$1' \text{ Longitude } = \frac{a\cos\phi}{\left(1-e^2\sin^2\phi\right)^{1/2}} \sin 1' \qquad \ldots \mathbf{3.12}$$

0315. Spheroid - Geodesic

Just as a *Great Circle* gives the shortest distance between two points on a *Sphere*, a *Geodesic* is the shortest line between two points on the *Spheroidal* Earth. See Para 0540a.

0316-0319. Spare.

SECTION 2 - DATUMS

0320. Vertical and Horizontal Datums

Chart data is referenced to a *Vertical Datum* and a horizontal *Geodetic Datum*. Charted depths and heights are referenced to a *Vertical Datum*; this subject is covered in detail at Chapter 10. The horizontal *Geodetic Datum* defines the *Spheroidal* shape of the Earth used to establish the positional framework (ie *Latitude / Longitude*, *Grids* etc).

0321. Geodetic Datums

A position given solely in terms of *Latitude* and *Longitude* will NOT define a unique point on the Earth's surface (see Fig 3-8 below). To define a point unambiguously, it is also necessary to quote the *Geodetic Datum* (and the particular *Spheroid* utilised) to which the position is referred. Fig 3-8 illustrates the difference on the Earth's surface between the same *Latitude* and *Longitude*, but referenced to two different *Geodetic Datums*.

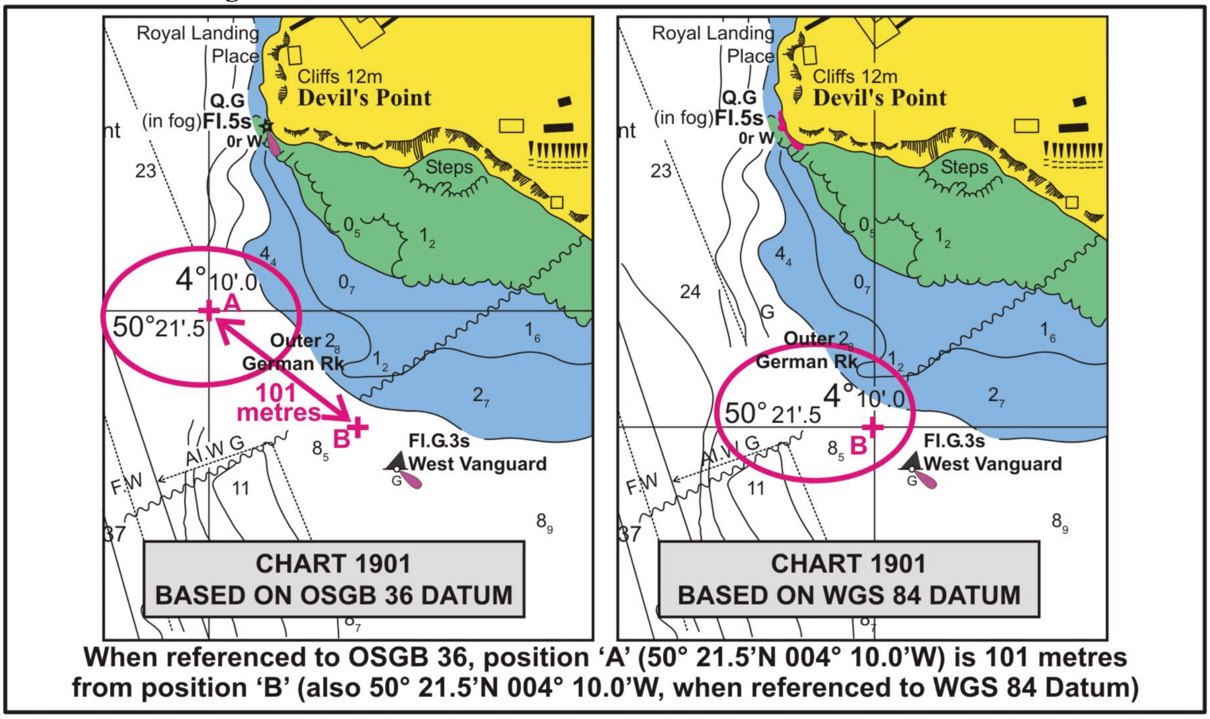

Fig 3-8. Example of the Difference for the Same Position Referred to Different Datums

a. **Choosing a Geodetic Datum.** Classically, a *Geodetic Datum* can be considered as the surface to which positions are referred. A (horizontal) *Geodetic Datum* utilises a specific *Spheroid* which best fits the *Geoid* over the area of interest. This is because the *Geodetic* measurements, (angles obtained by theodolite, and heights obtained by spirit levelling), are obtained with respect to the local gravity surface - the *Geoid*. However, since calculations are performed on the *Spheroid*, it is necessary that the *Spheroid* and *Geoid* be in close agreement to minimise errors in calculation.

b. **Local Geodetic Datums.** Thus, in the simple case of a *Local Geodetic Datum*, an accurate position may be defined by precise astronomic observations, with a *Spheroid* chosen to fit the *Geoid* exactly at this point (see 'rugby ball / orange peel' analogy at Note 3-3 overleaf). *Local Geodetic Datums* thus deal only with discrete parts of the Earth's surface and are only loosely related to the Earth's centre of mass.

(0321) c. **Classical Regional Geodetic Datums.** For 'classical' *Regional Geodetic* networks, the differences between observed astronomic position and *Geodetic* position computed through the triangulation network at several points may be minimised, thus ensuring an overall minimal separation between the *Geoid* and the *Spheroid*. Like *Local Geodetic Datums,* 'classical' *Regional Geodetic Datums* deal only with discrete parts of the Earth's surface, are only loosely related to the Earth's centre of mass and suffer from the 'rugby ball / orange peel' analogy at Note 3-3 below. See also Para 0321e - 'Modern Regional Datums Tied to WGS 84' (below) and Table 3-1 (opposite).

Note 3-3. *A crude analogy for a Local or Regional Geodetic Datum is to consider the chosen Spheroid as a rugby ball and the Local Geodetic Datum as a piece of orange peel placed over it where the position and orientation fitted best. The precise observed position would then be a pin stuck into the orange peel at the appropriate location.*

d. **World Geodetic System (WGS).** The *World Geodetic System (WGS)* is a combined *Spheroid* and *Datum.* It is global in coverage and directly related to the Earth's centre of mass.

- **Concept.** A satellite-derived *World Geodetic Datum / Spheroid* is defined by the system in which the satellite orbit parameters are given. These in turn are dependent upon the precise coordinates of the satellite tracking stations, the geopotential model for the Earth's gravity field and a set of constants including GM (the Earth's mass 'M' times the gravitational constant 'G').

- **WGS 84.** From this satellite reference system data, a universal (worldwide) *Spheroid* and *Datum* was established. *WGS 1984* ('*WGS 84*') is the current system and is NATO's preferred *Spheroid / Datum*; it has replaced *WGS 72*.

- **GNSS: GPS and Equivalents.** The adoption of *WGS 84* has made it possible to define absolute position anywhere in the world by common *Geodetic Latitude* and *Longitude* for use with *Global Navigation Satellite Systems* (*GNSS*) [eg *Global Positioning System* (*GPS*) or equivalents].

e. **'Modern' Regional Datums Tied to WGS 84.** Due to satellite technology, centimetric distance measurement is now possible over many thousands of *nautical miles* and the motion of tectonic plates becomes *Geodetically* significant. Thus *Regional Geodetic Datums* tied to *WGS 84 Datum* have been established at a specific time epoch and then held fixed in relation to the tectonic plate for the region. For most practical navigational purposes, these 'modern'*Regional Geodetic Datums* may be considered equivalent to *WGS 84*; the differences are only significant for the most precise uses.

- **Movement of European Tectonic Plate - ETRS 89 Regional Datum.** The European tectonic plate is moving at approximately 2.5cm / year in relation to *WGS 84*. A 'modern' *European Regional Geodetic Datum* (*ETRS 89*) was established in 1989 where *WGS 84* coordinates of the fundamental points were observed and held fixed. Since 1989, *WGS 84* and *ETRS 89* have drifted apart by 2.5cm / year and are now (2008) 0.47metres apart (increasing).

- **Other Modern Regional Datums.** Other examples of modern *Regional Geodetic Datum*s are *NAD 83* (N. America) and *SIRGAS* (S. America). Other similar *Regional Geodetic Datums* are now (2008) being established.

0322. Multiplicity of Geodetic Datums

a. **Common Geodetic Datums.** Numerous *Geodetic Datums* have been defined throughout the world from past centuries to the present time. Some of the more common *Geodetic Datums*, the *Spheroids* associated with them and their areas of application are at Table 3-1 (below). The *International (1924) Spheroid* is used for the calculation of distances in (*UKHO*) 'Admiralty Distance Tables' (NP 350) and 'Ocean Passages for the World' (NP 136). All calculations in this book are based on *WGS 84.*

b. **Geodetic Datums Worldwide.**

- **NATO Data.** NATO lists those *Geodetic Datums* and *Spheroids* which may be applicable for mapping and charting products within the area of NATO interest (and hence is also of interest for naval operations in support of land based activities, eg *Naval Gunfire Support [NGS]* or amphibious warfare). While the stated aim is to adopt the *WGS84 Datum* and *Spheroid* for all such products, it may be some time before this objective is fully achieved.
- **UKHO Data.** The *Admiralty List of Radio Signals* (*ALRS*) Volume 2 provides further detail on this subject, including the number of *UK Hydrographic Office (UKHO*) charts currently based on each *Datum*.

Table 3-1. Common Datums and Associated Spheroids with Areas of Application

Datum & (Type)	Spheroid	(*a*) Equatorial Radius (*b*) Polar Radius	Flattening (*f*) & Reciprocal (*1 / f*) $f = (a-b) / a$	Eccentricity (*e*) & Squared (e^2) $e^2 = (a^2-b^2) / a^2$	Remarks
OSGB 36 (Regional)	Airy 1830	**(*a*) 6,377,563.396m** (*b*) 6,356,256.909m	(*f*) 0.003340851 **(*1/f*) 299.324964**	(*e*) 0.081673374 (e^2) 0.006670540	Some remaining UK charts.
OSSN 80 (Regional)	Airy 1830	**(*a*) 6,377,563.396m** (*b*) 6,356,256.909m	(*f*) 0.003340851 **(*1/f*) 299.3249645**	(*e*) 0.081673374 (e^2) 0.006670540	Not for charts. OSGB 36 related.
ED 50 (Regional)	International 1924	**(*a*) 6,378,388m** (*b*) 6,356,911.946m	(*f*) 0.003367003 **(*1/f*) 297**	(*e*) 0.081991890 (e^2) 0.006722670	Some remaining European charts.
NAD 27 (Regional)	Clarke 1866	**(*a*) 6,378,206.4m** (*b*) 6,356,583.8m	(*f*) 0.003390075 **(*1/f*) 294.978698**	(*e*) 0.082271854 (e^2) 0.006768658	Some remaining USA charts.
Arc (Regional)	Clarke 1880	**(*a*) 6,378,249m** (*b*) 6,356,515m	(*f*) 0.003407561 **(*1/f*) 293.465**	(*e*) 0.0824834 (e^2) 0.006803511	Some remaining S. African charts.
WGS 72 (W'ldwide)	WGS 72 (1972)	**(*a*) 6,378,135m** (*b*) 6,356,750.520m	(*f*) 0.003352779 **(*1/f*) 298.26**	(*e*) 0.081818811 (e^2) 0.006694318	Being replaced by WGS 84.
NAD 83 (Regional)	GRS 80 (1980)	**(*a*) 6,378,137m** (*b*) 6,356,752.3141m	(*f*) 0.00335281068 **(*1/f*) 298.25722210**	(*e*) 0.0818191910 (e^2) 0.0066943800	Regional WGS 84 for N. America
ETRS 83 (Regional)	GRS 80 (1980)	**(*a*) 6,378,137m** (*b*) 6,356,752.3141m	(*f*) 0.00335281068 **(*1/f*) 298.25722210**	(*e*) 0.0818191910 (e^2) 0.0066943800	Regional WGS 84 for Europe
WGS 84 (W'ldwide)	WGS 84 (1984)	**(*a*) 6,378,137m** (*b*) 6,356,752.3142m	(*f*) 0.00335281066 **(*1/f*) 298.257223563**	(*e*) 0.0818191908 (e^2) 0.0066943800	WGS 84 is NATO preferred Datum.
Note: The defined parameters are 'a' and '1/f'; all others are derived values.					

0323. Geodetic Datum Shifts

a. **Datum Shifts.** Where two adjacent *Geodetic Datums* overlap, the same point on the Earth's surface will have two distinct sets of *Geodetic Coordinates*, one set in relation to each *Geodetic Datum* (see Fig 3-8). The difference between these two sets of coordinates is called a *'Datum Shift'*. These differences are primarily a result of the two different reference systems. The *Datum Shift* may not be consistent in magnitude and direction (ie it is likely to vary with geographic location).

b. **Datum Shift Information.** *Datum Shift* information is shown on navigational charts in the form of a 'Satellite-Derived Positions Note' and 'Overlapping Charts Note'. However, some charts exist for which the (horizontal) *Datum* or the relationship to *WGS 84* is unknown (some 1762 *UKHO* chart panels in 2008). These areas will have to be re-surveyed to modern standards before accurate *WGS 84* coordinates may be obtained for use with *GNSS* (ie *GPS* or equivalent).

Table 3-2. Examples of Common 'Datum Shifts'

From Datum	To Datum	Devonport Lat(")	Devonport Long(")	Rosyth Lat(")	Rosyth Long(")
WGS84	WGS72	-0.10N	-0.55E	-0.09N	-0.55E
WGS84	ED50	+3.40N	+5.16E	+2.75N	+5.82E
WGS84	OSGB36	-2.22N	+4.22E	+0.24N	+5.06E
OSGB36	ED50	+5.62N	+0.94E	+2.51N	+0.76E
OSGB80	ED50	+5.47N	+0.88E	+2.89N	+0.67E

Note 3-4. *See also Admiralty List of Radio Signals (ALRS) Volume 2, which provides further detail on this subject, including the number of UK Hydrographic Office (UKHO) charts currently based on each Datum.*

0324. Geodetic Datum Transformation Methods

Several techniques are available to transform positional information from a variety of *Geodetic Datums* onto a common reference *Geodetic Datum* (normally *WGS 84*).

a. **Non-Homogeneous Geodetic Datums - Inconsistencies.** Some of the earlier *Local* or *Regional Geodetic Datums* (eg *OSGB 36*) contain internal inconsistencies in *Scale* and *Azimuth*, and are thus known as *'Non-Homogeneous Geodetic Datums'*. These inconsistencies may be significant (eg for *Precise Navigation*) if calculating position transformations to other *Geodetic Datums,* particularly if using automatic systems.

b. **Published Geographical Shifts.** When the *Datum Shift* has been determined by comparison of positions common to both *Datum*s, a simple block shift in geographical coordinate values may be applied. As a result, many nautical charts referred to *Geodetic Datums* other than *WGS 84*, now show the *Datum Shift* between *WGS 84* and the horizontal *Datum* in which the chart is published. These *Datum Shifts* are quoted to an accuracy commensurate with the chart *Scale*, and should not produce an error capable of being plotted on the chart at that *Scale*.

c. **WGS 84 and WGS 72.** To plot *WGS 84* positions on *WGS 72* charts, the *WGS 84* positions must be moved by:

- **Longitude:** 0.554 seconds Westward.
- **Latitude:** 0.1455 cos *Latitude* + 0.0064 $\sin^2$*Latitude* seconds Southward. The distance varies between 17.1 metres at the *Equator* and zero at the *Poles.*

d. **Datum Conversions.** Further details may be found in NATO STANAG 2211 (available in the public domain). To transform positions between different *Geodetic Datums,* the *Geodetic Datum* coordinates of *Latitude* and *Longitude* are first converted into 3-dimensional *Cartesian Coordinates*. Using the 'Molodensky Equations', translations along the X, Y and Z axes, rotations around these axes and a *Scal*e factor are applied to the *Cartesian Coordinates*. The modified *Cartesian Coordinates* are then re-converted into the second *Geodetic Datum* coordinates of *Latitude* and *Longitude.*

- **Homogeneous Geodetic Datums.** For *Homogeneous Geodetic Datums,* an average set of values for X, Y and Z axis rotation will provide sufficient accuracy in *Datum* transformation. This process may be easily automated.

- **Non-Homogeneous Geodetic Datums.** For *Non-Homogeneous Geodetic Datums,* average values may not always be adequate. Two methods are commonly available to overcome this:
    - **Localised Values.** One method is to use localised values for X, Y and Z axis rotations. This method suffers from the disadvantage that no single value can be used in automated systems and that it must interrogate a digital record of the contours; these values should never be extrapolated.
    - **Multiple Regression Equations (MRE).** Another method is to use *Multiple Regression Equations (MRE)*. This method is appropriate for determining *Datum Shifts* for land based applications, but is NOT valid for maritime applications since the *MREs* become rapidly unstable outside the area for which they were defined.

0325-0329. Spare.

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SECTION 3 - EARTH MODELS FOR NAVIGATION SYSTEMS

0330. Earth Models - Spheroidal, Spherical and Flat Earth

a. **Types of Earth Models.** A variety of Earth models may be used in navigation systems. These fall into three categories:

- A *'Spheroidal* Earth'
- A *'Spherical* Earth'
- A *'Flat Earth'* or *'Tangential Plane'*

b. **Choice of Earth Models in Navigation Systems.** The *Spheroidal* Earth model is the closest approximation to the 'true' Earth shape, and is the most suitable surface on which to base precise long-range calculations in navigation systems. The other Earth models are less accurate approximations of the ideal model and will give less accurate results, particularly when dealing with long-range contacts beyond horizon-distance. The size of these errors should be compared with those of the *Spheroidal* Earth model.

0331. Spheroidal Earth Models

Numerous *Spheroids* are in existence and standardisation is therefore necessary.

a. **Adoption of WGS 84 Spheroid.** The *Global Positioning System* (*GPS*) uses *WGS 84,* which combines functions of both *Datum* and *Spheroid.* The preferred NATO *Geodetic Datum* and *Spheroid* for mapping and charting products in the NATO maritime operational areas is *WGS 84*. *WGS 84* was adopted for all relevant Royal Navy applications from 1991. *WGS 84* has been adopted by the *IMO* for *WECDIS / ECDIS / ECS* equipments and for *Electronic Navigation Charts* (*ENCs*) / *Raster Navigation Charts* (*RNCs*) used with them. The *Automatic Identification System* (*AIS*) also uses *WGS 84.*

b. **Differences Between Spheroids.** The correct *Spheroid* associated with the *Geodetic Datum* to which *Geodetic Coordinates* are referred must always be used in all calculations, otherwise some quite large and unexpected errors can be introduced. In *Naval Gunfire Support* (*NGS*), *Geodetic Latitude* and *Longitude* coordinates may require to be converted to *Grid* coordinates to be compatible with adjacent land mapping. See Example 3-1 (below) for an indication of possible errors if incorrect *Datum* / *Spheroid* information is applied to *Geodetic / Grid* conversions.

Example 3-1. In a gunnery exercise, a target position on the North East corner of Garvie Island at 58° 37.09' N, 004° 52.20' W (*WGS 84*) translates into a *British National Grid* position of 233418E 973594N when using (correctly) the *Airy Spheroid (1830)* / *OSGB36 Datum* (see Fleet Chart F6681WGS). If no special (Fleet) chart overprinted with various *Grids* is available for a particular area, the transformation calculation has to be carried out mathematically; some transformation facilities are contained within *WECDIS* and may be included in *ECDIS* equipments. However, if the wrong *Spheroid / Datum* combination is selected for the transformation, a significant error can occur (eg in the above example, the incorrect use of, say, the *International Spheroid (1924)* instead of *Airy Spheroid (1830)* will result in a *British National Grid* position 19 metres East and 288 metres North of the correct position).

0332. Spherical Earth Models

Spherical Earth models, as their name suggests, assume a perfectly *Spherica*l Earth. Position calculations are therefore based on *Spherical* trigonometry. The errors introduced by this assumption and method of calculation are principally, but not exclusively, dependent upon the adopted value of *Spherical* Earth radius, the *Latitude* of own ship, and the distance and direction to the observed (or reported) contact. Table 3-3 below indicates the likely maximum size of error induced by assumption of a *Spherical* Earth, by comparison with the *WGS 84 Spheroid,* when dealing with contacts at long range.

Table 3-3. Errors from a Spherical Earth Calculation versus WGS 84 Spheroid

Range of Contact (n. miles)	Positional error (n. miles) at Latitude 0°	Positional error (n. miles) at Latitude 75°
50	0.1	0.3
100	0.1	0.7
250	0.3	1.7
500	0.5	3.4
1000	1.1	6.5
2000	1.9	11.3

Note 3-6. *In the above calculations and tabulated results no account has been taken of the height above (or below) the reference surface (Spheroid, Sphere, Tangential Plane) of own platform or the target. This in itself will lead to further range errors, in addition to those directly due to the use of approximate Earth models.*

0333. Flat Earth Models

a. **Flat Earth Assumptions.** *Flat Earth* models assume a plane Earth and their *Cartesian Coordinate* reference systems adopt a 2-dimensional concept. These 2-dimensional *Cartesian Coordinate* systems have axes arranged to lie along True (or Magnetic) North and due East from an arbitrarily chosen *Grid Origin*, which may not always coincide with own ship's position.

b. **Errors Associated with Flat Earth Assumption.** The errors introduced by the *Flat Earth* assumption are dependent on the specific algorithms implemented within individual navigation systems. However, in general, the size of error will be a function of both the distance from the *Grid Origin* and the *Latitude* of the *Grid Origin*. Thus errors in bearing and distance increase with distance from the *Grid Origin* and with increase in the *Latitude* of the *Grid Origin*.

CHAPTER 4

PROJECTIONS AND GRIDS

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CHAPTER 4

PROJECTIONS AND GRIDS

0401. Scope of Chapter

Chapter 4 introduces *Projections* and *Grids* and their application in navigation. Detailed information on *Projections,* including their mathematical derivation, is at Appendix 4.

0402-0409. Spare

SECTION 1 - PROJECTION CONCEPTS AND PRINCIPLES

0410. Transposing Three Dimensions to Two Dimensions

a. **Requirement for Projections.** The Earth is a 3-dimensional *Spheroidal* object and a chart is a 2-dimensional plane. A *Projection* is a method of representing a *Spheroidal* surface on a plane. It is usually expressed as a mathematical formula for converting *Spheroidal* geographical co-ordinates to plane co-ordinates on charts or maps. Provided it is suitable, a *Projection* may be used to represent any portion of the Earth's surface.

b. **Distortion.** A *Spheroidal* surface CANNOT be fitted exactly on to a plane and, except over very small *Area*s, all *Projections* will contain some distortion. When the outline of three identical circular *Area*s from different parts of the Earth's surface are each *Projected* from a point of origin at the centre of the Earth on to a plane chart, they are represented by a quite different sizes and shapes (see example at Fig 4-1 below).

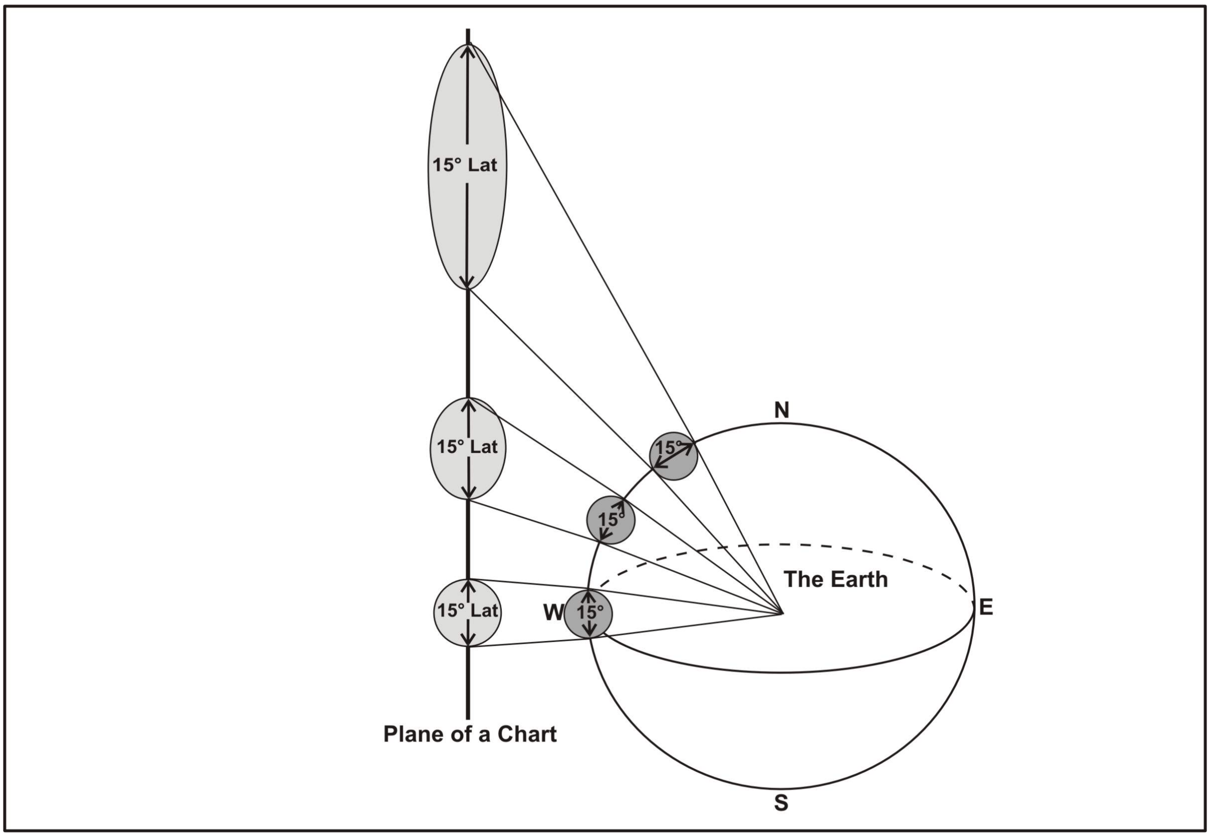


Fig 4-1. Example of Distortion of Areas of the Earth's Surface on a Chart

0411. Distortion in Projections

a. **Properties of Projections.** The distortion of a *Projection* must involve some or all of the following four interrelated properties:

- **Shape.**
- **Bearing.**
- **Scale.**
- **Area.**

b. **Choice of Projection Properties.** *Projections* can be devised which will eliminate or reduce to negligible proportions some of these distortions, while keeping the others within reasonable (and thus usable) limits. The choice of *Projection* for a chart of map is thus governed by the specific purpose for which the chart is intended.

c. **Mariners' Chart Projection Property Requirements.** Presentation of the correct *Area* of land on a chart is not usually a priority for the mariner, although it may be important for other users. Ideally, the mariner requires a chart which will provide the correct *Shape* of land, and correct *Bearings* and distances (*Scale*). Unfortunately these 3 requirements CANNOT be met in one single *Projection*, and a compromise must be made by accepting a very close approximation to all three (*Shape*, *Bearing*, *Scale*), or satisfaction of two (usually *Shape* and *Bearing*) at the expense of the third (*Scale*).

d. **Orthomorphism.** An *Orthomorphic* (or *Conformal*) *Projection* is one in which, with certain compromises (see Para 0411e below), *Shape*, *Scale* and *Bearings* are correctly represented. These properties are the ones needed by mariners, because, if distortion of *Shape* occurs, then distortion of the *Compass Rose* (ie *Bearing*) must also occur. A compass rose on a chart which is NOT *Orthomorphic* will NOT be circular, nor will its graduation be uniform, and so it would be impossible to lay off courses and bearings correctly. *Mercator*, *Transverse Mercator, Inverse (Oblique) Mercator, Lamberts Conical Orthomorphic, Skew Orthomorphic* and *Stereographic* charts (see Fig 4-3b) are *Orthomorphic*. In *Orthomorphic* chart *Projection*s, the following apply:

- **Shape.** The *Shape* of the land correctly represents that of the Earth's surface, at least over small *Area*s.

- **Scale.** At any specific point on that chart or map the *Scale* is the same in all directions. However, the same *Scale* may NOT apply to the whole chart.

- **Bearing.** The *Parallel of Latitude* and *Meridian* of *Longitude* at any point are at right angles to each other. Angles around any point on that chart or map, and hence *Bearings*, are correctly represented.

e. **Orthomorphism Compromises.** No *Projection* can meet the mariner's ideal requirement for perfect *Shape*, *Bearing* and *Scale* over large *Areas*. On a *Mercator* chart of the world, (see Fig 4-2 opposite), the *Area* around Cape Farewell (Greenland) is just as correctly shown for *Shape* as is the Amazon Estuary (South America), but Greenland as a whole 'appears' to have about the same *Area* as South America, whereas it is actually about one-tenth. This is because the *Scale* in the (near-*Polar*) Greenland *Area* is quite different from the *Scale* being used for *(Equatorial)* South America on the same chart, and this affects *Area* properties. The change of *Scale* can also be seen (at Fig 4-2) in the expanding intervals towards the *Poles* between *Parallels of Latitude*.

(0411e continued)

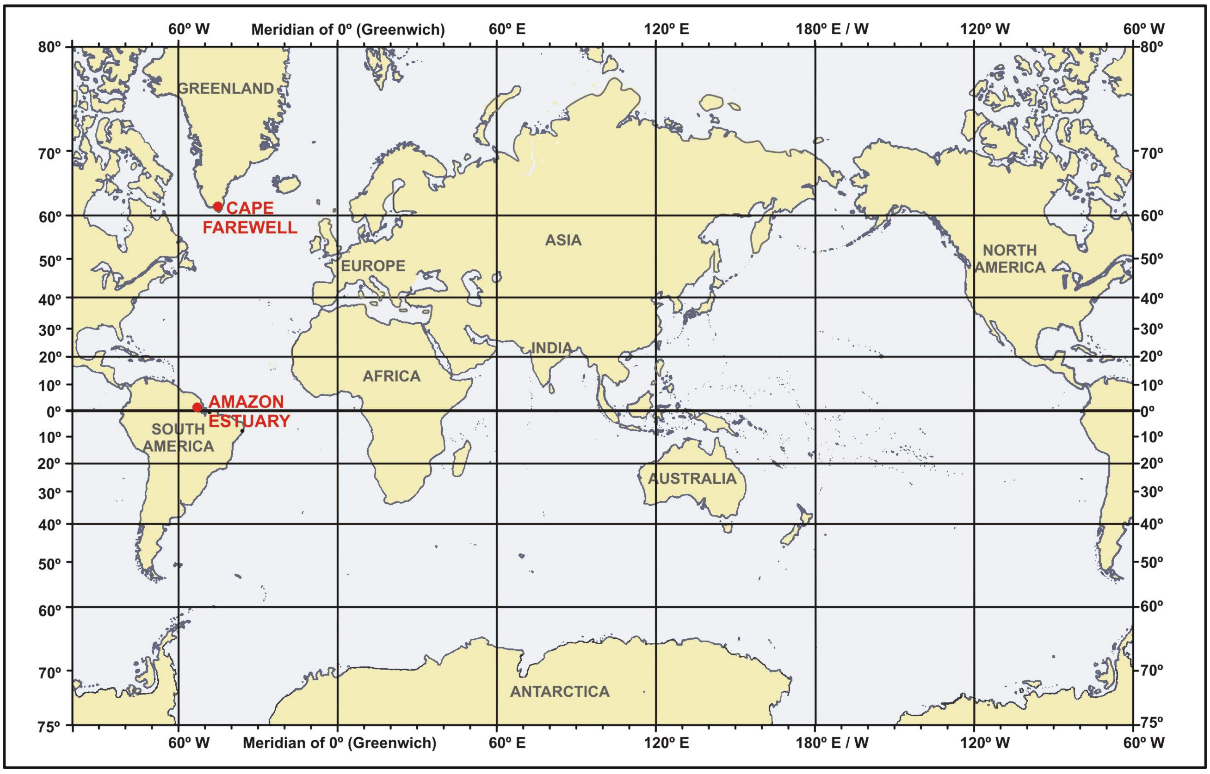


Fig 4-2. Mercator Chart of the World (*showing Scale Error increasing with Latitude*)

(0411) f. **The 'Flat Earth' Concept.** Over a limited *Area* (12 miles radius from a point) the Earth may be assumed to be flat for all practical purposes, as the errors introduced by this assumption are less than those resulting from the measurement of angles and distances. A simple sketch survey plan over a small *Area* may be constructed on *Flat Earth* principle by transferring observed ranges and bearings directly to a sheet of squared paper (see Chapter 18). However, at a distance of 50 miles from a point, the errors introduced by assuming a *Flat Earth* become navigationally and operationally significant; they increase rapidly with distance (see Para 0333).

0412. Projection Graticules and Grids

a. **Graticules.** A *Graticule* is the network of lines representing the *Parallels of Latitude* and *Meridians of Longitude* in a *Projection*.

b. **Grids.** A *Grid* is a reference system of rectangular *Cartesian Coordinates* obtained when a *Projection* is applied either to the whole world or a part of it. *Grids* are described in detail at Paras 0450-0453.

0413. Spherical Projections

If the Earth were considered to be *Spherical*, *Projections* could be created by wrapping a plane surface around an imaginary *Sphere*, switching a light on at its centre (or other position) and projecting features from the Earth's surface onto the plane. Although the Earth is *Spheroidal* and different methods are used in practice (see Para 0414 overleaf), the above analogy is adequate to explain the concepts and basic properties of the *Projections* in the examples below.

a. **Tangent and Secant Projections.** Using the above analogy, Fig 4-3a (opposite) shows six common examples of fitting plane surfaces around an imaginary *Sphere*.

- **Tangent Projections.** In the left column of Fig 4-3a (Examples 1, 3 and 5) the surfaces touch the *Sphere* along a circle or at a point. This type are known as '*Tangent Projections'*.
- **Secant Projections.** In the right column of Fig 4-3a (Examples 2, 4 and 6) the plane surfaces have been sunk into the *Sphere*; in Examples 2 and 4 they cut the *Sphere* twice and in Example 6 once. This type are known as '*Secant Projections*.

b. **Areas of Accuracy and Distortion.** If the detail on the *Spheres* at Fig 4-3a (opposite) are now projected on to a plane surface from a point on the axis of the cone, cylinder or plane circle, there will be no distortion of *Scale* along the lines where contact is made with the *Sphere* (shown by bold, grey-filled dashed lines). Elsewhere there is distortion of some sort or another, which will persist when the planes are unwrapped and laid flat.

c. **Points of Projection.** In Examples 1, 2, 3 and 4 of Fig 4-3a, the point from which the *Projection* takes place is usually the centre of the *Sphere*. With Examples 5 and 6 it may take place from anywhere on an axis at right angles to the plane, but usually from either the centre *B* of the *Sphere*, or from the opposite *Pole A*. The *Projections* at Fig 4-3a are usually referred to as follows:

- **Example 1:** '*Conical* with one *Standard Parallel*'.
- **Example 2:** '*Conical* with two *Standard Parallels*.
- **Example 3:** '*Cylindrical* with one *Standard Parallel* at the *Equator*'.
- **Example 4:** '*Cylindrical* with two *Standard Parallels*.
- **Example 5:** '*Zenithal'* projected from *A* – '*Stereographic*'.
- **Example 6:** '*Zenithal*' projected from *B* – '*Gnomonic*'.

d. **Orientation of Cones, Cylinders and Plane Circles.** Although for simplicity the examples in Fig 4-3a are oriented 'North-Up', the cones, cylinders and plane circles could equally well be inclined at any angle to the Earth's axis, and in practice, this does occur.

(0413 continued)

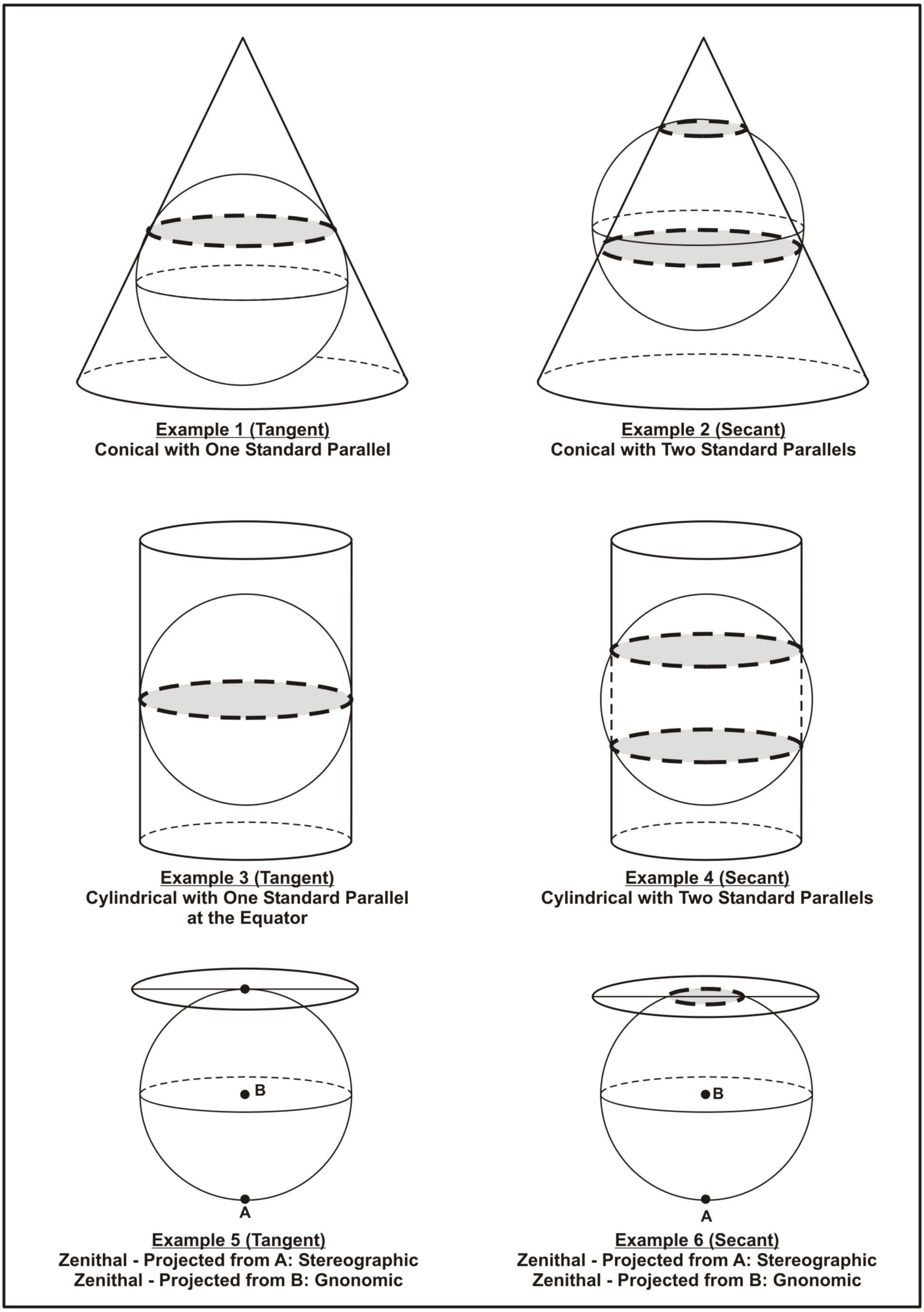


Fig 4-3a. Examples of Commonly Used Spherical Projections

0414. Spheroidal Projections

None of the '<u>perspective</u>' *Projection*s at Fig 4-3a (previous page) are *Orthomorphic* for the *Sphere* (except Examples 5 and 6 when *Projected* from *A*), and none at all are *Orthomorphic* for the Earth's *Spheroidal* shape. To overcome this, completely '<u>mathematical</u>' *Projections* have been devised, analogous to the perspective ones at Fig 4-3a, but with their formulae adjusted to ensure that some have *Orthomorphic* and others '*Equal Area*' properties, as required for their intended task. The main types of *Spheroidal Projection* for charts and *Grids* are explained in outline at Paras 0414a-g (opposite and overleaf) and summarised at Fig 4-3b (below). Details of mathematical constructions are at Sections 2-5 of this Chapter (Paras 0420-0450).

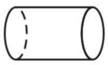
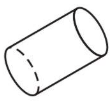
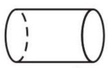
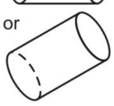
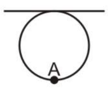
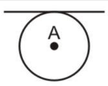
	PROJECTION	TYPE	ORTHOMORPHIC	GRIDS	USED FOR SURVEYS	USED FOR CHARTS	REMARKS
A	Lamberts Conical Orthomorphic		Yes	Very many, with one or two Standard Parallels	Yes	Yes, but not many	Used for aeronautical charts. Unsuitable at high Latitudes, where Modified Lambert Conformal is used instead. Superseded by UTM for land mapping.
B	Mercator		Yes	Not often used, Grids can be made from Meriodinal navigation tables.	Yes, on small scales	Yes	Used for most small scale Admiralty charts.
C	Transverse Mercator		Yes	—	Yes	Yes	Used for large-scale charts.
				National Grid of Great Britain	Yes	No	Based on Airy's Spheroid.
				UTM (Universal Transverse Mercator)	Yes	No	Worldwide between 84°N and 80°S
				Many Others	Yes	No	
D	Skew Orthomorphic		Yes	Malaysia, Borneo and Madagascar	Yes	No	Used mainly for land surveys and designed to keep scale errors small over the area covered by the grids
E	Inverse (Oblique) Mercator	 or 	Yes	Not often used	No	Yes	Excellent for air charts of Polar areas. A very close relation of the Transverse and Skew Orthomorphic.
F	Stereographic		Yes	Universal Polar Stereographic.	Yes	Yes (Polar Charts)	Good for surveys between the Pole and 80°. Not so good for small scale charts.
G	Gnomonic		No	Not used	No	Yes	Used for small scale Great Circle charts
H	Polyconic		No	Polyconic Projection Tables.	Yes	Yes	The so-called 'Gnomonic' of old large scale UKHO plans. Largely superseded by Transverse Mercator.

Fig 4-3b. Commonly Used Projections and Grids

(0414) a. **Lambert's Conical Orthomorphic Projection.** *'Lambert's Conical Orthomorphic Projection*' (see Fig 4-3b, Line A) is a modification of the *Conical Projection* with one or two *Standard Parallels* (see Fig 4-3a, Examples 1 and 2).

- **Orthomorphism.** The *Parallels* (of *Latitude*) other than the *Standard Parallels* appear as circular arcs, concentric with the *Standard Parallels*, but distances between them are chosen so that the *Projection* is *Orthomorphic*.
- **Scale Manipulation.** To achieve *Orthomorphic* properties, the *Scale* along the *Meridian* at any place must be equal to the *Scale* along the *Parallel* at that place. The *Scale* along the *Meridian* cannot now be uniform but must be adjusted to the *Scale* along the *Parallels*. The *Scale* is correct only along the *Standard Parallels*; if there are two of these, the *Scale* is smaller between them and it becomes increasingly large outside. The *Latitude* covered by the *Projection* is limited so that the *Scale* error does not become unacceptable.
- **Great Circles.** *Great Circles* are very nearly straight lines on this *Projection*.
- **Uses.** This *Projection* is widely used for aeronautical charts. It cannot be used in high *Latitudes* where *Modified Lambert's Conformal* is used instead. It can also be used for mapping countries with a large extent in *Longitude* but not much in *Latitude*, but has largely been superseded for this purpose by the *Universal Transverse Mercator Projection* (see Fig 4-3b / Para 0414c).
- **Derivatives.** The *Mercator Projection* is a derivative of *Lambert's Conical Orthomorphic Projection.*

b. **Mercator Projection.** The *Mercator Projection* (see Fig 4-3b, Line B) is described in detail at Paras 0420-0425, but a brief summary is as follows.

- **Scale Expansion.** *Scale* units are minutes of *Longitude* measured along the *Equator*. The *Scale* expands as *Latitude* increases (see Fig 4-2).
- **Meridional Parts.** The *(Cartesian Coordinate) Grid* is not normally shown, although accurate calculations are generally carried out in terms of *Meridional Parts* which form the unit of the *Grid.*
- **Properties.** The *Mercator Projection* is *Orthomorphic* and is a special case of *Lambert's Conical Orthomorphic Projection* in which the *Equator* is used as the *Standard Parallel*. Its properties are:
  - *Rhumb Lines* on the Earth appear as straight lines on the chart.
  - Angles between *Rhumb Lines* are the same on the Earth's surface / chart.
  - The *Equator* (a *Rhumb Line* & *Great Circle*) appears as a straight line.
  - *Parallels of Latitude* appear as straight lines parallel to the *Equator*.
  - *Meridians* appear as straight lines perpendicular to the *Equator*; use of a *Meridional Grid* to plot Eastings / Northings is unnecessary.
  - A straight line on the chart joining two points does NOT represent the shortest distance between them, unless it is also *Great Circle*. A *Great Circle* which is not a *Meridian* or the *Equator* will appear as a curve.
  - Increased spacing of adjacent *Parallels* away from the *Equator* leads to *Scale* magnification which increases with *Latitude* (see Fig 4-7).
  - The *Shape* of charted features is correctly drawn for <u>small</u> *Areas* but those of <u>large</u> features are distorted as *Latitude* increases.

(0414b continued)

- **Uses.** The *Mercator Projection* is extensively used for standard *UKHO* nautical charts at *Scales* smaller than 1:50,000, up to *Latitudes* 80˚N / 80˚S. However, for large *Scale* nautical charts (1:50,000 and greater), the *Transverse Mercator Projection* is generally used instead (see Paras 0430-0431).

- **Shape on a Mercator Chart - Scale Error Increasing with Latitude.** On the Earth, *Meridians* converge (see Fig 4-4 - left diagram), so land masses on a *Mercator Projection* chart will be increasingly distorted in an east-west direction proportional to their distance from the *Equator*, until at the *Poles* their sizes would be infinite. In order to preserve the correct *Shape*, the *Parallels of Latitude*, which are equally spaced on the Earth's surface (see Fig 4-4 - right diagram) must be increasingly spaced towards the *Poles* on the *Mercator Projection* chart until at the *Poles* the *Latitude Scale* is infinite. This distortion is governed by the secant of the *Latitude*. Thus, on a *Mercator Projection* chart of the world (see Fig 4-2 at Para 0411), Greenland appears as broad as Africa at the *Equator*, although the latter is three times wider; this fact becomes apparent once the distance is measured at the *Latitude Scale* in the vicinity of the two *Areas*.

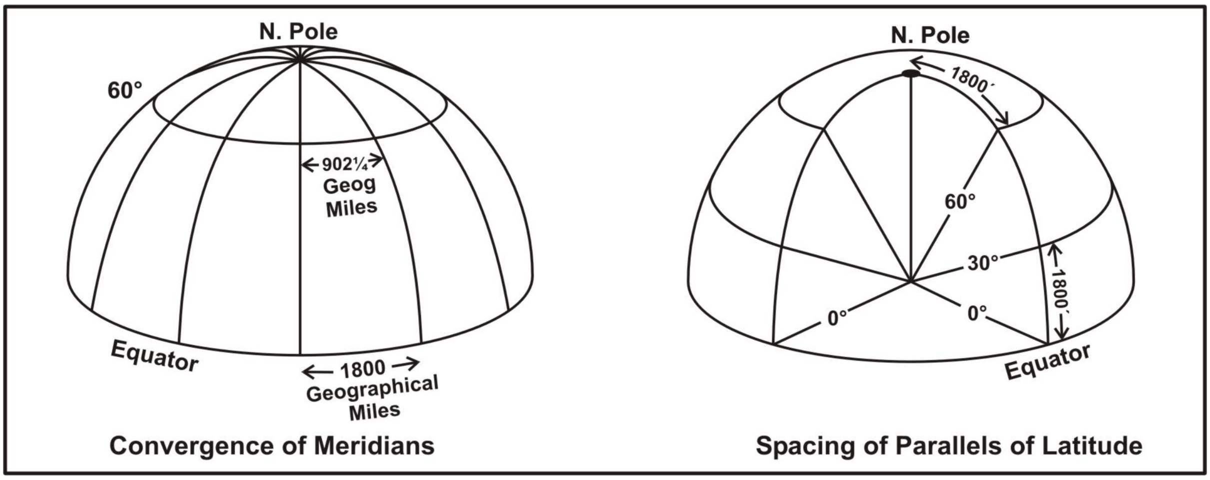


Fig 4-4. Earth's Surface - Convergence of Meridians and Spacing of Parallels

- **Practical Measurement of Distance on Mercator Projection Charts.** For the reasons stated above, distances should ALWAYS be measured using the *Latitude Scale* at the *Latitude* of the position concerned. The *Longitude Scale* must NEVER be used for measuring distances on a *Mercator Projection* chart.

- **Use of the Rhumb Line as Course.** If two places on the Earth's surface are joined by a *Rhumb Line* and a ship steers along that line, the direction of the ship's head will remain the same throughout the passage (see Para 0202a). This direction is measured by the angle from the *Meridian* to the *Rhumb Line*, measured clockwise from 0˚ to 360˚, and is the *Course*. The *Rhumb Line* itself is often called the *Course*. On the Earth's surface, a continuous *Rhumb Line* (except 090˚/270˚) will spiral towards the *Pole*. The *Mercator Projection* chart shows the ship's track as a straight line between the starting-point and destination; the measurement of this straight line gives the steady *Course*.

(0414b continued)

- **Plotting of Great Circle Tracks.** A *Great Circle* which is not a *Meridian* or the *Equator* will appear as a curve on a *Mercator Projection* chart (see Fig 4-5 below). Thus *Great Circle* tracks are normally plotted on a *Gnonomic* chart and the *Waypoints* transferred to a *Mercator Projection* chart.

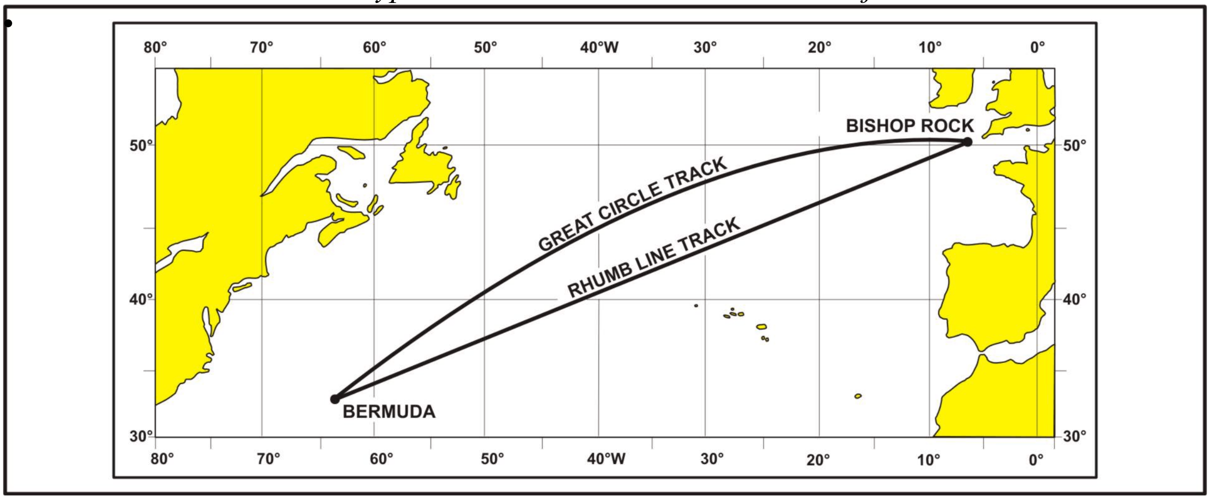


Fig 4-5. Mercator Projection of North Atlantic - Rhumb Line / Great Circle Tracks

(0414) c. **Transverse Mercator Projection.** The *Transverse Mercator Projection* (see Fig 4-3b, Line C) is described in detail at Paras 0430-0431, but a brief summary is as follows.

- **Construction.** The *Transverse Mercator Projection* may be considered as a standard *Mercator Projection* (see Fig 4-3a, Example 3) with the 'cylinder' turned through 90˚. The *Central Meridian* and the *Equator* plot as straight lines; all other *Meridians* and *Parallels* plot as curves.

- **Properties.** The *Transverse Mercator Projection* is *Orthomorphic*.
  - **Scale Expansion and Direction.** The *Scale* between *Meridians* expands as *Longitude* increases away from the *Central Meridian.* However, due to the large *Scale* used, these *Meridians* will appear as straight lines to the user, and for all practical purposes, straight lines can be used to plot all *Bearings* and direction lines on the chart.
  - **Central Meridians.** To minimise the effects of *Scale* expansion, coverage of a *Transverse Mercator Projection* chart is restricted to ±3˚ of *Longitude* from the *Central Meridian.*
  - **High Latitudes.** The *Transverse Mercator Projection* does not suffer from the high *Latitude* difficulties of standard *Mercator Projections*.

- **Uses.** The *Transverse Mercator Projection* is used for:
  - Most *UKHO* large *Scale* charts of 1:50,000 or larger (ie covering a small *Area*) as well as for land maps.
  - Most land maps (including UK Ordnance Survey maps and NATO military maps).
  - *Polar* charts and maps, although the *Polar Stereographic Projection* (see Para 0414d overleaf) is more commonly used for this purpose.

(0414) d. **Stereographic Projection (Universal Polar Stereographic Projection).** The point of origin of a *Stereographic Projection* (see Fig 4-3b, Line F) may be anywhere; however, as this projection is normally only used in *Polar* regions, only the *Universal Polar Stereographic Projection* is considered here.

- **Construction.** For the *Universal Polar Stereographic Projection,* the *Meridians* and *Parallels of Latitude* are *Projected* on to a plane tangential to the *Pole*, the centre of *Projection* being the opposite *Pole* (see Fig 4-3a, Example 5). *Meridians* appear as straight lines originating from the *Pole*, *Parallels of Latitude* appear as concentric circles radiating outwards from and centred on the *Pole*.

- **Orthomorphic Properties.** The *Universal Polar Stereographic Projection* is *Orthomorphic*.

- **Great Circles.** *Great Circles* (except *Meridians*) are not projected as straight lines, although in practice, little accuracy is lost by plotting them as such.

- **Uses.** The *Universal Polar Stereographic Projection* is used for Polar charts and *Orthomorphic* maps of *Polar* regions.

e. **Gnomonic Projection.** The *Gnomonic Projection* (see Fig 4-3b, Line G) is described in detail at Paras 0440-0442, but a brief summary is as follows.

- **Construction.** The *Gnomonic Projection* projects the Earth's surface from the Earth's centre onto a tangent plane. The *Gnomonic Projection* is only applied to a *Sphere* which represents the Earth.

- **Properties.** The *Gnomonic Projection* is NOT *Orthomorphic* and has the following properties:
  - **Great Circles.** *Great Circles* are represented by straight lines on this *Projection*.
  - **Parallels of Latitude.** *Parallels of Latitude* are curves.
  - **Meridians.** The *Meridians* will not be parallel unless the tangent point is on the Equator.
  - **Rhumb Lines.** *Rhumb Lines* will be shown curves, not as straight lines.
  - **Tangent Point and Distortion.** Angles are also distorted, except at the tangent point. The farther a point on the chart is away from the tangent point, the greater will be the distortion. It is therefore impossible to take courses and distances from a *Gnomonic Projection* chart.
  - **Equal Areas.** The *Gnomonic Projection* does NOT have *Equal Area* properties.

- **Uses.** The distortion of the *Gnomonic Projection Graticule*, which gives neither *Orthomorphic* nor *Equal Area* properties, makes it quite unsuitable for general *Navigation* purposes. Its usage is limited entirely to plotting *Great Circles* as straight lines, usually in order to obtain *Great Circle Waypoints*. It was also used historically to plot long range radio beacon *Bearings*, although this usage has almost completely disappeared.

f. **Skew Orthomorphic Projection.** Instead of a *Central Meridian*, a central *Great Circle* passing through the axis of the country is used as the line of contact for the *Skew Orthomorphic Projection* (see Fig 4-3b, Line D). It is used mainly for land surveys.

(0414) g. **Polyconic Projection.** The *Polyconic Projection* (see Fig 4-3b, Line H) is a modification of the simple *Conical Projection*.

- **Construction.** The *Central Meridian* of the *Area* to be displayed is divided correctly for intervals of *Latitude*, but each *Parallel* is constructed as if it were the *Standard Parallel* of a simple *Conical Projection* (see Fig 4-6 below). The *Parallels* are arcs of circles, the radii of which steadily increase as the *Latitude* decreases. The *Central Meridian* is a straight line, although the other *Meridians* are curved.
- **Orthomorphic and Equal Area Properties.** The *Polyconic Projection* is neither *Orthomorphic* nor *Equal Area*, so it is unsuitable for large *Areas*.
- **Advantages.** The main advantage *Polyconic Projection* is that if small *Areas* are shown on this *Projection*, with each small *Area* covering the same amount of *Longitude*, the sheets on which the geographical *Graticules* are drawn fit exactly along their northern and southern edges. For ordinary purposes, the sheets also fit along their eastern and western edges, although the join here is a 'rolling fit' as the *Meridians* are curved (see Fig 4-6 below).
- **Modified Form.** In slightly modified form, the *Meridians* can be made to project as straight lines, and this modified form was used in practice.
- **Uses.** The modified *Polyconic Projection* is suitable for topographical maps which, individually covering a small *Area*, combine to cover a large one. However, it has been superseded by the *Transverse Mercator Projection*. A few old *UKHO* harbour plan charts may still exist with the legend '*Gnomonic Projection*', but they almost certainly used the modified *Polyconic Projection*.

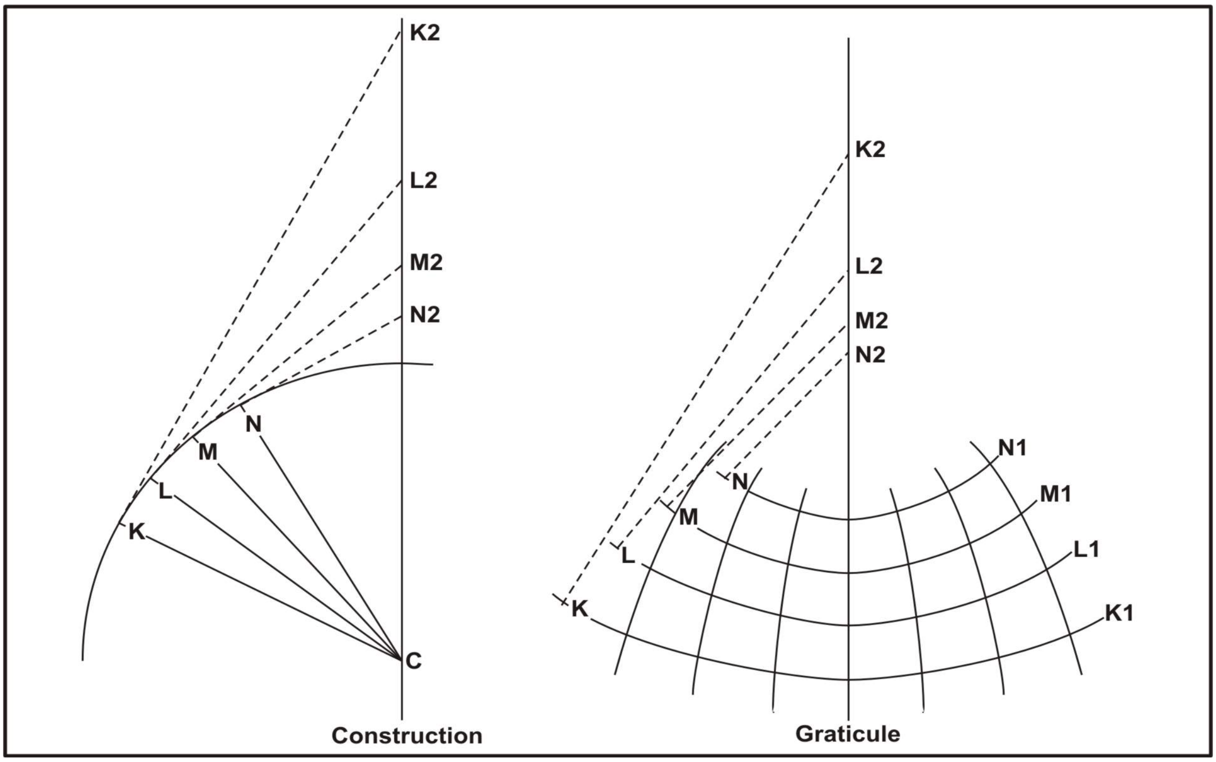


Fig 4-6. Polyconic Projection

Paras 0415-0419. Spare

SECTION 2 - MERCATOR PROJECTION FOR CHARTS

0420. Principles of the Mercator Projection

An introduction to the *Mercator Projection* is at Para 0414b. It is a '*Cylindrical Orthomorphic Projection*' and in practice, it is a 'mathematical' rather than a 'perspective' projection.

a. **Mercator Projection - 'Perspective' Concept and 'Mathematical' Reality.** The 'perspective' concept of the *Mercator Projection* is to imagine a cylinder of paper wrapped around the Earth, with the cylinder's and Earth's axes coincident, and their surfaces in contact only at the *Equator*; the Earth's surface is then projected from the Earth's centre onto the cylinder (see Fig 4-5a below). When unrolled flat, the image on the paper cylinder will represent a *Mercator Projection.* In reality, the 'perspective' method is not used; instead the *Mercator Projection* is produced by 'mathematical' formulae which are adjusted to ensure that the resulting charts are *Orthomorphic*. Different formulae apply for the *Sphere* (see Para 0422) and *Spheroid* (see Para 0531a).

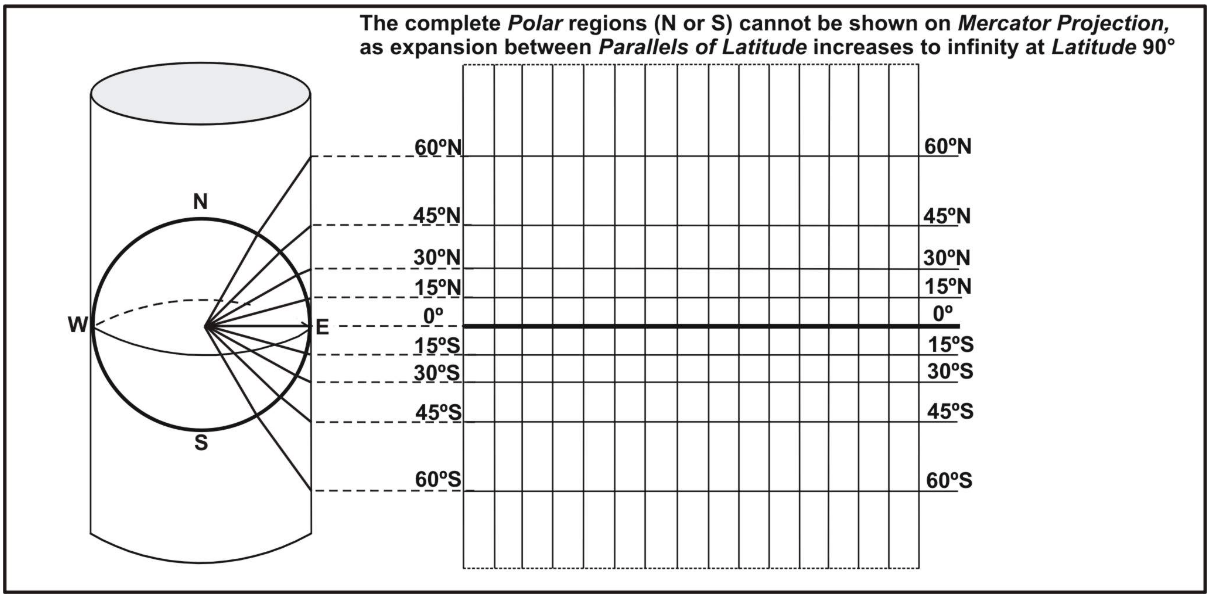


Fig 4-7. Simplified Diagram of Mercator Projection

b. **Properties of Mercator Projection.** The properties of the (*Orthomorphic*) *Mercator Projection* are listed at Para 0414b:

c. **Uses.** The uses of *Mercator Projection* are listed at Para 0414b.

d. **History of the Mercator Projection.** The idea of the *Mercator Projection* belongs to Gerhard Kremer, a Fleming who adopted the name Mercator. Kremer used the *Graticule* derived from the *Projection* in the world map which he published in 1569. However, the *Graticule* was inaccurately drawn above the *Parallels* of 40˚, and there was no mathematical explanation of the *Projection*. In 1599, a mathematical explanation was established by Wright who correctly calculated the positions of the *Parallels* and published the results. The *Mercator Projection* chart came into general use among navigators in about 1630, but the first complete mathematical description of it was not available until 1645, when Bond published the logarithmic formula for it.

0421. Mathematical Analysis of the Mercator Projection

a. **Mercator Projection - Concept.** The *Mercator Projection* is introduced at Para 0414b and is “a special case of the *Lambert Conical Orthomorphic Projection* in which the *Equator* is used as the *Standard Parallel* ”. Para 0420 and Fig 4-7 opposite) provide an outline of the principles of the *Mercator Projection*.

b. **Mercator Projection - Governing Criteria.** The requirements for the *Mercator Projection* are governed by two criteria:

- In a cylindrical *Projection* based on the *Lambert Conical Orthomorphic Projection,* the *Constant of the Cone* must be zero (see Para 0421d).
- The *Projection* must retain *Orthomorphic* properties (see Para 0421e).

c. **Mercator Projection - Detailed Mathematical Analysis.** Amplifying Para 0420 and Fig 4-7 (opposite) in more detail, Fig 4-8 (overleaf) takes the *Equator* as the *Latitude* of the origin ϕ_0 .

- *RO* is a *Central Meridian* and is equal in length to $V_o \cot \phi$, where V_o is the *Radius of Curvature* at right angles to the *Meridian* at *O* for the *Spheroid* of the Earth in use, and ϕ is the *Latitude* of *O*.
- As the cotangent of 0˚ is infinity, *R* recedes northwards (or southwards) to infinity (see Fig 4-8 overleaf); this can also be seen from Fig 4-7 (opposite) if *Latitude* is extrapolated to 90˚. Thus the complete *Polar* regions can never be shown on *Mercator Projection* charts as expansion between *Parallels of Latitude* expands to infinity at *Latitude* 90˚.
- The angle between *True North* and *Grid North* becomes zero for this *Projection*, thus there is no convergence.
- OP_o coincides with *Grid* East, all *Parallels* become straight lines parallel to OP_o and, since *Convergence* is zero, all the *Meridians* are parallel *Grid North*.
- One minute of *Longitude* measured along the *Equator* (or *Standard Parallel*) as the unit of the *Grid* makes this *Projection* suitable for navigation use.

d. **Constant of the Cone.** The quantity *Sin* ϕ_0 is the *Constant of the Cone* (where ϕ is the *Latitude* of O) and it is a constant for any given *Latitude* of the point of origin. When the *Equator* is the point of origin:

$$\sin \phi_0 = \sin 0^\circ = 0$$

e. **Orthomorphism and Rhumb Lines.** The *Scale* along a *Meridian* in the neighbourhood of a point in *Latitude* ϕ is stretched by the same amount ($\sec \phi$) as the *Scale* along the *Parallel* through that point.

- **Orthomorphism.** In view of this, and that the *Meridians* and *Parallels* on the *Mercator Projection* are at right angles, the *Projection* must be *Orthomorphic*.
- **Rhumb Lines.** The *Parallels* are spaced at increasing intervals as they approach the *Poles* (see Fig 4-7 opposite), while the *Meridian*s are spaced equally. Thus any straight lines (including diagonals) make a constant angle with the *Meridian*s with no distortion; they are thus *Rhumb Lines*.

(0421c continued)

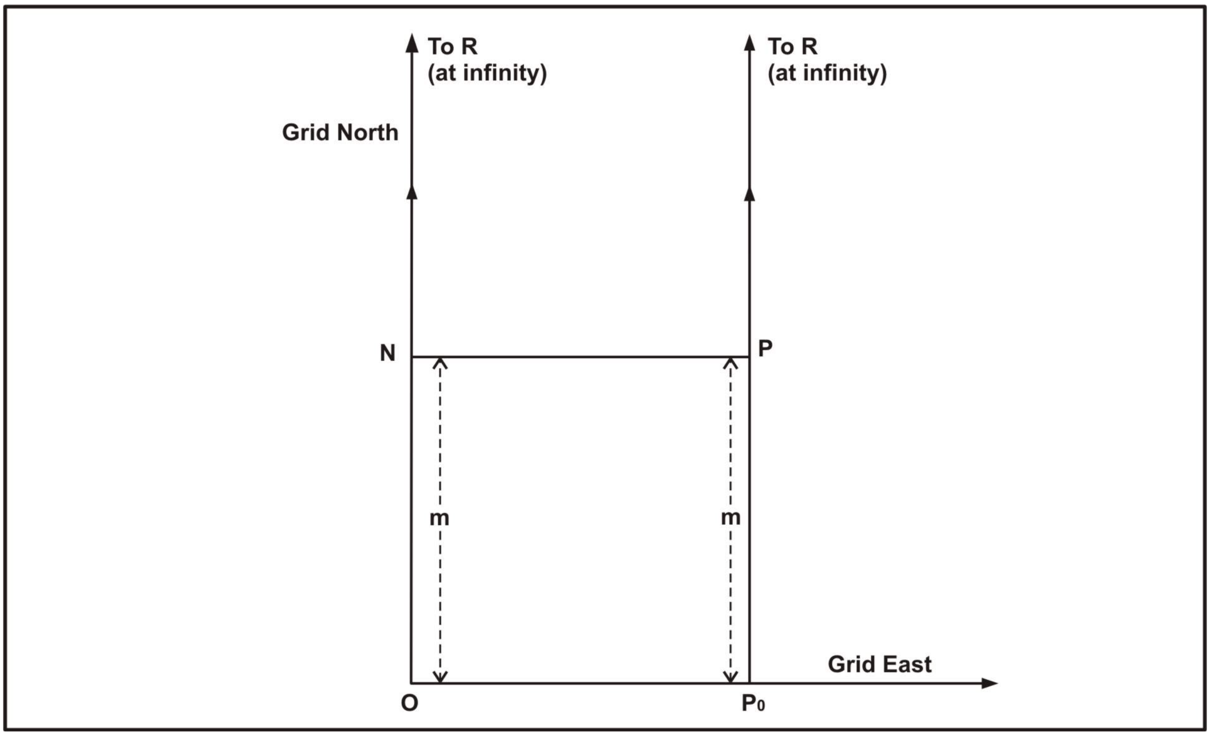


Fig 4-8. Detailed Analysis of Mercator Projection

(0421) f. **Use of Longitude Scale When Constructing a Mercator Chart.** As *Meridians* of the *Mercator Graticule* are straight lines at right angles to the *Equator*, the *Longitude Scale* is the same everywhere and provides a means of measurement in the *Graticule*.

- Let the *Scale* of any *Mercator Projection* chart be *x* millimetres to 1' of *d.long*.
- *Departure* ≃ *d.long* cos *Lat* (see Note 4-1 below), thus *Departure* on the chart represented by '*x* millimetres' approximates to *1'* cos *Lat* (ie one *Sea Mile* in that particular *Latitude* is represented by *x* sec *Lat* millimetres on the chart).
- **Therefore, the *Scale* of *Latitude* and distance at any part of a *Mercator Projection* chart is proportional to the secant of the *Latitude* of that part; thus the amount of distortion in any *Latitude* is also governed by the secant of that *Latitude* (see Example 4-1 below)**.
- The *Latitude Scale* cannot be used to draw the *Parallel* in its correct position on the *Graticule* because it is continually being stretched as the *Latitude* increases. **Thus for this purpose, the spacing of any *Parallel* from the *Equator* must be calculated in units of the *Longitude Scale*.**

Example 4-1. On a *Mercator Projection* chart (see Fig 4-2) , Greenland (70˚N) appears to have the same width as Africa (0˚), although Africa is in reality three times as wide as Greenland. This is not unexpected, as: *sec 70˚ ≃ 3.*

Note 4-1. *This formula is correct for the Sphere but only approximates for the Spheroid. The precise length of one minute of Longitude is given by formula (3.12) at Para 0314.*

(0421) g. **Measurement on the Chart.** *Mercator Projection* charts are graduated along the sides for *Latitude* and along the top and bottom for *Longitude*.

- **Latitude Scale and Distance.** The length of the *Rhumb Line FABCT* is the distance between them (see Fig 4-9); FF_1, AA_1, BB_1 etc are *Parallels* of *Latitude*. The distance between any two points must be measured on the *Latitude Scale* between their *Parallels* of *Latitude* (ie *FA* between F_1 and A_1, *AB* between A_1 and B_1 etc). If *FT* is less than 100', no appreciable error is made by measuring it on the *Scale* roughly either side of its middle point.
- **Longitude Scale.** The *Longitude Scale* is used only for laying down or taking off the *Longitude* of a place, never for measuring distance.

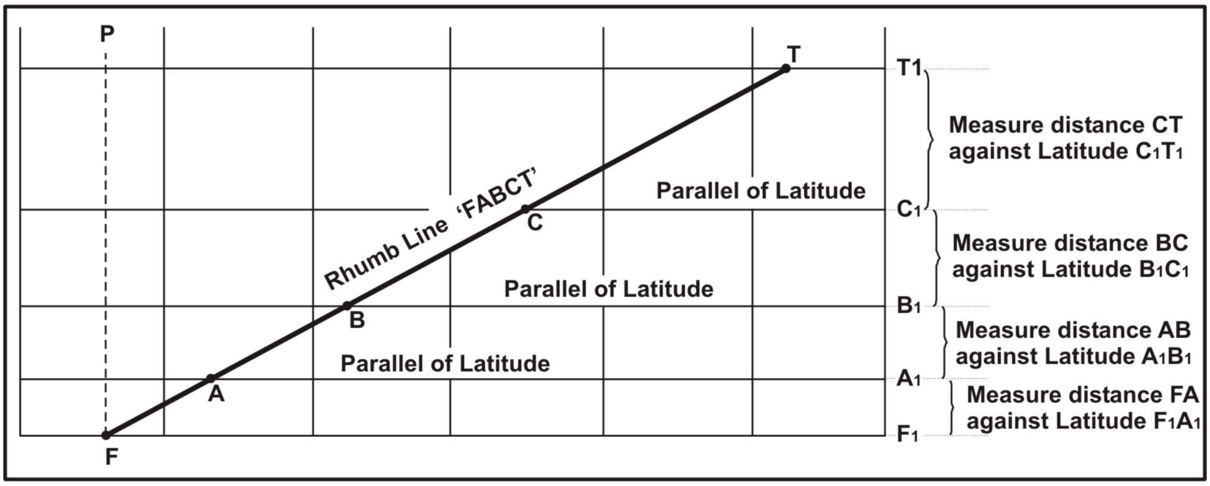


Fig 4-9. Measurement of Distance on a Mercator Chart

0422. Meridional Parts of a Mercator Chart

a. **Latitude and Distance Scale Distortion.** Para 0414b / Fig 4-4 and Para 0420a / Fig 4-7 established that the *Latitude* (distance) *Scale* of a *Mercator Projection* chart continually increases as it recedes from the *Equator*, until at the *Pole* it becomes infinite; thus the complete *Polar* regions cannot be shown on a *Mercator Projection* chart. Para 0421f (opposite) established that the *Latitude* (distance) *Scale* at any part of a *Mercator* chart is proportional to the secant of the *Latitude* of that part.

b. **Meridional Parts.** The *Latitude Scale* of a *Mercator Projection* chart affords no ready comparison with the fixed *Longitude Scale*. In Fig 4-9 (above), the tangent of the course-angle *PFT* cannot be *PT* (measured on the fixed *Longitude Scale*) divided by *FP* (measured on the *Latitude Scale*), because the units of *Latitude* measurement are continually changing. For the ratio *PT / FP* to be valid, *PT* and *FP* must be measured in the same fixed units. The length of 1 minute of arc of the fixed *Longitude Scale* provides this unit of measurement and is called a '*Meridional Part*'. Thus:

The Meridional Parts of any Latitude are the number of 'Longitude Units' in the length of a Meridian between the Parallel of that Latitude and the Equator. A 'Longitude Unit' is the length on the chart representing one minute of arc in Longitude.

Meridional Parts are lengths measured on the *Mercator Projection* chart (usually called '*Chart Lengths*') and should NOT be confused with distance on the Earth's surface, which is expressed in *Sea Miles* or *Nautical Miles* (see Para 0422c / Example 4-2).

(0422) c. **Number of Meridional Parts for Any Latitude.** The number of *Meridional Parts* for any *Latitude* may be found from formula (4.1) for the *Sphere* (see Para 0422e below) and formulae (5.21a/b) for the *Spheroid* (see Para 0531a). They are also tabulated in Norie's Nautical Tables (NP 320) which uses the Clarke 1880 *Spheroid* ('compression' of 1/293.465 [ie based on '*1/Flattening*' = 293.465 at Para 0322 Table 3-1]). Example 4-2 shows their use in calculating the length of the *Meridian* on a *Mercator Projection* chart (ie the *Chart Length*) between a *Parallel of Latitude* and the *Equator.*

Example 4-2. If the *Longitude Scale* on the *Mercator Projection* chart is 1 degree (ie 60 *Meridional Parts*) to 10 mm, what is the length of the *Meridian* between the *Parallel* of 45˚ 00' N and the *Equator*, when measured on the chart in millimetres?

From NP 320, for 45˚ 00' there are 3013.38 *Meridional Parts*. Thus the distance on the chart (ie the Chart Length) for the *Meridian* is 10 x 3013.38 / 60 = 502.23mm.

A simplistic and incorrect calculation of 45 x 10 = 450mm is entirely wrong.

d. **Difference of Meridional Parts (DMP).** Where the two positions are both remote from the *Equator* (eg *A* and *K* in Fig 4-10a, and *a* and *k* in Fig 4-10b opposite), their relative position may be determined by the difference between the individual *Meridional Parts* for *K* and the individual *Meridional Parts* for *A*, which gives the number of *Longitude Units* in the length of a *Meridian* between the two *Parallels of Latitude* through *A* and *K*. **This length *mk* (see Fig 4-10b opposite) is usually referred to as the '*Difference of Meridional Parts*' and written as '*DMP*'.** See also Para 0510-0511.

e. **To Find the Meridional Parts of any Latitude on a Sphere.** Fig 4-10a (opposite) represents a part of the Earth's *Spherical* surface (see Note 4-2 overleaf for *Spheroidal* data). Position *F* is a point on the *Equator* and *FT* is the *Rhumb Line* joining it to position *T*. Fig 4-10b (opposite) shows this same *Rhumb Line* as the straight line *ft* on a *Mercator Projection* chart.

- If *TQ* is now divided into *n* small lengths α, so that ($n\alpha$) is equal to the *Latitude* of *T*, the arcs of *Parallels of Latitude* drawn through the points of division are equally spaced and, with the *Meridians*, form a series of small triangles *FAX*, *ABY*, ... If α is so small that these triangles may be considered plane, they are equal in **all** respects, since:

$$\begin{aligned} FX &= AY = \ldots = \alpha \\ FXA &= AYB = \ldots = \text{one right angle} \\ XFA &= YAB = \ldots = \text{the course} \\ \therefore AX &= BY = \ldots \end{aligned}$$

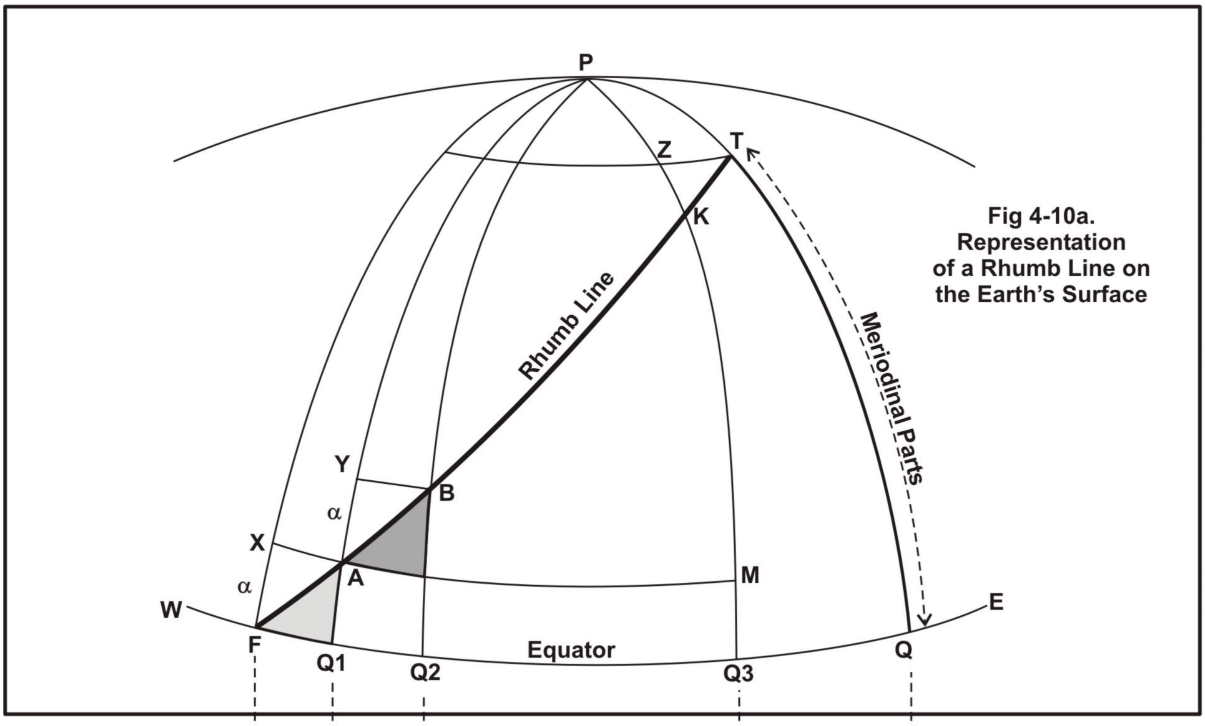
- Since these small arcs recede in size in succession from the *Equator*, the *Meridians* which bound them are spaced successively farther apart. Hence:

$$FQ_1 < Q_1Q_2 < \ldots$$

- A comparison of Figs 4-10a and 4-10b shows that the small triangles on the Earth are all 'equal'(ie equal angles and linear dimensions), but when drawn on the chart, they are only 'similar'(ie equal angles only).

- The triangles in Fig 4-10b progressively increase in size as they recede from the *Equator*, and this is emphasised by progressively darker shading. This increase in size can be found by considering two similar and corresponding triangles.

(0422e continued)



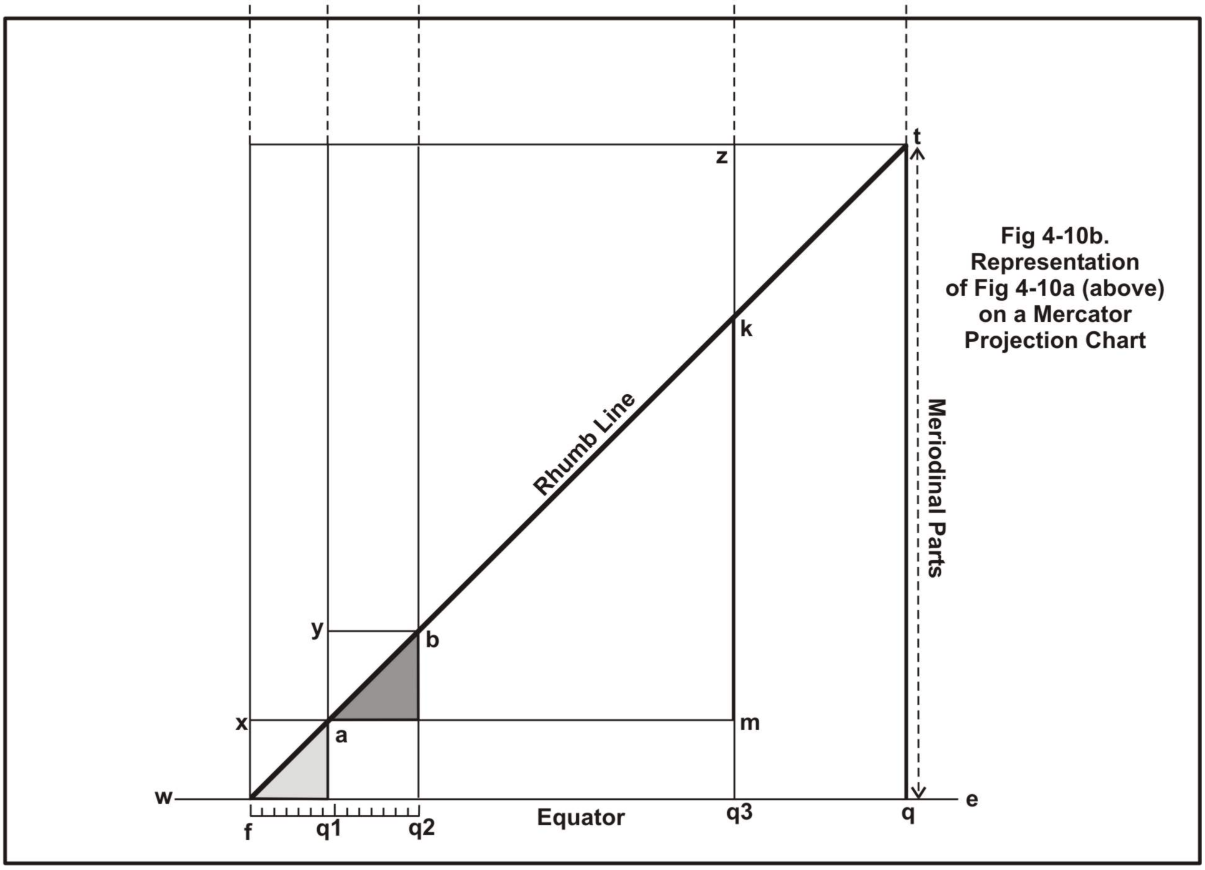


Fig 4-10a (top figure). Representation of a Rhumb Line / Mer Parts on the Earth's Surface

Fig 4-10b (bottom figure). Representation of Fig 4-10a on a Mercator Projection Chart

(0422e cont) Thus:

$$\frac{fx}{FX} = \frac{ax}{AX} = \frac{FQ_1}{AX} = sec\ \ Lat\ A$$

$$fx = FX\,sec\ Lat\ A$$

$$= \alpha\,sec\ \alpha$$

- Similarly, by considering the triangles *ABY* and *aby*:

$$ay = \alpha\,sec\ 2\ \alpha$$

- But *qt*, the length of the *Meridian* between the *Parallel of Latitude* through *t* and the *Equator,* is the sum of all the elements *fx*, *ay* . . . *kz*. That is:

$$qt = \alpha\ (\sec\alpha + \sec 2\alpha + \sec 3\alpha + \ldots + \sec n\alpha)$$

$$qt = 7915.7045\,log_{10} tan\left(45° + \frac{T°}{2}\right) \qquad \textbf{. . . 4.1}$$

Note 4-2. *Formula (4.1) gives the number of Meridional Parts in the Latitude of T for a Sphere (see also Appendix 3). It is a simplified version of the equivalent Spheroidal formulae (5.21a/b) at Para 0531a, but with the corrections for Spheroidal Eccentricity 'e' ignored; the proof of formulae (5.21a/b) are at Appendix 5. As stated at Para 0422c, Norie's Nautical Tables (NP 320) gives the Meridional Parts for the Clarke 1880 Spheroid ('compression' of 1 / 293.465).*

0423. Constructing a (Small Scale) Mercator Chart of the World

a. **Calculating the Longitude Scale.** As there is no *Latitude* or distance distortion at the *Equator* on a *Mercator Projection* chart, the base on which the chart is constructed must be the line representing the *Equator*, and convenience governs the length of this line. Suppose it is 720 mm (about 28 in). Then the *Longitude Scale* must be:

$$\frac{Length\ of\ Equator\ in\ degrees}{Length\ of\ base\ in\ millimetres} = \frac{360}{720} = \tfrac{1}{2}°\ \ Longitude\ per\ millimetre$$

½˚ of *Longitude* at the *Equator* equates to 30 *Meridional Parts*; more conveniently this can be expressed as 5˚ of *Longitude* or 300 *Meridional Parts* to 10 mm. Vertically the *Scale* will be the same, 300 *Meridional Parts* to 10 mm.

b. **Placing of Meridians.** If it is required (for example) to draw *Meridians* every 20˚ of *Longitude*, the *Equatorial* line must be divided into eighteen (20˚ x 18 = 360˚) equal parts, each of 40 mm (or 1200 *Meridional Parts*) in width. The perpendiculars drawn through the points of division will be the *Meridians*. For simplicity (see Note 4-3 below), the *Prime Meridian* (*Greenwich Meridian,* 0˚ *Longitude*) has been placed in the centre of Fig 4-11 (opposite); thus with a total coverage of 360˚ of *Longitude*, the extreme left-hand *Meridian* will be 180˚W and the extreme right-hand *Meridian* will be180˚E.

Note 4-3. *In practice, the Prime Meridian (0˚ Longitude) is normally placed off-centre to the left with some overlap (usually 40˚) of Longitude overall (ie 400˚ coverage), in order to show the Atlantic Ocean, Indian Ocean and Pacific Ocean in an unbroken view, with North America shown in full at one side of the chart and South America at the other (see example at Fig 4-2).*

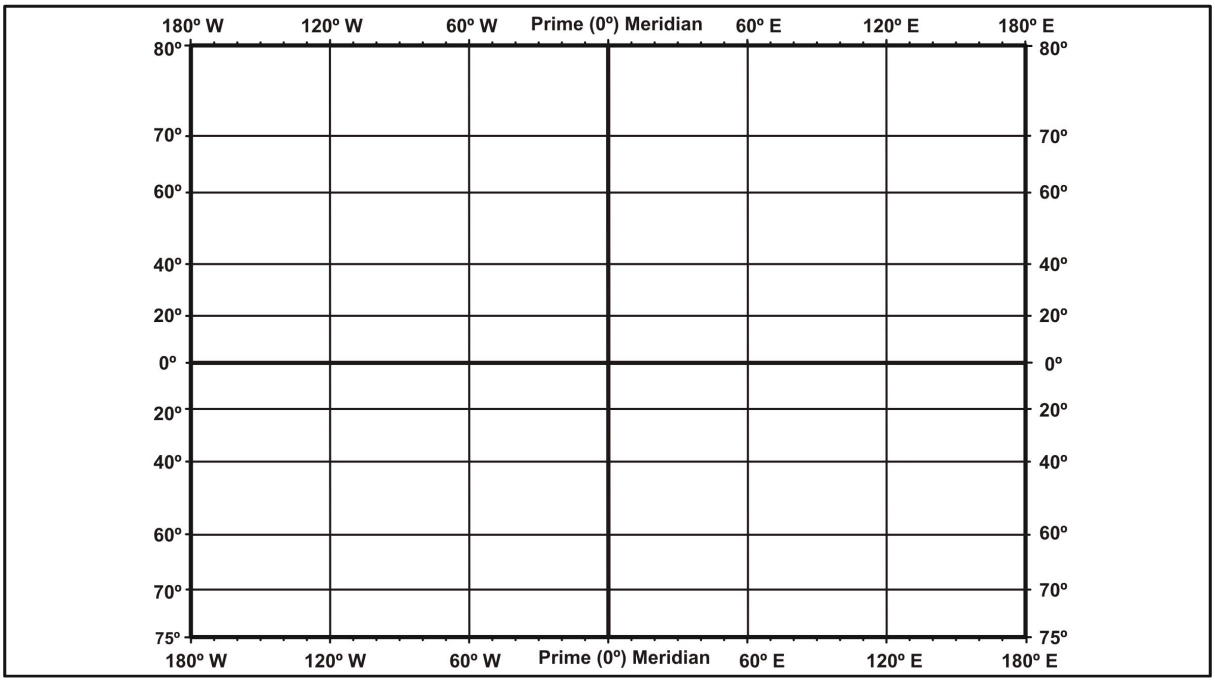


Fig 4-11. Graticule for a Mercator Chart of the World (Centred on the Prime Meridian)

(0423) c. **Placing of Parallels of Latitude.** *Meridional Parts* for deciding the positions of the *Parallels of Latitude* may be extracted from Norie's Nautical Tables (NP 320), although it uses the Clarke 1880 *Spheroid* (see Para 0422c).

- **Parallels of Latitude for 20˚.** From NP 320, there are 1217.14 *Meridional Parts* between the *Parallels of Latitude* for 20˚ and the *Equator*. At a *Scale* of 300 *Meridional Parts* to 10 mm, the *Parallels of Latitude* for 20˚ must be drawn on the chart 1217.14 ÷30 = 40.57mm either side of the *Equator*.
- **Parallels of Latitude for 40˚.** Similarly, there are 2607.64 *Meridional Parts* between the *Parallels of Latitude* for 20˚ and the *Equator* and so the *Parallels of Latitude* for 40˚ are drawn 2607.64 ÷ 30 = 86.92mm from the *Equator*.
- **Other Parallels of Latitude.** In the same way the other *Parallels of Latitude* are drawn. On the *Graticule* formed (see Fig 4-11 above), it is possible to insert the position of any place, if the *Latitude* and *Longitude* are known.

d. **Choice of Spheroids when Constructing a Mercator Projection Chart.**

- **Errors Induced by Using Norie's Tables - Clarke 1880 Spheroid.** The table of *Meridional Parts* in Norie's Nautical Tables (NP 320) uses the (Clarke 1880 *Spheroid*). This will produce a chart based on the Clarke 1880 *Spheroid* and induce an error when plotting positions referred to *WGS 84* (ie *GPS*) on the chart. However, on a very small *Scale* world-map such as this, for practical purposes of plotting, the induced error will be negligible.
- **Use of Spheroid Meridional Parts Formulae for WGS 84.** To construct a *Mercator Projection* chart based on *WGS 84*, the appropriate *Meridional Parts* for this *Spheroid* need to be calculated using general formula (5.21a) at Para 0531a, which is customised for *WGS 84* at formula (5.21b) at Para 0531a.

0424. Constructing a (Larger Scale) Mercator Chart of Part of the World

To show small portions of the Earth in detail, a larger *Scale* is needed with only the relevant portion of the chart being constructed.

a. **Equator Not Included on Chart.** If the *Equator* is not included on the chart, the *Chart Lengths* between successive *Parallels of Latitude* on the chart (in millimetres, appropriate to the *Scale* chosen) are found from the *Difference of Meridional Parts* (*DMP*) for the corresponding *Parallels of Latitude* (see Para 0422d).

b. **Construction.** To construct a *Mercator Projection Graticule* (see Fig 4-12 opposite) from 142˚E - 146˚E and 45˚N - 49˚N at a *Scale* of 1˚ of *Longitude* to 30mm:

- At this *Scale*, 1' *Longitude* equates to 30÷60 = 0.5mm.
- The difference between 142˚E - 146˚E is 4˚, and at a *Scale* of 1˚ of *Longitude* to 30mm, the base-line at the bottom of the chart representing the *Parallel of Latitude* 45˚N is thus 30 x 4 = 120 mm in width.
- The limiting *Meridians* of 142˚E and 146˚E will be perpendiculars erected on this line at its two ends. The *Meridians* of 143˚E, 144˚E, 145˚E will be spaced equally between the limiting *Meridians* (see Fig 4-12 opposite).
- The length in millimetres between the *Parallels of Latitude* of 45˚N to 49˚N can be established from the *DMPs* at Table 4-1 (below).

Table 4-1. 'Chart Lengths' (mm) between Parallels of Latitude of 45˚N to 49˚N

Latitude	Meriodinal Parts (*from Norie's Tables - See Note 4-4 opposite*)	DMP	'Chart Lengths' between Parallels (DMP x 0.5)
49˚N	3364.41	---	
48˚N	3274.13	90.28	45.14 mm
47˚N	3185.59	88.54	44.27 mm
46˚N	3098.70	86.89	43.45 mm
45˚N	3013.38	85.32	42.66 mm

c. **Intermediate Divisions.** To create intermediate divisions on the *Graticule*, for both *Longitude* and *Latitude Scales*, the *Chart Lengths* between *Meridians*, and those between *Parallels of Latitude* may be calculated for smaller units (eg 10' of *Longitude* between *Meridians*, and 10' of *Latitude* between *Parallels*). This division is easily effected on the *Longitude Scale*, because it is fixed. On the *Latitude Scale*, a new table of *Meridional Parts* for every 10' of *Latitude* between 45˚ and 49˚is needed (not shown).

d. **Completed Graticule.** Fig 4-12 (opposite) shows the complete *Graticule* for this example. Each rectangle, whatever its dimensions in millimetres, represents a part of the Earth's surface bounded by *Meridians* 1˚ apart in *Longitude,* and *Parallels* 1˚ apart in *Latitude*. Although the *Chart Lengths* between these *Parallels of Latitude* vary from 42.66mm to 45.14mm as shown, each *Chart Length* represents a distance of 60 *Sea Miles* on the Earth's surface. The actual distance in *Sea Miles* between the *Meridians* depends on the *Latitude* in which it is measured on the chart (41.82 *Sea Miles* at 46˚N and 40.27 *Sea Miles* at 48˚N), and may be obtained by measurement against the *Latitude Scale*, or from formulae (3.12) and (3.9).

(0424d continued)

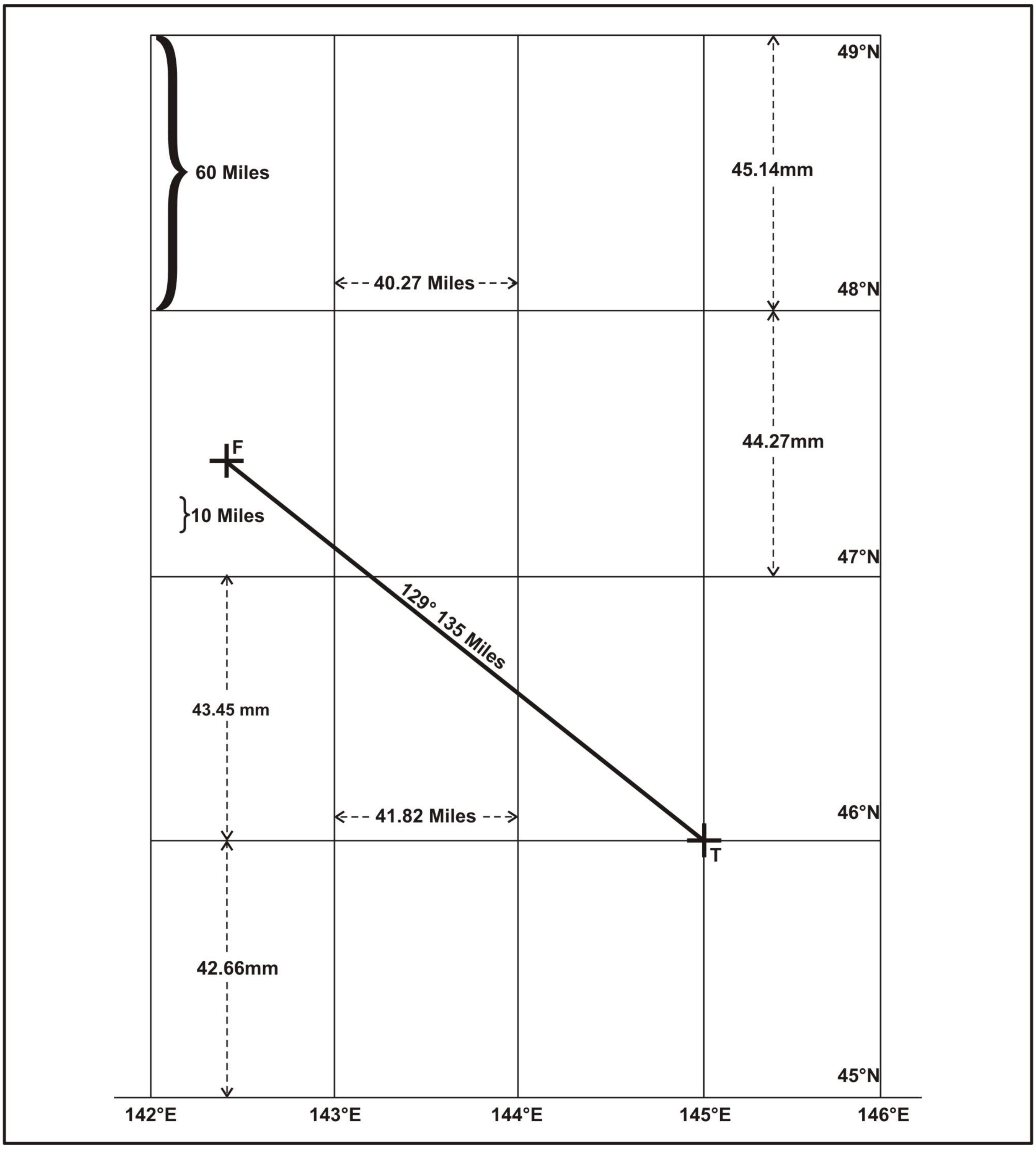


Fig 4-12. Construction of a (Larger Scale) Mercator Chart of Part of the World

(0424) e. **Measuring Distances on the Chart.** The distance between any two points must be measured on the *Latitude Scale* between their *Parallels* of *Latitude* (see Para 0421g / Fig 4-9). Using this method, the distances between points *F* and *T* on Fig 4-12 (above), measured on the *Latitude Scale* between 46˚ and 48˚, is found to be 135 miles.

Note 4-4. *The Meriodinal Parts at Table 4-1 (opposite) were taken from Norie's Nautical Tables (NP 320), so that the reader can easily duplicate them. Although NP 320 uses the Clarke 1880 Spheroid (not WGS 84), as it is the DMP which is actually used for construction, in practice little difference in the Graticule Shape is caused. To establish Meridional Parts for WGS 84, use the Spheroidal formulae at Para 0531a.*

0425. Great Circle Tracks on a Mercator Chart

Rhumb Lines appear as straight lines on a *Mercator Projection* chart. In general, *Great Circles* will appear as curves; the exceptions to this are the *Equator* and *Meridians,* which appear as straight lines perpendicular to each other. See also Paras 0414b / Fig 4-5 and Para 0441.

a. **Appearance of Great Circles.** On a *Mercator Projection* chart, the limiting *Great Circles* are the *Equator* which appears as a horizontal line, and any 'double' *Meridians*, which appear as two separate vertical lines 180˚ apart. Any other *Great Circles* passing through their points of intersection must appear as two curves with vertices towards the *Poles* (see Fig 4-13 below).

b. **Great Circles in One Hemisphere.** In Fig 4-13 (below), the *Great Circle* joining points *F* and *T* will always lie on the *Polar* side of the *Rhumb Line* joining them.

- **Small Latitude Difference and Large Longitude Difference.** When the difference of *Latitude* between *F* and *T* is small and the difference of *Longitude* large (ie a broadly East - West course), the difference between the two tracks is considerable.

- **Large Latitude Difference and Small Longitude Difference.** When the difference of *Latitude* between *F* and *T* is large and the difference of *Longitude* small (ie a broadly North - South course), the difference between the two tracks is small and is usually insignificant.

c. **Great Circles in Two Hemispheres.** In Fig 4-13 (below), as points *A* and *B* lie on opposite sides of the *Equator,* the *Rhumb Line* almost coincides with the *Great Circle*. The difference between the two tracks is small and is usually insignificant.

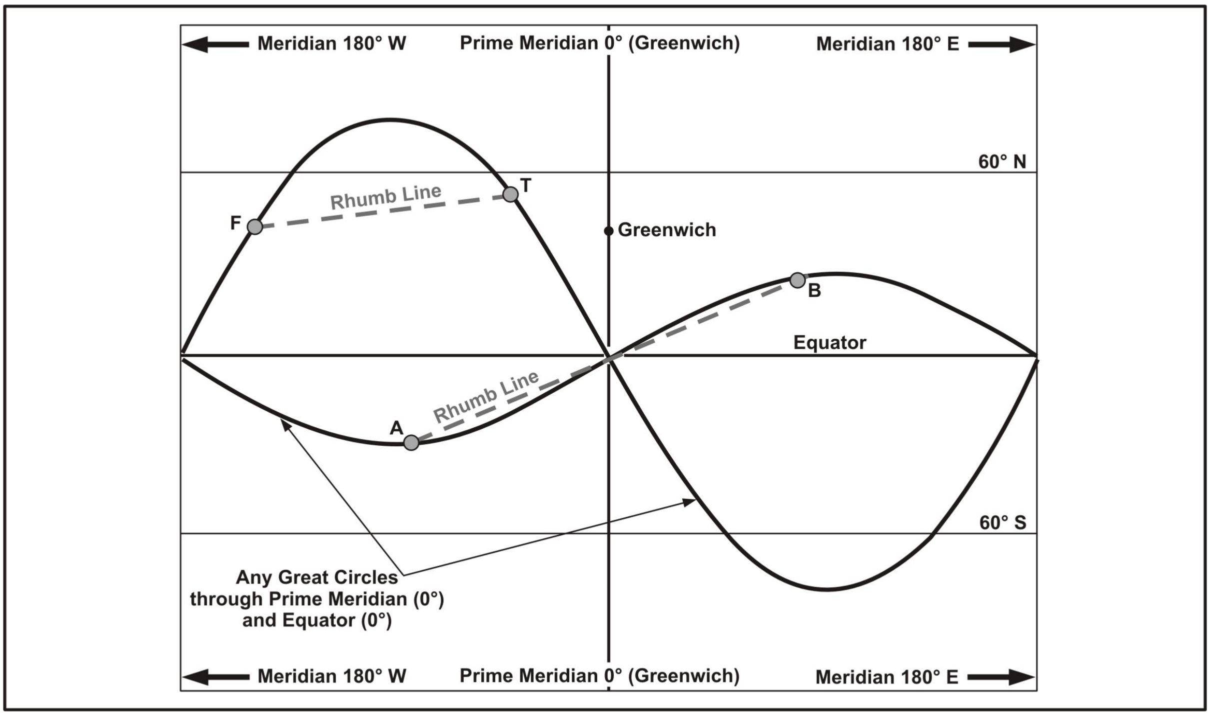


Fig 4-13. Great Circle Tracks on a Mercator Projection Chart

0426-0429. Spare.

SECTION 3 - TRANSVERSE MERCATOR PROJECTION FOR CHARTS

0430. Transverse Mercator Projection - Concept

The *Transverse Mercator Projection* (sometimes known as the *Gauss Conformal Projection*) is essentially a *Mercator Projection* with the 'cylinder' turned through 90˚; it is *Orthomorphic*. However, the resulting appearance of the *Graticule* is markedly different (see Fig 4-14 below and compare with Fig 4-2 / Fig 4-11). A list of *Transverse Mercator Projection*'s construction and uses is at Para 0414c, together with a list of its properties; an explanation of the mathematics for geographical / *Grid* conversions is at Appendix 4.

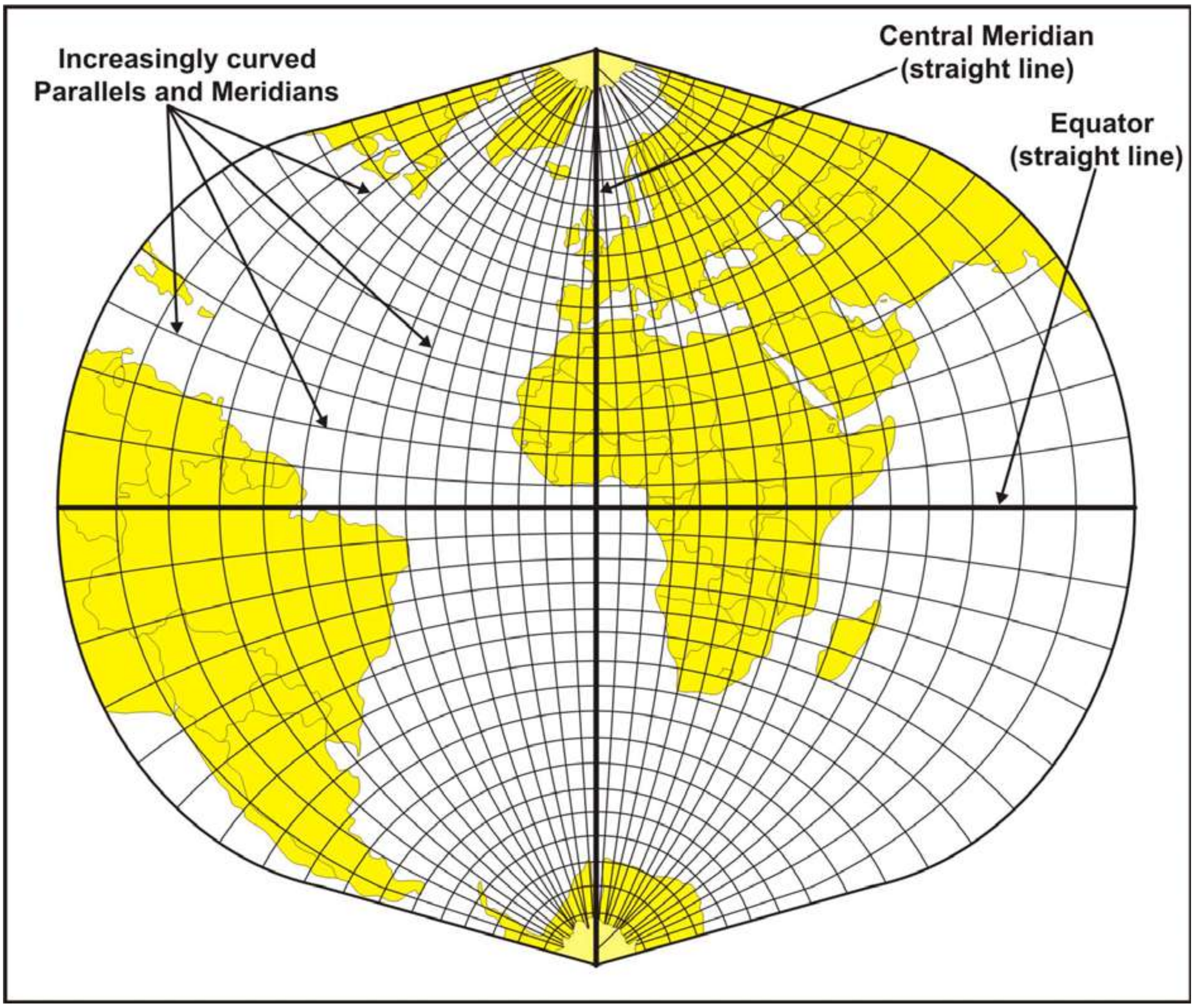


Fig 4-14. Global Application of the Transverse Mercator Projection

0431. Transverse Mercator Projection - Principles

a. **Central Meridian.** In the *Transverse Mercator Projection,* the line of contact with the cylindrical *Projection* surface is known as the *Central Meridian* and plots as a straight line; the *Equator* also plots as a straight line (see Fig 4-14 above and Fig 4-15 overleaf). All other *Meridians* and *Parallels of Latitude* plot as curves.

b. **Scale Expansion.** Adjacent *Meridians* (of *Longitude*) plot further apart as *Longitude* increases from the *Central Meridian*. This is analogous to the *Scale* expansion between *Parallels* with increasing *Latitude* in the *Mercator Projection.*

(0431) c. **Central Meridians and Zones of Longitude.** To minimise the effects of *Scale* expansion, a *Transverse Mercator Projection* chart is normally restricted to ±3˚ of *Longitude* from the *Central Meridian* (see Fig 4-15 below). Beyond these limits, a new chart must be constructed with a *Central Meridian* in a new *Zone* of *Longitude*.

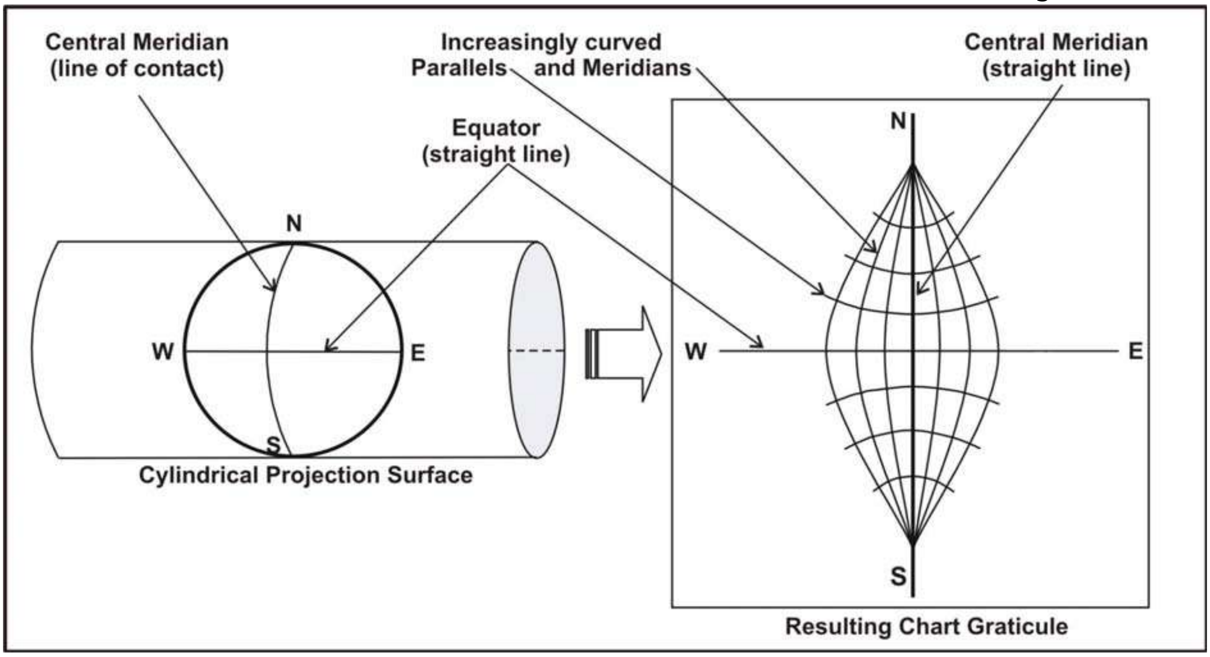


Fig 4-15. Transverse Mercator Projection - Limited Longitude Coverage

d. **High Latitudes.** By restricting its use to a narrow band of *Longitude*, the accuracy constraints imposed by high *Latitude* working on *Mercator Projections* are overcome. Although it is possible to depict the *Polar* regions on the *Transverse Mercator Projection*, in practice, better *Projections* exist for coverage of these *Areas*.

e. **Properties and Uses.** The properties and uses of *Transverse Mercator Projection* charts are listed at Para 0414c.

f. **Universal Transverse Mercator (UTM) Grid.** Due to the *Meridians* (except the *Central Meridian*) and *Parallels of Latitude* (except the *Equator*) plotting as curves, a *Grid* with Eastings and Northings is needed for the rapid identification of positions. To exploit the properties of the *Transverse Mercator Projection* globally, the *Universal Transverse Mercator* (*UTM*) *Grid* has been devised with globally standardised *Central Meridians*. These standardised *Central Meridians* are spaced at 6˚ intervals based on initial *Central Meridians* of 3˚W and 3˚E (ie on either side of the *Prime* (*Greenwich*) *Meridian*). *Projections* for charts using the *UTM Grid* are constructed for the appropriate *Central Meridian*. See details at Para 0451.

g. **UK Transverse Mercator (UKTM) Grid.** Charts and maps (including Ordnance Survey maps) using the *British National Grid* are constructed on the *UK Transverse Mercator* (*UKTM*) *Projection* with a (non-standard) *Central Meridian* of 2˚W and *Grid Origin* 49˚N 2˚W. The *Grid* is then offset to give a *False Origin* located to the South and West of UK (see details at Para 0452).

0432-0439. Spare.

SECTION 4 - GNOMONIC PROJECTION FOR CHARTS

0440. Gnomonic Projection - Concept

An introduction to *Gnomonic Projection* is at Para 0414e / Fig 4-3b and an explanation of its application is at Para 0622. To establish the *Waypoint*s for a *Great Circle* track, it is very helpful to show the *Great Circle* on a chart as a straight line. This also allows radio direction-finder *Bearings* to be plotted as straight lines, although this usage has almost completely disappeared.

a. **Gnomonic Projection - Construction.** The *Gnomonic Projection* achieves the representation of *Great Circles* as straight lines by projecting the Earth's surface from the Earth's centre onto the tangent plane (see Note 4-5 below). To minimise distortion, the tangent point is chosen to be at the centre of the *Area* to be shown on the chart.

b. **Gnomonic Projection - Example.** An example of a *Gnomonic Projection Graticule* is at Fig 4-16 (below). The tangent point has been placed on the *Equator* and at *Longitude* 0˚; the *Graticule* is symmetrical about the *Meridian* through this tangent point, which is independent of the *Longitude*. The *Longitude Scale* can thus be adjusted as required (eg *UKHO* Chart 5029).

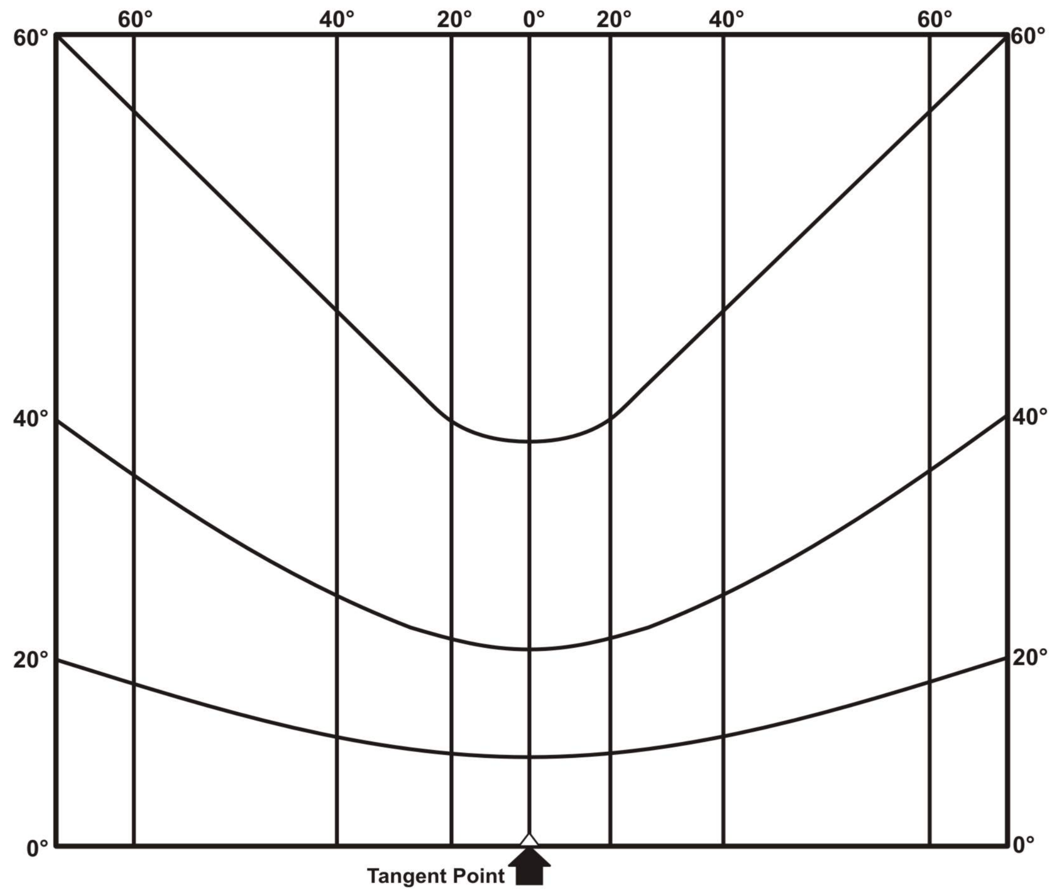


Fig 4-16. Example Gnomonic Projection Graticule (*Similar to UKHO Chart 5029*)

Note 4-5. Gnomonic Projection - Type. *The Gnomonic Projection is a Zenithal Projection (from position 'B' at Para 0413, Fig 4-3a Example 5), and is based on a* *<u>Sphere</u>* *which represents the Earth.* *<u>It is NOT a Spheroidal Projection</u>.*

(0440) c. **Representation of Great Circles as Straight Lines.** A *Great Circle* is defined at Para 0110c as: "... the intersection of a *Spherical* surface and a plane which passes through the centre of the *Sphere*". As one plane will always cut another in a straight line, all *Great Circles* will appear on the chart as straight lines.

d. **Properties and Uses.** The properties and uses of *Gnomonic Projection* charts are listed at Para 0414e.

e. **Theory.** The mathematical theory of the *Gnomonic Projection* is at Appendix 4.

0441. Transfer of a Great Circle Track from a Gnonomic to a Mercator Projection Chart

a. **Transfer Procedure.** To transfer of a *Great Circle* track (eg *FT* in Fig 4-17), from a *Gnomonic* to a *Mercator* chart, note the *Latitude* / *Longitude* of convenient *Waypoints A, B, C* ...etc on the line *FT* and mark them on the *Mercator Projection* chart. This will produce a series of *Rhumb Lines*, closely approximating a curve. See also Para 0208e.

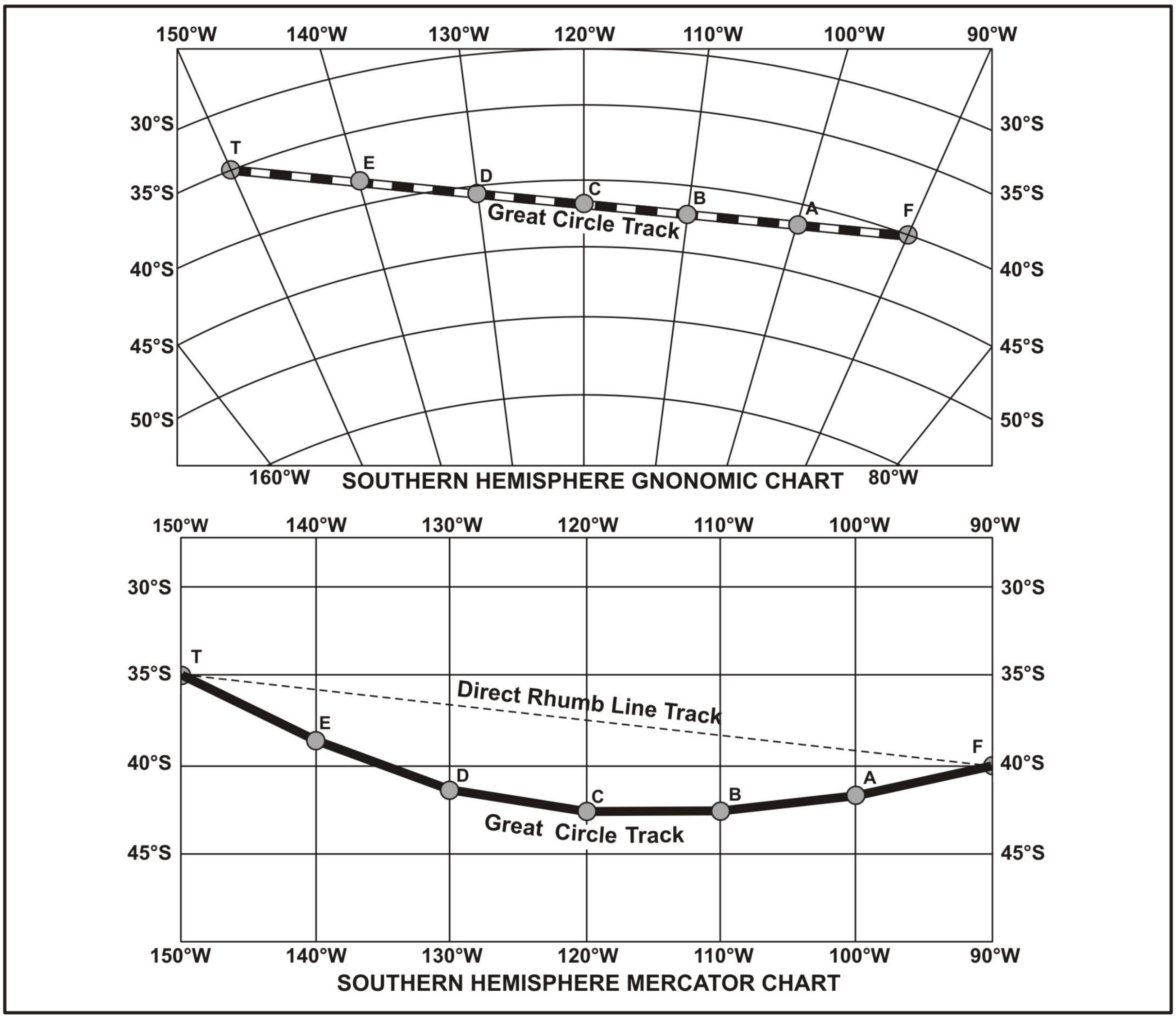


Fig 4-17. Great Circle Track on Gnomonic and Mercator Projection Charts (Southern Hemisphere Example)

(0441) b. **Adjusting Courses.** The *Rhumb Line* courses between transferred *Waypoints* may not be multiples of full (integer) degrees (see Fig 4-17 opposite). The *Waypoints* should thus be adjusted to approximate the *Great Circle* track, while achieving convenient whole-degree courses. If manual steering is intended (as opposed to auto-pilot) the track should ideally be adjusted to multiples of 5˚ (ie 295˚, 300˚ etc).

c. **Same and Opposite Sides of the Equator.** When *F* and *T* lie on the same side of the *Equator* (solid line at Fig 4-18 below), the procedure is as at Para 0441a opposite. When *F* and *T* lie on opposite sides of the *Equator* (eg *F* being North and *T* South in Fig 4-18 below), if the tangent point is on the *Equator* (eg *UKHO* Chart 5029), the same chart can be used (a *Gnomonic Projection* chart of both hemispheres is symmetrical about the *Equator)*. In practice, this is rarely necessary as in these case*s,* the *Rhumb Line* almost coincides with the *Great Circle*; the difference between the two tracks is small and is usually insignificant (see Para 0425c). However, if required to do so (see dashed lines at Fig 4-18 below), the following geometrical construction suffices:

- Mark the position of *T* as if it were in the Northern hemisphere.
- Join *F* to *K*, the point on the *Equator* which has *T*'s *Longitude*.
- Join *T* to *H*, the point on the *Equator* which has *F*'s *Longitude*.
- Drop a perpendicular *RQ* on the *Equator* from *R*, the point where *FK* cuts *TH*.
- Draw *FQ* and *QT*.
- Then *FQ* is the *Great Circle* track in the Northern hemisphere, and *QT* is the reflection of its continuation South of the *Equator*. Points on *QT* may therefore be treated as if they were in the Southern hemisphere.

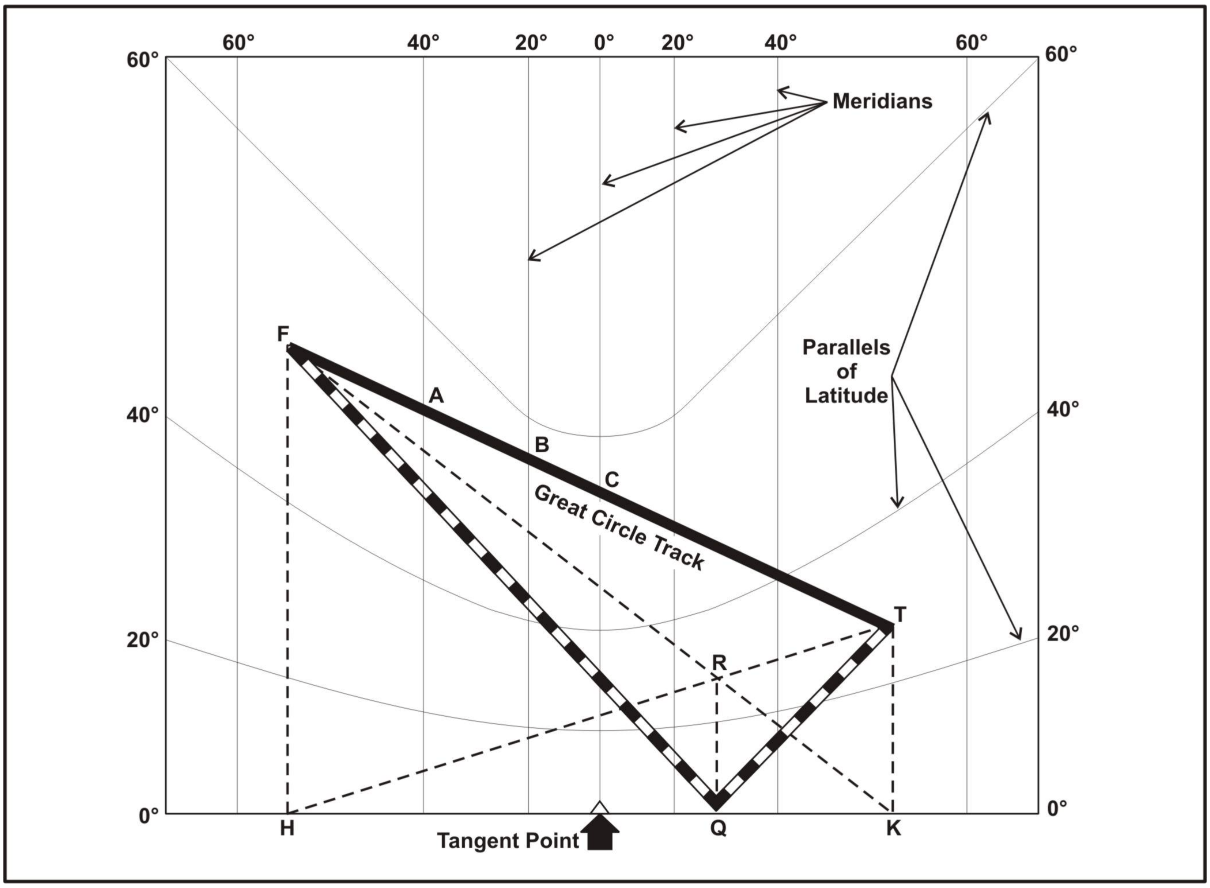


Fig 4-18. Great Circle Track on One / Both Sides of Equator - Gnomonic Projection Chart

0442. Composite Tracks

A *Composite Track* may easily be constructed using a *Gnomonic Projection* chart (see Para 0441, previous pages). Fig 4-19 below shows three tracks - a *Rhumb Line*, a *Great Circle* track and a *Composite Track* - all on a *Mercator Projection* chart.

a. **Composite Track on a Gnomonic Projection Chart.** A *Composite Track* is formed by two *Great Circle* arcs joined at their *Vertices* by the '*Safe Parallel*' *of Latitude* (see Para 0209b / Fig 2-12). On a *Gnomonic Projection* chart, the two *Great Circle* tracks are shown as straight lines and the '*Safe Parallel*' *of Latitude* is shown as a curve (see single *Great Circle* track example at Fig 4-17 facing previous page) .

b. **Composite Track on a Mercator Projection Chart.** On a *Mercator Projection* chart, the appearance of *Composite Track* is the reverse of the *Gnomonic Projection* chart presentation. The two *Great Circle* tracks are shown as curves, and the '*Safe Parallel*' *of Latitude* is shown as a straight line (see Fig 4-19 below).

c. **Transferring Composite Track Waypoints.** *Waypoints* for the two separate *Great Circle* elements of the *Composite Track* may be transferred from a *Gnomonic Projection* chart to a *Mercator Projection* chart (as shown at Para 0441 / Fig 4-17 and Para 0208e / Fig 2-11).

d. **Calculation.** The calculation of the *Composite Track* is at Para 0522.

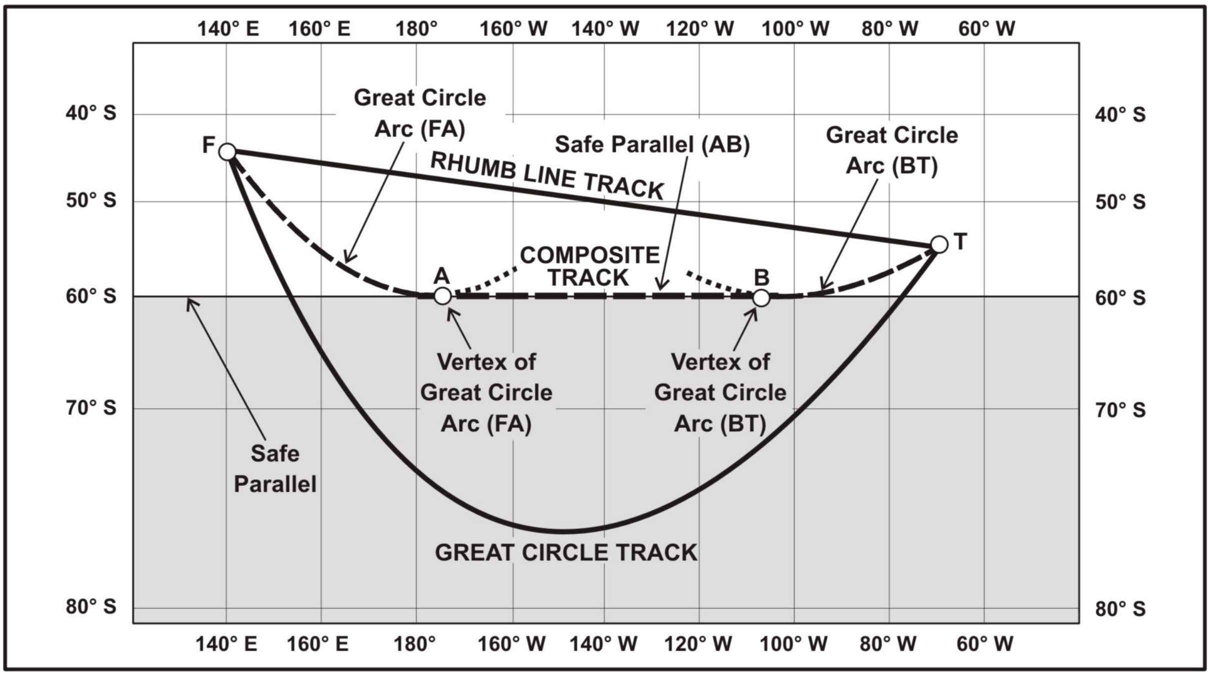


Fig 4-19. Rhumb Line, Great Circle and Composite Track on a Mercator Chart

0443-0449. Spare.

SECTION 5 - GRIDS

0450. Grid Reference Systems - Concept

a. **Grids - Definition and Properties.** A *Grid* is a reference system of rectangular *Cartesian Coordinates* obtained when a *Projection* is applied to a part or all of the world. The *Grid* will have all the properties of the *Spheroid* and *Projection* used and may have some special ones peculiar to itself. Several *Grids*, all different, may be based on the same *Spheroid* and *Projection*.

b. **Grids and Geographical Graticules.** An example of a *Transverse Mercator Projection Grid* with a (geographical) *Graticule* superimposed is at Fig 4-20 below. A *Grid* equates to a large piece of graph paper, graduated in suitable *Grid* units North (Northings) and East (Eastings) from the *Grid Origin*. Distances West and South of the *Grid Origin* are given negative values of Eastings and Northings respectively, but this can be avoided by using a *False Origin*.

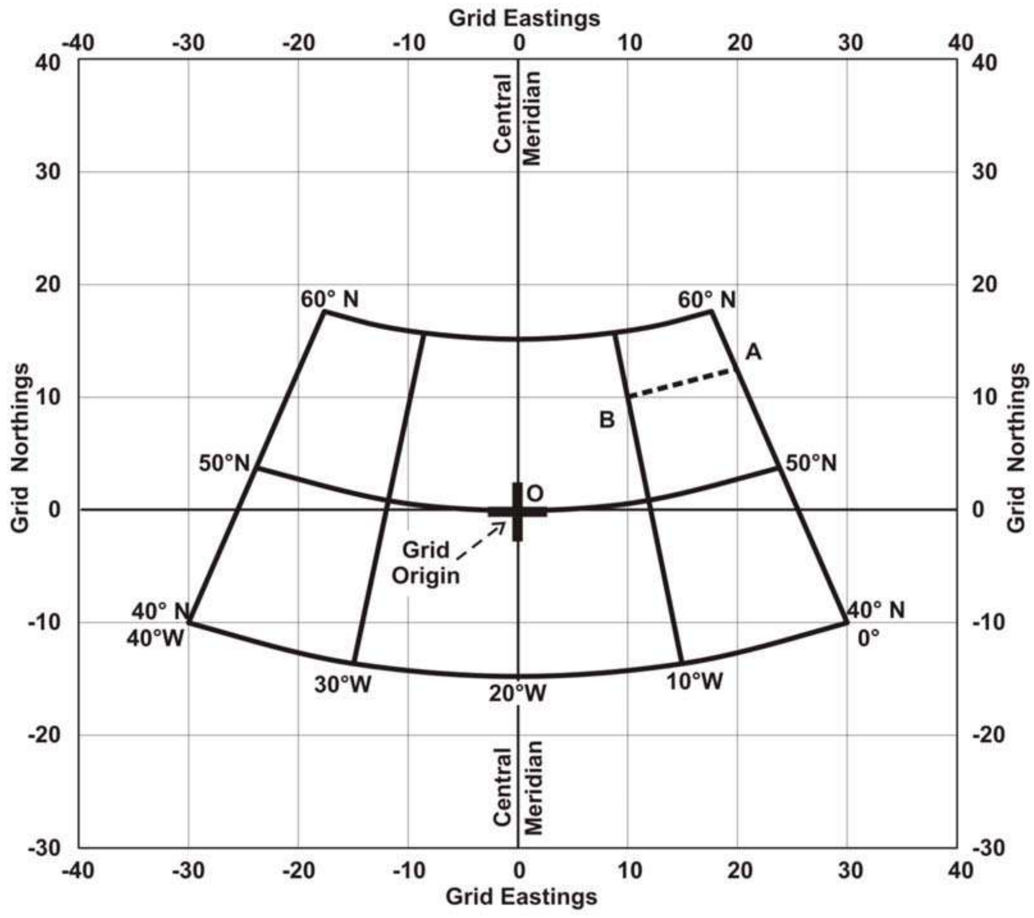


Fig 4-20. Generic Grid with Geographical Graticule Superimposed

c. **Plotting the Graticule on the Grid.** The intersections of *Meridians* (of *Longitude*) and *Parallels of Latitude* are converted into *Grid Eastings* and *Grid Northings*; these are then plotted as individual points on the *Grid* and joined by smooth curves to form the geographical *Graticule*. To simplify this conversion, a set of tables or a computer program may be created, based on the *Spheroid* and *Projection* used.

(0450) d. **Scale Factors.** At the *Grid Origin* (position *Grid* 0000 0000, or 50˚N 20˚W at Fig 4-20 previous page), the distortion in distance and direction is at a minimum. This distortion increases with distance from *Grid Origin* and can be represented by a *'Scale Factor'*, defined as:

Scale Factor = Grid Length ÷ Spheroidal Arc

The *Scale Factor* is applied to both Eastings and Northings. *Grid Length* and *Spheroidal Arc* are illustrated at at Fig 4-21 (below). *Scale Factors* for the <u>Universal</u> *Transverse Mercator Projection* (*UTM*) are at Fig 4-22 (below).

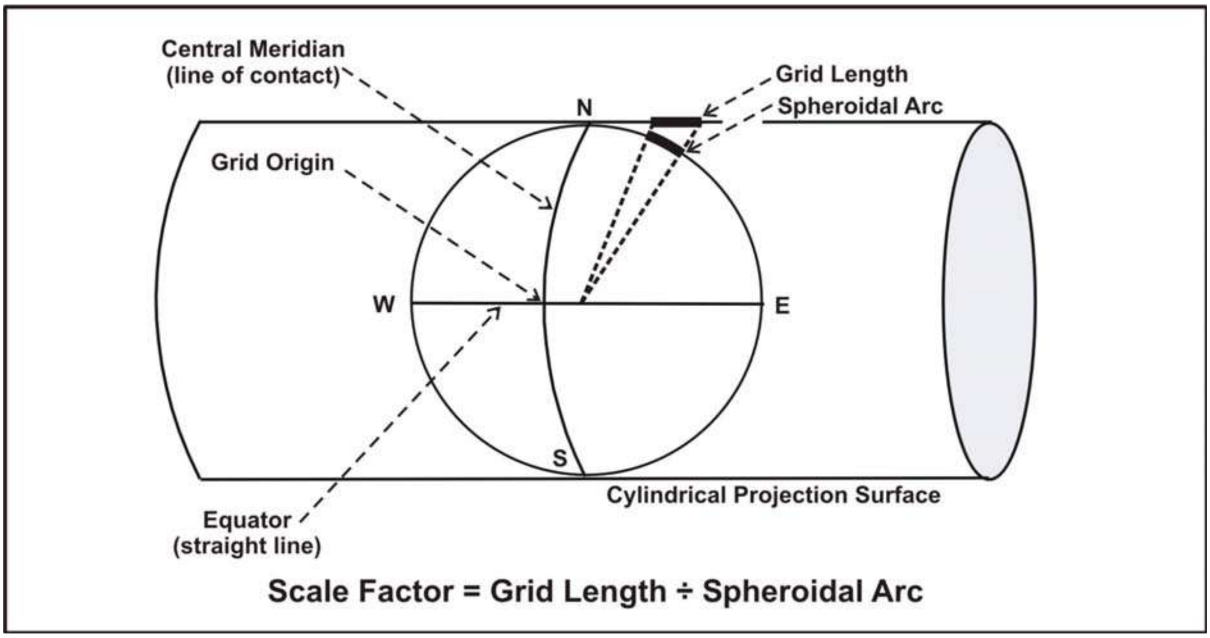


Fig 4-21. Grid Length, Spheroidal Arc & Scale Factor (<u>Transverse</u> Mercator Projection)

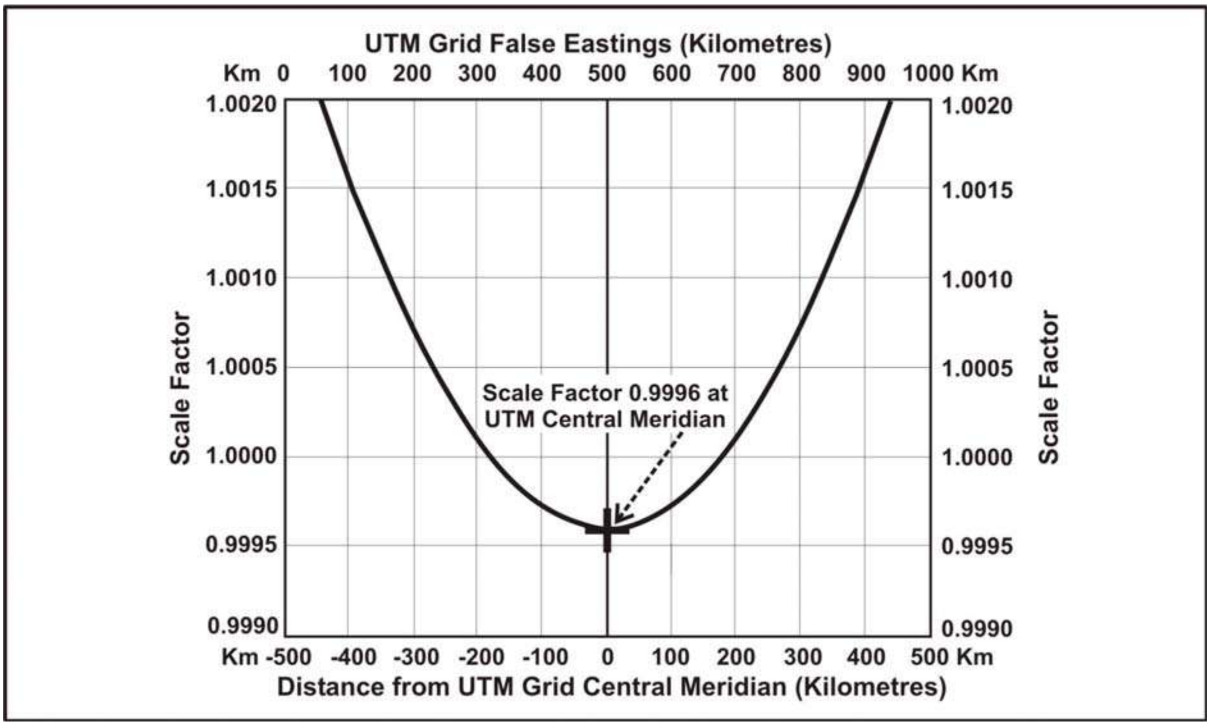


Fig 4-22. Scale Factors for <u>Universal</u> Transverse Mercator (UTM) Projection

(0450) e. **Grid Convergence and Grid Orientation.** On a *Transverse Mercator Projection* map / chart, the *Grid* does not always point North (see Fig 4-23 below); this also applies in principle to other *Grids* using other *Projections*. At any point on the *Grid*, the angle between the true *Meridian* (ie true North) and the *Grid North* line **(measured <u>from</u> True North),** is known as the *Grid Convergence* (*C*) in navigation and as *Grid Orientation* in gunnery. It varies as follows:

- **Transverse Mercator Projection.** On *Transverse Mercator Projection*, *Grid Convergence* increases with distance from the *Central Meridian*.
- **Polar Stereographic and Other Projections.** *Grid Convergence* varies with the *Projection* in use and can be as much as 180˚(eg *Polar Stereographic*).
- **Mercator Projection.** On *Mercator Projection* charts, *Grid Convergence* / *Grid Orientation* is zero everywhere (ie *Grid North always coincides with Meridians*). If a *Transverse Mercator Grid* is <u>superimposed</u> on a *Mercator Projection* chart, *Grid* distortions will occur away from the chart borders.

Part of the *Transverse Mercator Projection Grid* at Fig 4-20 containing the points *A* and *B*, is shown enlarged at Fig 4-23 below. AP_1 and *BP* are the *Meridians* through *A* and *B* respectively and they are both very slightly curved (see Fig 4-14). AX_1 and *BX* both define the direction of *Grid North* at positions *A* and *B* respectively.

C°, the *Grid Convergence* at *B* = angle *PBX*
C_1°, the *Grid Convergence* at *A* = angle P_1AX_1

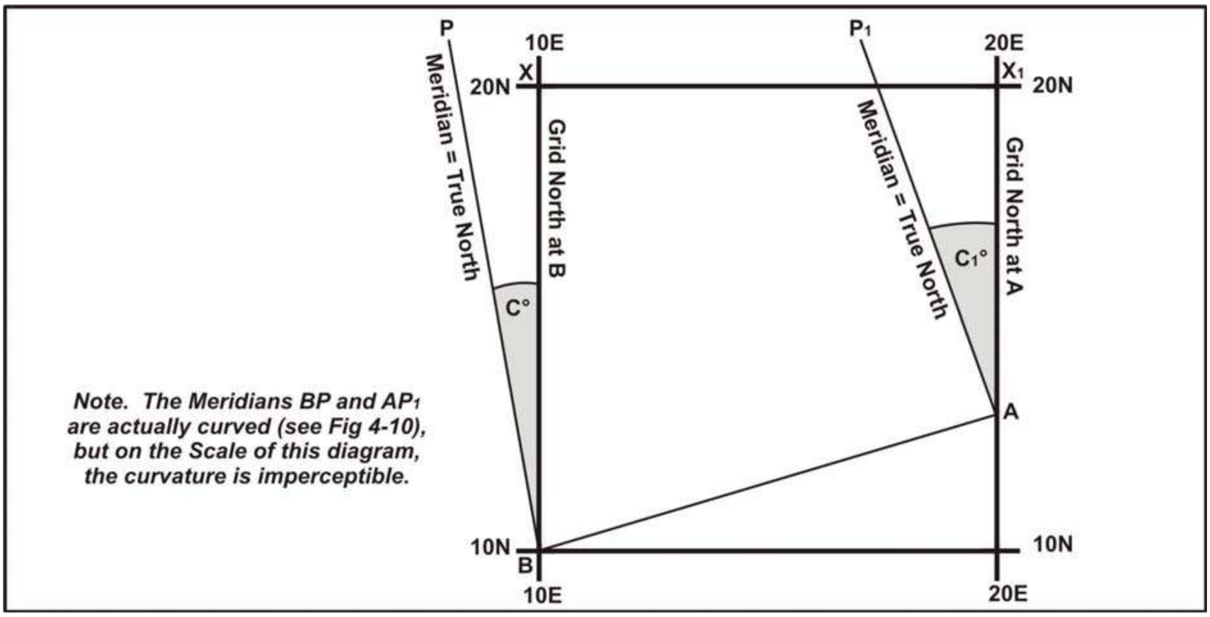


Fig 4-23. Transverse Mercator Projection Grid Convergence at 'A' and 'B' from Fig 4-20

f. **Calculating Grid Convergence and Grid Orientation.** On Military and Ordnance Survey (OS) maps, the difference between *True North* and *Grid North* is printed for specified positions - in the centre on Military Maps and in the corners on OS Maps. At any intermediate position, interpolation is required. Care must be taken when using Ordnance Survey Maps, as these show the difference of *True North* **from** *Grid North* (ie the <u>opposite</u> of the definition at Para 0450e above).

Example 4-3. If *<u>Grid</u> North* is 2˚ west of *<u>True</u> North* then *Grid Convergence* is 2˚ West.
Example 4-4. If *<u>True</u> North* is 2˚ east of *<u>Grid</u> North*, *Grid Convergence* is 2˚ West.

0451. Universal Transverse Mercator (UTM) Grid

The *Transverse Mercator Projection* was introduced at Paras 0430-0431 and was used to provide examples for the concept of *Grids* at Para 0450. The *Universal Transverse Mercator* (*UTM*) *Grid* is used extensively worldwide for military and civil maps. Detailed *UTM Grid* arrangements are contained in NATO 'STANAG 2211'(available in the public domain); the following sub-paragraphs contain a summary of this information.

a. **Scope of the UTM Grid - Zones of Longitude.** The *UTM Grid* is split into *UTM Zones* (or strips) of *Longitude* 6˚ wide (although there are a small number of NATO anomalies to this - see Para 0451d opposite). The *UTM Zones of Longitude* are numbered from 1 to 60, with *Zone 1* covering from 180˚ to 174˚ W (ie starting at the *International Date Line*), counting eastwards to *Zone 60* (174˚ E to 180˚) Although 6˚ of *Longitude* wide, the geographical width of each *UTM Zone* varies with *Latitude* (ie at the *Equator* each *Zone* is 360 miles wide, reducing to 62 miles wide at 80˚ *Latitude)*. A *UTM Zone* includes the West boundary but NOT the East boundary.

b. **Standardised Central Meridians.** Each *UTM Zone* is bisected by a *Central Meridian* (see Table 4-2 below). *UTM Grid Central Meridians* are spaced at 6˚ of *Longitude* intervals (except for some NATO anomalies - see Para 0451d opposite), with first / last *Central Meridians* at 177˚W and177˚E (ie on either side of the *International Date Line*); two of them will lie at 3˚W and 3˚E of the *Prime* (*Greenwich*) *Meridian*).

Table 4-2. Longitude, UTM Zones and Central Meridians

Longitude	180˚ 174˚W 168˚W 162˚W 156˚W 150˚W 144˚W 138˚W 132˚W 126˚W 120˚W 114˚W 108˚W 102˚W 96˚W 90˚W 84˚W															
UTM Zone	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Central Meridian	177˚ W	171˚ W	165˚ W	159˚ W	153˚ W	147˚ W	141˚ W	135˚ W	129˚ W	123˚ W	117˚ W	111˚ W	105˚ W	99˚ W	93˚ W	87˚ W

Longitude	84˚W 78˚W 72˚W 66˚W 60˚W 54˚W 48˚W 42˚W 36˚W 30˚W 24˚W 18˚W 12˚W 6˚W 0˚ 6˚E 12˚E															
UTM Zone	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
Central Meridian	81˚ W	75˚ W	69˚ W	63˚ W	57˚ W	51˚ W	45˚ W	39˚ W	33˚ W	27˚ W	21˚ W	15˚ W	9˚ W	3˚ W	3˚ E	9˚ E

Longitude	12˚E 18˚E 24˚E 30˚E 36˚E 42˚E 48˚E 54˚E 60˚E 66˚E 72˚E 78˚E 84˚E 90˚E 96˚E 102˚E 108˚E															
UTM Zone	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
Central Meridian	15˚ E	21˚ E	27˚ E	33˚ E	39˚ E	45˚ E	51˚ E	57˚ E	63˚ E	69˚ E	75˚ E	81˚ E	87˚ E	93˚ E	99˚ E	105˚ E

Longitude	108˚E 114˚E 120˚E 126˚E 132˚E 138˚E 144˚E 150˚E 156˚E 162˚E 168˚E 174˚E 180˚											
UTM Zone	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Central Meridian	111˚ E	117˚ E	123˚ E	129˚ E	135˚ E	141˚ E	147˚ E	153˚ E	159˚ E	165˚ E	171˚ E	177˚ E

(0451) c. **Scope of the UTM Grid - Latitude Bands.** The *UTM Grid* covers *Latitude* 80˚S to 84˚N, and splits this *Area* into *Bands* of *Latitude* 8˚ high, except for the most Northerly *Band* (72˚-84˚N) which is 12˚ high. Each *(Latitude) Band* is given a letter, starting at 'C' in the Southern hemisphere through to 'X' in the Northern hemisphere, but omitting the letters 'I' and 'O' (see Table 4-3 below).

Table 4-3. Latitude and UTM Zones (*shown from South to North*)

Southern Hemisphere			**Northern Hemisphere**	
Letter	**Latitude Band**		**Letter**	**Latitude Band**
C	80˚S-72˚S		N	0˚-8˚N
D	72˚S-64˚S		P	8˚N-16˚N
E	64˚S-56˚S		Q	16˚N-24˚N
F	56˚S-48˚S		R	24˚N-32˚N
G	48˚S-40˚S		S	32˚N-40˚N
H	40˚S-32˚S		T	40˚N-48˚N
J	32˚S-24˚S		U	48˚N-56˚N
K	24˚S-16˚S		V	56˚N-64˚N
L	16˚S-8˚S		W	64˚N-72N
M	8˚S-0˚		**X**	**72˚N-84˚N**

d. **NATO UTM Grid - Zone Anomalies.** NATO *UTM Grid* charting and mapping anomalies in the Northern hemisphere *Bands* 'V' and 'X' that do NOT comply with the normal convention for *UTM Zones* are at Tables 4-4a/b (below). These anomalies affect SW Norway, Denmark and the Svalbard Island between 56˚N and 64˚N (*Band* 'V'), and Norway between 72˚N and 84˚N (*Band* 'V', just south of the permanent pack ice line). NATO mapping in these *Area*s is based upon the following modified *UTM Zones*.

Table 4-4a. NATO UTM Grid - Band 'V'(56˚N-64˚N) - Zone Anomalies

UTM Zone	**Band Affected**	**Zone Limits (Longitude)**	**Central Meridian**
31	V (56˚N - 64˚N)	0˚ - 3˚E	3˚E
32	V (56˚N - 64˚N)	3˚E - 12˚E	9˚E

Table 4-4b. NATO UTM Grid - Band 'X' (72˚N-84˚N) - Zone Anomalies

UTM Zone	**Band Affected**	**Zone Limits (Longitude)**	**Central Meridian**
32, 34, 36	X (72˚N - 84˚N)	Zones 32, 34, 36 not used	Not applicable
31	X (72˚N - 84˚N)	0˚ - 9˚E	3˚E
33	X (72˚N - 84˚N)	9˚E - 21˚E	15˚E
35	X (72˚N - 84˚N)	21˚E - 33˚E	27˚E
37	X (72˚N - 84˚N)	33˚E - 42˚E	39˚E

e. **100,000 Metre Square Identifiers.** Two-letter identifiers are also provided to identify the 100,000 metre squares within a 6˚ *Zone* / 8˚ *Band.* These are not normally needed when using computerised *Grid* conversions (see Paras 0451g/i).

(0451) f. **UTM 'True' Grid Origin and 'False Origin.** The *UTM Grid Zone* number (see Para 0451a) and *Band* letter (see Para 0451c) may be combined to provide a single unique identifier for a *UTM Grid* 'block' (normally a 6˚ wide *Zone* of *Longitude* and 8˚ high *Band* of *Latitude*). The *Zone* number always precedes the *Band* letter (eg '30U').

- **'True' Grid Origin.** Each *Zone / Band* 'block' (eg '30U') has a 'True' *Grid Origin* at the intersection of its *Central Meridian* (*Longitude*) and the *Equator*. If a *Grid* value of zero was assigned to the 'Eastings' and 'Northings' of the 'True' *Grid Origin*, then *Grid* 'Eastings' for positions West of the *Central Meridian* would always have negative values, and in the Southern hemisphere, all *Grid* 'Northings' would have negative values.

- **Grid 'False Origin'.** To overcome negative values, each *Zone / Band* 'block' (eg '30U') is assigned a *Grid* '*False Origin*'. To achieve this, a '*False Easting*' and '*False Northing*' are assigned, but the '*False Northing*' is different in the Northern and Southern hemispheres (see Para 0451g below).

g. **Calculating Global UTM Grid References.** The method of calculating *UTM Grid* references from the *False Origin* (see Fig 4-24 below) is as follows:

- **Both Hemispheres - Eastings.** In both hemispheres, a *False Origin* is placed 500,000m West of the *Central Meridian* (which thus has a '*False Easting*' of 500,000m East). *UTM Grid Easting*s are counted eastwards across *Zone*, although they do not start at zero due to *Zone* width. At *Zone* boundaries, <u>Eastings are re-set</u> for the *Central Meridian* / *False Origin* of the new *Zone*.

- **Northern Hemisphere - Northings.** In the Northern hemisphere, the '*False Origin*' is on the *Equator* (ie *'False Northings'* are zero). *UTM Grid Northings* are counted northwards (irrespective of *Band* boundaries) from the *False Origin* at the *Equator; UTM Grid* <u>*Northings* increase as *Latitude* increases</u> **away** from the *Equator* and *False Origin* (eg 16˚N 3˚E has a Northing of 1,768,935m; 80˚N 3˚E has a Northing of 8,881,585m).

- **Southern Hemisphere - Northings.** In the Southern hemisphere, the '*False Origin*' is placed 10,000,000m South of the *Equator*, close to the *South Pole* (ie the *Equator* has a *'False Northing'* of 10,000,000m North). *UTM Grid Northings* are counted northwards (irrespective of *Band* boundaries) from the *False Origin*. *UTM Grid* <u>*Northings* decrease as *Latitude* (South) increases</u> **away** from the *Equator* but **towards** the *False Origin* (eg 16˚S 3˚E has a *Grid Northing* of 8,231,064m; 80˚S 3˚E has a *Grid Northing* of 1,118,416m).

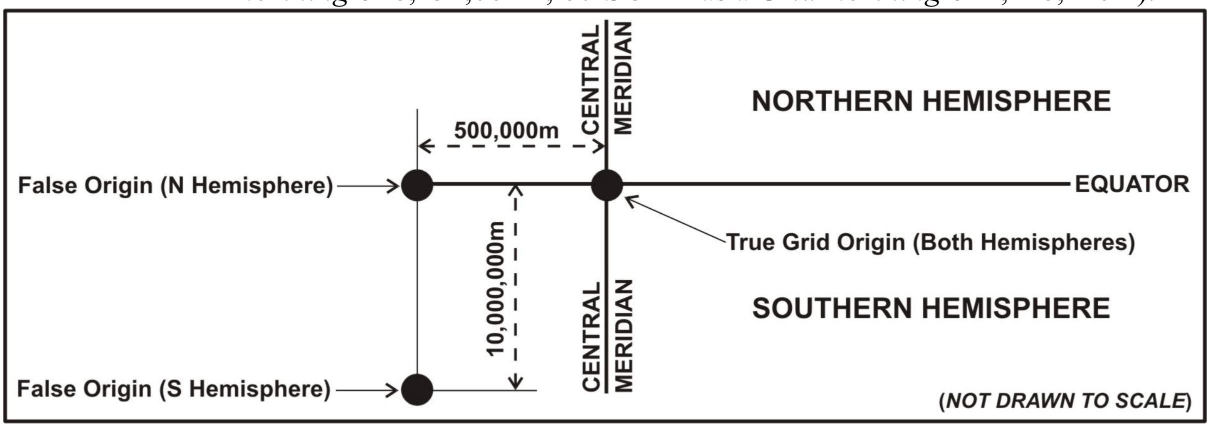


Fig 4-24. UTMG True and False Grid Origins (N & S Hemispheres)

(0451) h. **Local UTM Grid References (4, 6 and 8 Figures).** Although the *Grid Easting* and *Northing* coordinates are measured in metres from the *False Origin*, when using maps, the user normally requires only the 'local' 6-Figure or 8-Figure *Grid* reference. On most maps and charts the smallest *Grid* division shown is the 1 kilometre (1000 metres) square (see Fig 4-25 below); however, so that there can be no doubt, the legend of the map or chart will normally specify the graduations shown.

- **Grid Coordinates.** To specify *Grid* coordinates of a point, the *Eastings* (measured from the western edge of the graduations) are stated first, followed by the *Northings* (measured from the southern edge of the graduations).

- **4-Figure Grid References.** Using the 1 kilometre square graduations, a 4-figure grid reference (eg '0588' at Fig 4-25 below) will specify a position to an accuracy of 1000 metres; it is in fact an 'area' rather than a 'point'.

- **6-Figure Grid References.** By estimating tenths in the 1 kilometre square graduations, a 6-figure grid reference (eg 'White Head Light' at *Grid* '040874' in Fig 4-25) will specify a position to an accuracy of 100 metres.

- **8-Figure Grid References.** By estimating hundredths in the 1 kilometre square graduations, an 8-figure grid reference (eg 'White Head Light' at *Grid* '03988743' in Fig 4-25 below) will specify a position to an accuracy of 10 metres. This level of accuracy is difficult to measure without a large *Scale* map or chart, but can be obtained by observation on the ground with a hand-held *DGPS* receiver, or by computer transformation of accurate *Latitude* and *Longitude* positions (see Para 0451i overleaf).

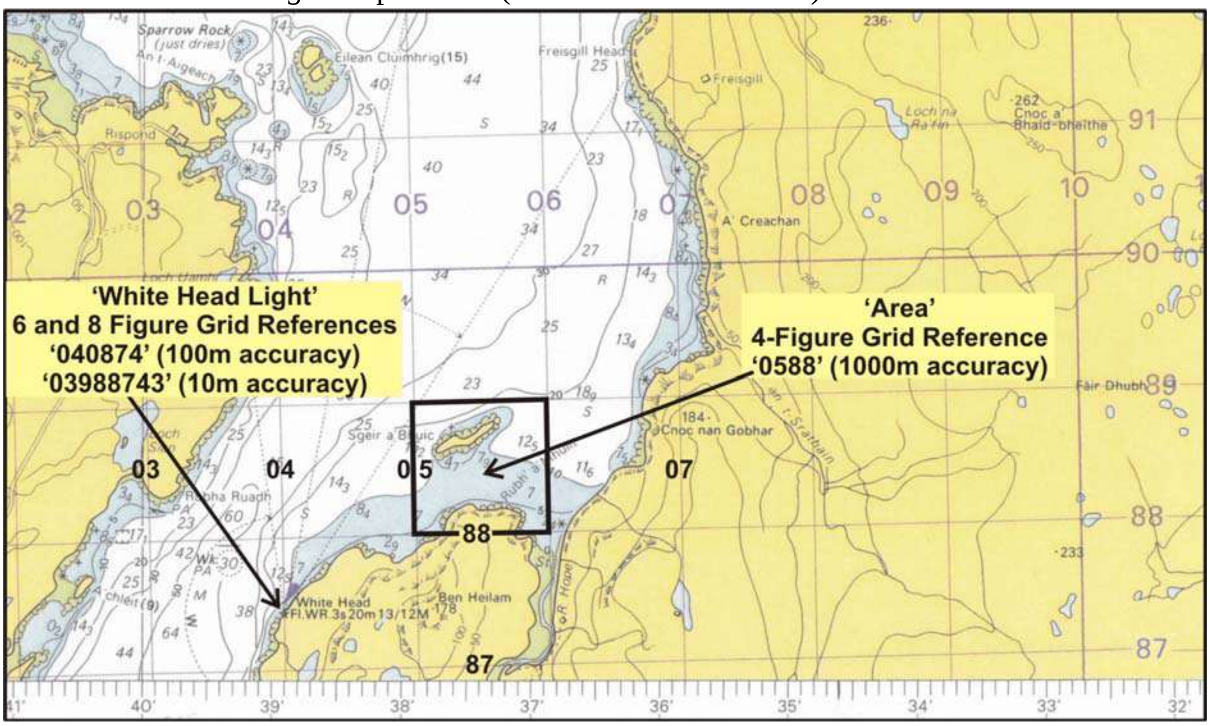


Fig 4-25. UTM - 4, 6 and 8 Figure Grid References

Note 4-6. *The UTM Grid in Fig 4-25 is clearly not aligned exactly North-South, but is offset by the Grid Convergence / Grid Orientation due to this chart showing an Area some distance from the Central Meridian (see details at Para 0450e).*

(0451) i. **UTM Grid Conversions - 'Silent' Figures.** When converting from *Latitude / Longitude* to *UTM Grid* using computerised navigation systems (eg *ECDIS* or *WECDIS*) or even navaids (eg *DGPS / GPS* receivers), the full *Grid* coordinates to the nearest hundredth of a metre are normally given. The two-letter identifiers used to identify the 100,000 metre squares are normally replaced by figures (see Paras 0451e/g). This generates potentially confusing additional figures in addition to the 6 or 8 figure *Grid* references at Para 0451h (previous page). When converting from *UTM Grid* to *Latitude / Longitude*, to avoid ambiguity, care must be taken use the full *UTM Grid* reference and to specify the *Central Meridian* and hemisphere.

- **White Head Light Example.** In the case of the example of White Head Light (58˚ 31.01'N 004˚ 38.90'W [*WGS 84*]) at Para 0451h / Fig 4-25 (previous page), the full *UTM Grid* coordinates to the nearest hundredth of a metre (as provided by *WECDIS*) are: 403982.17 (Eastings), 6487431.13 (Northings).

- **'Silent' Figures - 'The Key'.** The key to decoding these coordinates is that the figures to the left of the decimal point (in order to the left) are: metres / tens of metres / hundreds of metres / thousands of metres etc.

- **6-Figure Grid References.** Thus, if a 6-figure *Grid* reference (ie accurate to 100 metres) is needed, ignore the first two figures to the left of the decimal point (ie metres / tens of metres), take the next three figures (rounded to the nearest significant figure) and ignore the figures further left to obtain *Grid* reference '040874' (see Table 4-5a below - significant figures underlined):

Table 4-5a. Decode of Full Grid Reference to 6-Figure Grid Reference

	Full Grid Reference (1/100th metre accuracy)	**6-Figure Grid Reference (100 metre accuracy)**
Eastings	403982.17	040 (rounded up)
Northings	6487431.13	874 (rounded down)

- **8-Figure Grid References.** Thus, if an 8-figure *Grid* reference (ie accurate to 10 metres) is needed, ignore the first figure to the left of the decimal point (ie metres), take the next four figures (rounded to the nearest significant figure) and ignore the figures further left to obtain *Grid* reference '03988743'(see Table 4-5b below - significant figures underlined):

Table 4-5b. Decode of Full Grid Reference to 8-Figure Grid Reference

	Full Grid Reference (1/100th metre accuracy)	**8-Figure Grid Reference (10 metre accuracy)**
Eastings	403982.17	0398 (rounded down)
Northings	6487431.13	8743 (rounded down)

- **'Silent' Figures on the Map / Chart.** The map or chart will normally show the 'silent' figures in small print at the edge of the map / chart where major *Grid* intersections occur. Some reference to these 'silent' figures may also be mentioned in the map / chart legend.

0452. British (Ordnance Survey) National Grid

The *British National Grid* is used for Ordnance Survey mapping of UK; it differs from *UTM* only in that it has a non-standard *Central Meridian* / *Grid Origin*, a non-standard *False Origin* and the *UK Transverse Mercator* (*UKTM*) *Projection* used is referenced to the Airy *Spheroid* / OSGB 36 *Datum*, rather than to the *WGS 84 Spheroid / Datum*.

a. **UTM Grid Covering UK.** The *UTM Grid* covering UK has an inconvenient change of *Zone* on either side of the *Prime* (*Greenwich*) *Meridian*, which straddles UK. To the West, *Zone* '30U' has a *Central Meridian* of 3˚W and covers from 6˚W-0˚, while to the East, *Zone* '31U' has a *Central Meridian* of 3˚E and covers from 0˚-6˚E (see Fig 4-26 below). Thus to use *UTM Grid* in UK would mean using coordinate systems from several different *Zones*.

b. **British National Grid.** To overcome this difficulty, the *British National Grid* uses a *Grid Origin* of 49˚N 002˚ W with a non-standard *Central Meridian* of 2˚W. Non-standard *False Eastings* of 400,000 metres and *False Northings* of 100,000 metres are applied to give a *False Origin* located to the South and West of UK (see Fig 4-27 overleaf). This choice allows all of UK (except Northern Ireland) to be covered by a single (non-standard) *Zone* with positive *Grid* coordinates. Unlike *UTMG Zones* which are normally limited to a width of 6˚(*Longitude)*, the *British National Grid* is approximately 12½˚ wide at 61˚N and 10¾˚wide at 52˚N, thus leading to substantial *Grid Convergence* at the extremities. Due to national boundaries, the overall shape of the *British National Grid* is not entirely regular (see Fig 4-27 overleaf).

c. **Spheroid and Projection.** For historical reasons, the *British National Grid* is based on the *UK Transverse Mercator* (*UKTM*) *Projection* referenced to the Airy *Spheroid* / OSGB 36 *Datum*. Particular care must be taken when transferring *British National Grid* coordinates to *Latitude / Longitude* if a different *Spheroid* (eg *WGS 84*) is involved.

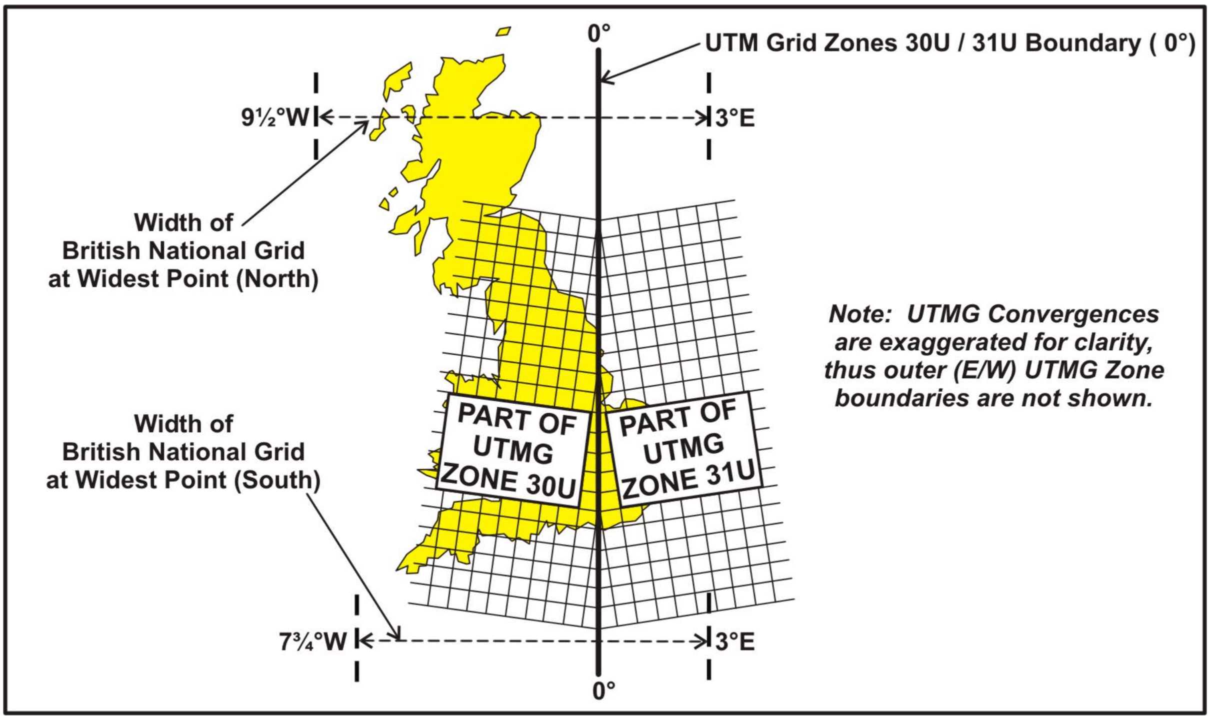


Fig 4-26. UTM Grid Zones Covering UK and British National Grid Widths

(0452) d. **Simplified Diagram of British National Grid.** A simplified diagram of the *British National Grid* (source STANAG 2211) is at Fig 4-27 below.

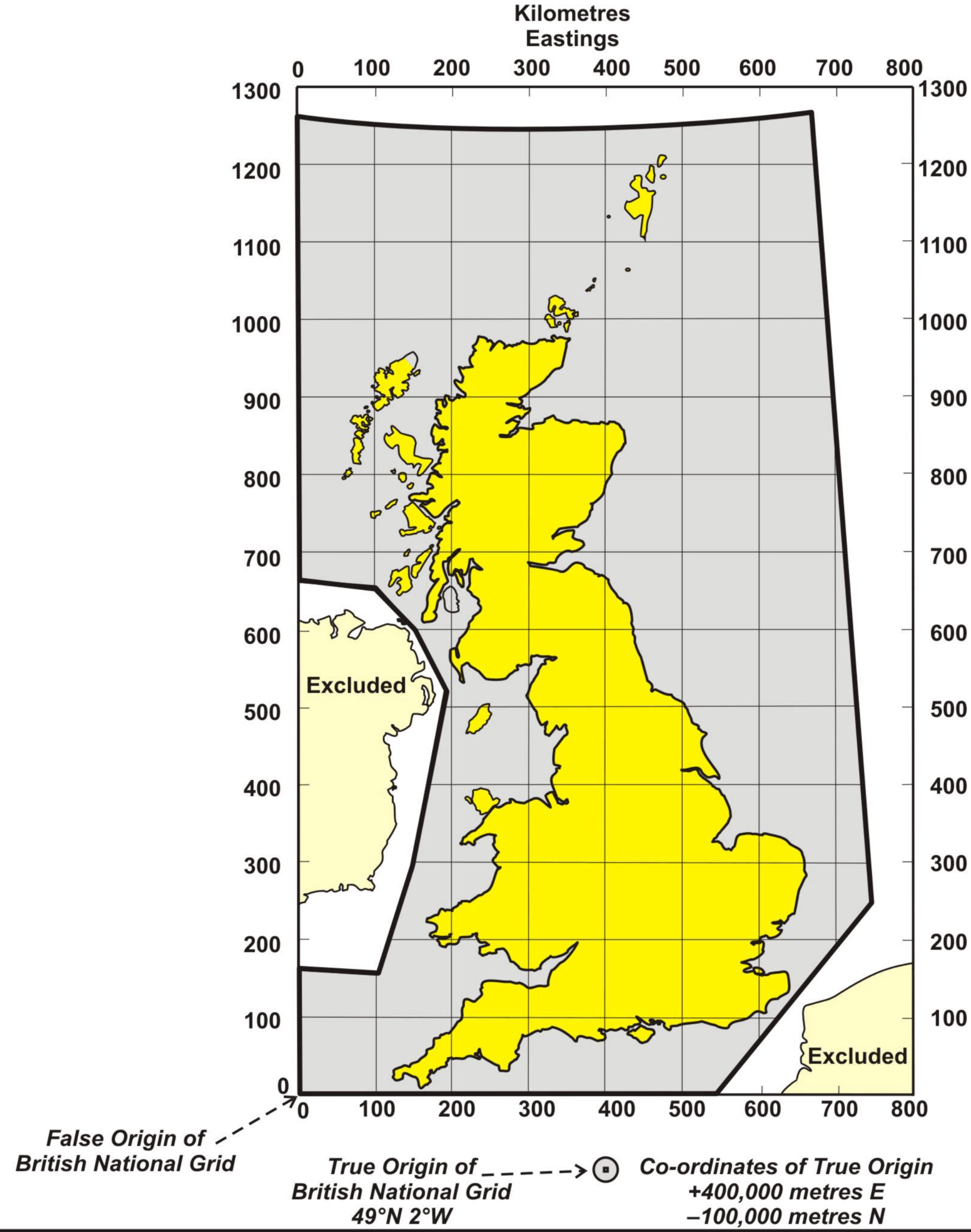


Fig 4-27. Simplified Diagram of British National Grid (*source STANAG 2211)*

0453. Other National Grids

For the same reasons that the *British National Grid* differs from *UTM Grid*, certain other nations have dedicated *Grids* with non-standard *Central Meridians*, *False Origins and Spheroids*; these include the *Irish Grid*, the *Nord Maroc Grid*, the *Sud Maroc Grid*, the *Nord Algerie Grid*, the *Sud Algerie Grid*, the *Nord Tunisie Grid* and the *Sud Tunisie Grid*. Details of the above *Grids* are contained in NATO STANAG 2211 (available in the public domain).

0454. Grivation

The term '*Grivation*' is occasionally used for the combination of (*Grid*) *Convergence* and *Magnetic Variation*, and is the angular difference between *Magnetic North* and *Grid North*.

CHAPTER 5

THE SAILINGS (2) - MORE COMPLEX CALCULATIONS

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CHAPTER 5

THE SAILINGS (2) - MORE COMPLEX CALCULATIONS

0501. Scope of Chapter

a. **Sailings (1).** Chapter 2 introduced *'Sailings* (1)' (ie *Parallel Sailing, Plane Sailing, Mean Latitude Sailing / Corrected Mean Latitude Sailing, Traverse Sailing* and brief overviews of *Mercator Sailing* and *Spherical / Spheroidal Great Circle Sailing*).

b. **Sailings (2).** Chapter 5 deals with the more complex calculations of '*Sailings* (2)', consisting of:

- *Spherical Mercator Sailing*.
- *Spherical Great Circle Composite Track* and *Vertex.*
- *Spheroidal Rhumb Line Sailing*.
- *Spheroidal Great Circle Sailing*.

0502-0509. Spare

SECTION 1 - SPHERICAL MERCATOR SAILING

0510. Mercator Sailing and Meridional Parts Overviews

A very brief overview of *Mercator Sailing* is given at Para 0207; a detailed explanation of *Meridional Parts* is at Para 0422.

a. **Meridional Parts.** The number of *Meridional Parts* on a *Meridian* of the *Sphere* for any *Latitude* ϕ is established at Para 0422e, as follows:

$$Meridional\ Parts = 7915.7045\ log_{10} tan\left(45° + \frac{\phi°}{2}\right) \qquad \text{. . . (formula 4.1)}$$

b. **Difference of Meridional Parts (DMP).** The term '*Difference of Meridional Parts* (*DMP*)' is explained at Para 0422d; an extract is repeated below for the convenience of readers:

(Extract fromPara 0422d): Where the two positions are both remote from the *Equator*, their relative position may be determined by the difference between their individual *Meridional Parts*, which gives the number of *Longitude Units* in the length of a *Meridian* between the two *Parallels of Latitude*. **This length is usually referred to as the '*Difference of Meridional Parts'* and written as '*DMP*'.**

0511. Spherical Rhumb Line Course and Distance From Meridional Parts

Spherical Mercator Sailing (*Rhumb Line) Course* and *Distance* may be found by *Spherical Meridional Parts* calculations, although in different circumstances the appropriate or inappropriate choice of a particular formula can affect the accuracy of the calculation.

a. **Difference of Meridional Parts.** From the explanation at Para 0422d (repeated at Para 0510b previous page), the number of *Meridional Parts* in the length *MT* of a *Meridian* on a *Mercator Projection* chart between the *Parallels of Latitude* through two points *F* and *T* (see Figs 5-1a/b below) is the '*Difference of Meridional Parts* (*DMP*)'.

b. **Course by Meridional Parts.** From Para 0511a (above), it follows that:

- **Same Hemisphere.** If *F* and *T* are on the same side of the *Equator* (see Fig 5-1a below), then:

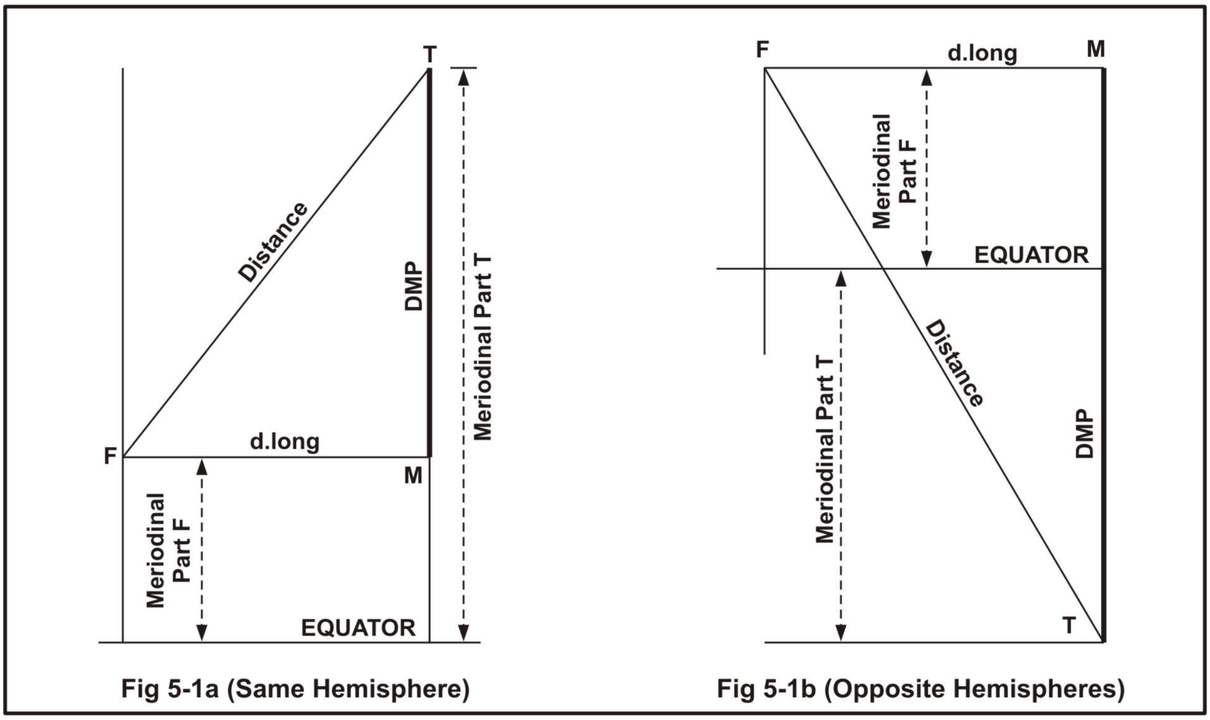
Meridional Parts TF = Meridional Parts T minus *Meridional Parts F* . . . **5.1**

- **Opposite Hemispheres.** If *F* and *T* are on opposite sides of the *Equator* (see Fig 5-1b below), then:

Meridional Parts TF = Meridional Parts T plus *Meridional Parts F* . . . **5.2**

- **Course.** From the triangle *FTM* (see Fig 5-1a/b below), it follows:

$$tan\ Course = \frac{FM}{MT} = \frac{d.long\ \ (E\ /\ W)}{DMP\ \ (N\ /\ S)} \qquad ...\ \mathbf{5.3}$$



Figs 5-1a/b. Difference of Meridional Parts - Same and Opposite Hemispheres

(0511) c. **Course by Departure.** An alternative method for finding the *Course* is by the *Departure* method (see Para 0204a):

$$\tan Course = \frac{Departure}{d.lat} \qquad \text{. . . (formula 2.4)}$$

d. **Comparison of Methods for Finding Course.** The relation between the two methods of finding the *Course* is shown at Fig 5-2 (below). The use of the *Departure* formula (2.4) involves finding a *Corrected Mean Latitude* (see Paras 0205b/c), if an error in the *Course* is to be avoided. For this reason, the *DMP* formula is preferred.

- **Course by Meridional Parts.** In the *Meridional Parts* method, *d.lat* (FM_1) is stretched into *DMP* (*FM*) and *d.long* (*MT*) remains unchanged, ie:

$$\tan Course = \frac{d.long}{DMP} \qquad \text{. . . (formula 5.3)}$$

- **Course by Departure.** In the *Departure* method, *d.lat* (FM_1) remains unchanged and *d.long* (*MT*) is compressed into *Departure* (MT_1), ie:

$$\tan Course = \frac{Departure}{d.lat} \qquad \text{. . . (formula 2.4)}$$

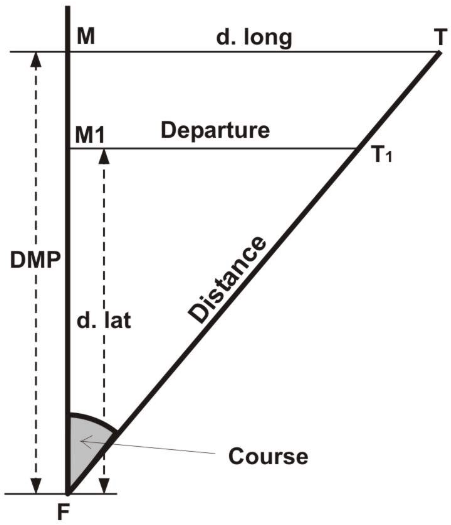


Fig 5-2. Comparison of Methods for Finding Course

e. **Finding Rhumb Line Distance - Method 1.** The *Course* angle obtained by formula (5.3) is exact, irrespective of the length of *Distance* (*FT*). The *Rhumb Line Distance*, as in *Plane Sailing*, is obtained from formula (2.3) re-arranged as follows:

$$Distance = d.lat \sec Course \qquad \text{. . . } \mathbf{5.4}$$

Formula (5.4) is quite satisfactory in use for *Course* angles approaching 90. There is, however, a fundamental weakness in this formula at *Course* angles between 60˚ and 90˚ because (see Note 2-1 at Para 0204a), small errors in the *Course* introduce increasingly large errors in the *Distance*. When using tables in these circumstances it is preferable to use the formula in Method 2 (see Para 0511f overleaf). See also Note 5-1 overleaf.

(0511) f. **Finding Rhumb Line Distance - Method 2.** Instead of Method 1 (Para 0511e previous page), the *Rhumb Line Distance* may also be obtained by formulae (5.5) or (5.6), particularly when the *Course* angle is between 60˚ and 90˚.

either: $$Distance = d.lat \frac{d.long}{DMP} \text{ cosec } Course \qquad \ldots \mathbf{5.5}$$

or: $$Distance = Departure \text{ Cosec } Course \qquad \ldots \mathbf{5.6}$$

Note 5-1. *The Rhumb Line Distance may also be found from formula (2.7) using a Corrected Mean Latitude, but as formula (2.7) can be manipulated into the Meridional Parts formula (5.3) by use of formulae (2.5) and (2.4), this will always give the same results for Course (and thence Distance) as formula (5.3).*

Example 5-1: Rhumb Line Course and Distance (Spherical Mercator Sailing)

What is the *Rhumb Line Course* and *Distance* by *Mercator Sailing* from F (45˚N, 140˚E) to T (65˚N, 110˚W) (the same positions as in the *Great Circle* Example 2-6 at Para 0211)?

$$d.long\ (FT) = 110°E = 6600'E$$
$$d.lat\ (FT) = 20°N = 1200'N$$

- **DMP (FT):**

... (formula 4.1)

$$Meriodinal\ Parts\ T = 7915.7045 \log_{10} \tan\left(45° + \frac{65°}{2}\right) = 5178.81$$

$$Meriodinal\ Parts\ F = 7915.7045 \log_{10} \tan\left(45° + \frac{45°}{2}\right) = 3029.94$$

$$DMP\ (FT) = 2148.87N$$

- **Course:**

$$\tan Course = \frac{d.long(E)}{DMP(N)} \qquad \ldots \text{(formula 5.3)}$$

$$= \frac{6600}{2148.87}$$

$$= 3.0713817$$

$$Course = N71.965457° = 072°$$

- **Distance:**

Either:

$$Distance = d.lat \sec Course \qquad \ldots \text{(formula 5.4)}$$

$$= 1200' \sec 71.965457° = 3876'.09$$

Or:

$$Distance = d.lat \frac{d.long}{DMP} \text{ cosec } Course \qquad \ldots \text{(formula 5.5)}$$

$$= 1200' \times 3.0713817 \text{ Cosec } 71.965457° = 3876.09'$$

0512-0519. Spare

SECTION 2 - SPHERICAL GREAT CIRCLE COMPOSITE TRACK / VERTEX

0520. Finding the Position of the Vertex of a Great Circle

The concepts of the *Vertex* and the *Composite Track* were introduced at Para 0209. The calculations of the *Vertex* and the *Composite Track* are given below.

a. **The Vertex.** The point at which a *Great Circle* most nearly approaches the *Pole* is called its *Vertex* - shown as '*V*' at Fig 5-3 (below). If a series of *Parallels of Latitude* are drawn, one *Parallel of Latitude* will touch the *Great Circle FT* at *V*. As the *Great Circle* and the *Parallel of Latitude* touch at *V* and the *Meridian PV* cuts the *Parallel of Latitude* at right angles, it also cuts the *Great Circle* at right angles; thus the *Spherical Triangles PFV* and *PTV* are both right-angled at V (ie angles *PVF* and *PVT* = 90˚) and can be easily solved mathematically.

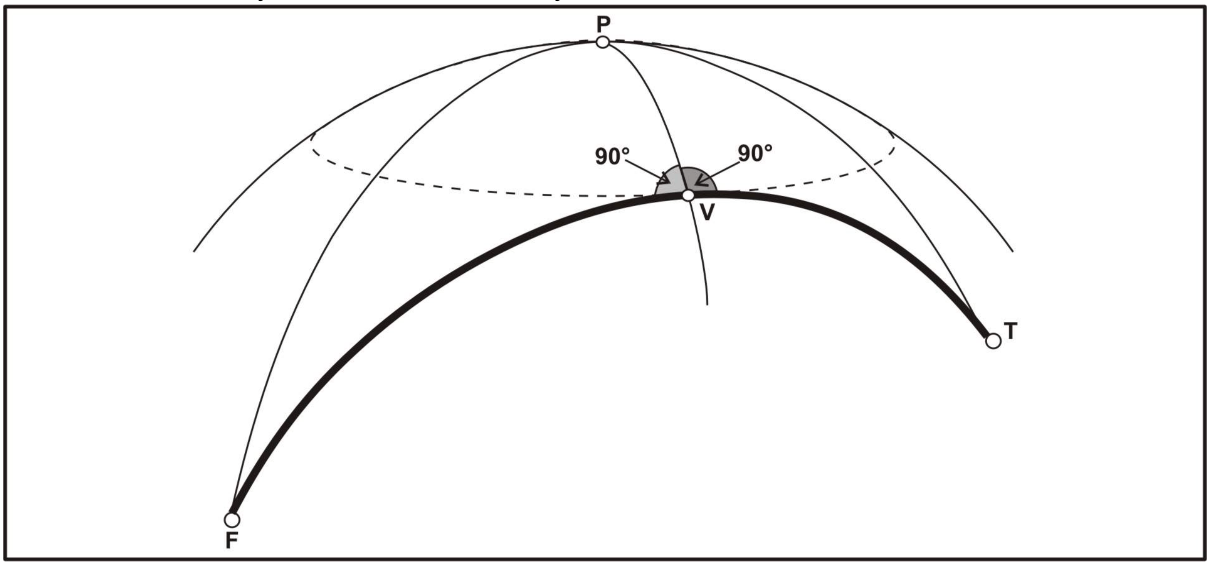


Fig 5-3. The Vertex (V) of a Great Circle (FT)

b. **Location of the Vertex.** The *Vertex* may NOT be located between *F* and *T*. Only one plane cuts the *Sphere* to create a *Great Circle* joining *F* and *T*. Depending on the positions of *F* and *T*, the *Vertex* (ie point at which it most nearly approaches the *Pole)* may be beyond *F* or *T* (eg if the final *Course* angle is < 90˚, the *Vertex* lies beyond *T*).

c. **Longitude of the Vertex.** The *Vertex Longitude* can be found from the formula:

$$\tan d.long\ VT = \tan Lat\ F \cot Lat\ T \operatorname{cosec} d.long\ FT - \cot d.long\ FT \quad \ldots \mathbf{5.8}$$

d. **Latitude of the Vertex.** The *Vertex Latitude* may be found from the formula:

$$\cot Lat\ V = \cot Lat\ F \cos d.long\ FV \quad \ldots \mathbf{5.9}$$

e. **Napier's Rules.** Alternatively, if the initial *Course* has been found, the position of *Vertex V* can be obtained from *Napier's Rules* (see Appendix 2 Para 10). Thus:

$$\cos Lat\ V = \cos Lat\ F \sin \text{initial Course} \quad \ldots \mathbf{5.10}$$

$$\tan d.long\ FV = \operatorname{cosec} Lat\ F \cot \text{initial Course} \quad \ldots \mathbf{5.11}$$

f. **Example.** An illustration of a *Vertex* calculation is at Example 5-2 (overleaf).

(0520f continued)
Example 5-2: Calculating the Position of the Vertex (Spherical Great Circle Sailing)

On the *Spherical Great Circle* F (45˚N, 140˚E) to T (65˚N, 110˚W) in Example 2-6, find the *Vertex* position.

From Napier's Rules:

- **Latitude of Vertex:**

$$\cos \text{Lat Vertex} = \cos \text{Lat F} \sin \text{initial Course} \quad \text{. . . (formula 5.10)}$$

$$= \cos \text{Lat } 45^\circ \sin 28.122305^\circ$$

$$\text{Lat Vertex} = 70.530896^\circ N = 70^\circ 31'.85N$$

- **Longitude of Vertex:**

$$\tan \text{d.long FV} = \text{cosec Lat F} \cot \text{initial Course} \quad \text{. . . (formula 5.11)}$$

$$= \text{cosec } 45^\circ \cot 28.122305^\circ$$

$$\text{d.long FV} = 69.297735^\circ E = 69^\circ 17'.86E$$

$$\text{Long Vertex} = 150^\circ.70227W = 150^\circ 42'.14W$$

0521. Calculating Great Circle Waypoints for a Mercator Chart

The simplest method of calculating *Great Circle Waypoints* for a *Mercator Projection* chart is to select and transfer them from a *Gnomonic Projection* chart (see Para 0441). However, if a *Gnomonic Projection* chart is not available, the *Waypoints* can be calculated with reference to the *Vertex* or to an intermediate *Meridian*. The latter method is used by HM Nautical Almanac Office's *NAVPAC* program (available from *UKHO* as DP330; see details at Paras 0210a / 0551a).

a. **Vertex Method.** There is no simple formula for finding where a *Great Circle* track cuts *Parallels of Latitude* without knowing the position of the *Vertex*. The '*Vertex* Method' is as follows

- **Vertex.** Find the position of the *Vertex V* [formulae (5.10), (5.11)].
- **Waypoints.** *Waypoints* are calculated from the following formula, where *G* is any position on the *Great Circle* joining *F* and *T* (see Fig 5-4 opposite). The results may be tabulated (see Example 5-3 opposite).

Either: $\cos \text{d.long VG} = \cot \text{Lat V} \tan \text{Lat G}$ . . . **5.12**

Or: $\tan \text{Lat G} = \tan \text{Lat V} \cos \text{d.long VG}$ . . . **5.13**

b. **Meridian Method.** The '*Meridian* Method' may be used to find the *Latitude* where a *Great Circle* track cuts an intermediate *Meridian*, without the need to find the position of the *Vertex*. However, if a number of intersections are required, it is simpler to use the *Vertex* method at Para 0521a (above).

$$\tan \text{Lat G} = \frac{\tan \text{Lat F} \sin \text{d.long GT} + \tan \text{Lat T} \sin \text{d.long FG}}{\sin \text{d.long FT}} \quad \text{. . . } \mathbf{5.14}$$

c. **Plotting.** Once calculated, the *Waypoints* may be plotted on the *Mercator Projection* chart and joined by means of a series of *Rhumb Lines* (see Para 0441b).

(0521 continued)

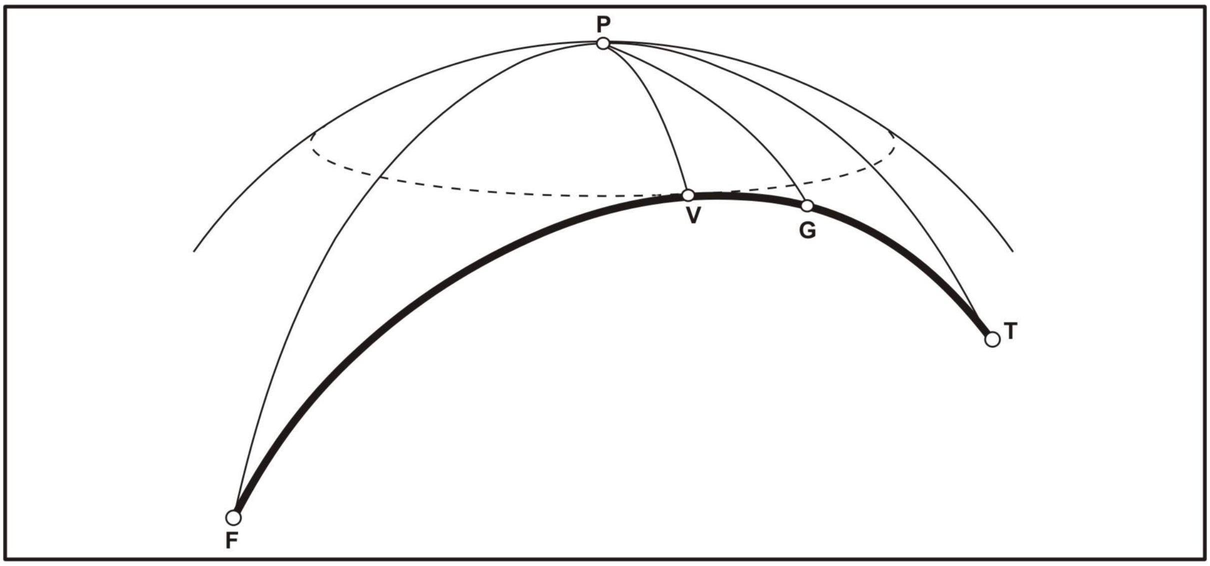


Fig 5-4. Calculating Great Circle Waypoints ('G', etc) for a Mercator Chart

Example 5-3: Calculating Spherical Great Circle Waypoints for a Mercator Chart

Find the *Latitude*s where the *Spherical Great Circle* track F (45˚N, 140˚E) to T (65˚N, 110˚W) used in Example 5-2 (opposite), cuts the *Meridian*s of 150˚E, 160˚E, 170˚E, 180˚, 170˚W, 160˚W, 150˚W, 140˚W, 130˚W, 120˚W. (F (45˚N, 140˚E), T (65˚N, 110˚W).)

- **Calculation and Tabulation.** Using formula (5.13) for a series of *Latitudes*, a Table / Tables (similar to Tables 5-1a/b, below) may be prepared.

Table 5-1a. Table of Calculated Great Circle Waypoints for a Mercator Chart

Longitude	150˚E	160˚E	170˚E	180˚	170˚W
VG (d.long)	59˚.298	49˚.298	39˚.298	29˚.298	19˚.298
Latitude *D*	55˚18'.1	61˚32'.3	65˚26'.9	67˚56'.1	69˚28'.0

Table 5-1b. Table of Calculated Great Circle Waypoints for a Mercator Chart

Longitude	160˚W	150˚W	140˚W	130˚W	120˚W
VG (d.long)	9˚.298	0˚.702	10˚.702	20˚.702	30˚.702
Latitude *D*	70˚17'.5	70˚31'.8	70˚12'.8	69˚17'.9	67˚39'.0

0522. Calculating the Composite Track

The concepts and definitions for the *Composite Track* were explained at Para 0209, to which reference should be made before proceeding further. In summary, the *Composite Track* is formed by two *Great Circle* arcs joined at their *Vertices* by the '*Safe Parallel*' *of Latitude*. It is normally used to avoid high *Latitude*s due to the danger of ice.

a. **High Latitude Situation.** Fig 5-5 (below) illustrates a high *Latitude* situation, showing:

- The *Safe Parallel [of Latitude]* (*LABM*).
- The *Great Circle* (*FLVMT*) joining *F* and *T*.
- The *Composite Track* (*FABT*) in which *FA* and *BT* are *Great Circle* arcs touching the *Safe Parallel* at *A* and *B*.
- *AB* is part of the *Safe Parallel* and comprises part of the *Composite Track*.

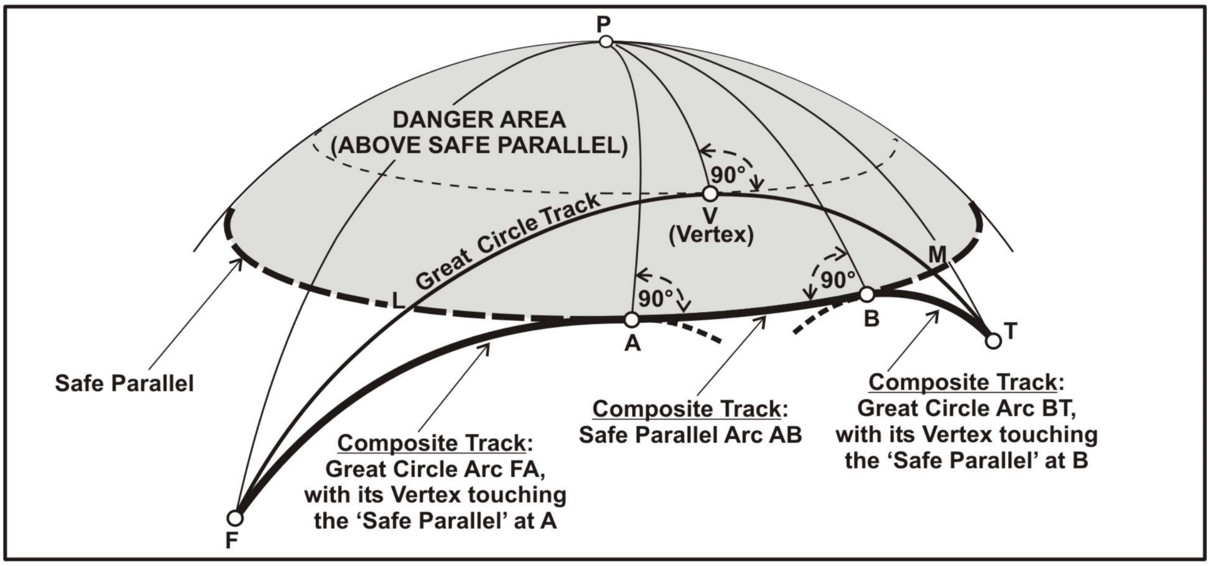


Fig 5-5. High Latitude Situation - The Composite Track (*Copy of Fig 2-12*)

b. **Calculation.** The positions of *A* and *B* are quickly found because the *Course* angles at *A* and *B* are right angles (*Meridians* and *Parallels of Latitude* intersect at 90˚). Also, along the *Parallel of Latitude AB,* the ship is steering a *Course* of 090˚/270˚.

- If the *Latitude* of the *Safe Parallel* (*LABM*) is ϕ :

$$\mathrm{AB} = d.long \cos \phi \qquad ..\ \mathbf{5.15a}$$

- Using the formulae for a *Spherical* right-angled triangle:

$$\cos PF = \cos PA \cos FA$$

$$\cos FA = \frac{\cos PF}{\cos PA} \qquad ...\ \mathbf{5.15}$$

$$\sin FPA = \frac{\sin FA}{\sin PF} \qquad ...\ \mathbf{5.16}$$

- Formula (5.15) gives the length of the *Great Circle* arc *FA*. Formula (5.16) gives the *d.long* between *F* and *A* by which the position of *A* may be found. *BT* may also be found in a similar manner.

(0522 continued)

Example 5-4: Calculating Spherical Composite Track

Find the *Spherical Composite Track Distance* F (45˚N, 140˚E) to T (65˚N, 110˚W), when a *Safe Parallel* of *Latitude* 67˚N is applied (see Fig 5-6 below). F and T are the same coordinates as in Examples 5-2 and 5-3.

- **Total Distance.** The total *Composite Track Distance* = FA + AB + BT

- **Distance FA:**

$$\cos FA = \frac{\cos PF}{\cos PA} = \frac{\cos 45°}{\cos 23°} \qquad \text{. . . (formula 5.15)}$$

$$FA = 39.809911° = 39°48'.6 = 2388.6 \text{ miles}$$

- **Distance BT:**

$$\cos BT = \frac{\cos PT}{\cos PB} \qquad \text{. . . (formula 5.15)}$$

$$BT = 10.075896° = 10°04'.6 = 604.6 \text{ miles}$$

- **Longitude A:**

$$\sin FPA = \frac{\sin FA}{\sin PF} = \frac{\sin 39.809911°}{\sin 45°} \qquad \text{. . . (formula 5.16)}$$

$$FPA = 64.882575° = 64°53'E$$

Longitude of A is thus: 180 + 40 - 64˚53′ = 155˚07′W.

- **Longitude B:**

$$\sin TPB = \frac{\sin TB}{\sin PT} = \frac{\sin 10°.075896}{\sin 25°} \qquad \text{. . . (formula 5.16)}$$

$$TPB = 24.454656° = 24°27'.3\ W$$

Longitude of B is thus: 110 + 24˚27′.3 = 134˚27′.3 W.

- **Distance AB:**

$$FPT = 40° + 70° = 110°$$

Latitude of A and B is 67˚N (*Safe Parallel of Latitude*)

$$\begin{aligned} AB &= APB \cos 67° \qquad \text{. . .(formula 5.15a)} \\ &= [FPT - (FPA + TPB)] \cos 67° \\ &= 20.662769° \cos 67° \\ &= 8.0735870° = 484.4 \text{ miles} \end{aligned}$$

- **Composite Track Distance FABT:**

$$FABT = 2388.6 + 604.6 + 484.4 \text{ miles} = 3477.6 \text{ miles}$$

- **Courses from F to A and B to T:** Course from F to A and B to T may be found by any of the methods listed at Para 0210.

0523-0529. Spare.

SECTION 3 - SPHEROIDAL RHUMB LINE SAILING

0530. Rhumb Line Methods and Accuracies

Provided that the *Meridional Parts* and the length of the *Meridional Arc* (eg *Arc 'EM'* in Fig 5-6 below), between the *Latitude*s of the two places are computed for the *Spheroid*, an accurate *Rhumb Line Course* and *Distance* may be calculated for any *Spheroid*. If *Spherical* formulae (ie Paras 0204-0205 and 0511) are used without correction for the *Spheroid*, the *Rhumb Line* solution will be inaccurate (up to 0.5% error), depending on *Course*, *Distance* and *Latitude*.

0531. Calculating the Spheroidal Rhumb Line Course and Distance

a. **Meridional Parts for the Spheroid.** *Meridional Parts* for the *Clarke Spheroid (1880)* are tabulated in Norie's Nautical Tables (NP 320). It is shown at Appendix 5 that the *Meridional Parts 'm'* may be evaluated for any *Spheroid*, from the formula:

$$m = \frac{10800}{\pi}\left[\log_e \tan\left(45° + \frac{L°}{2}\right) - e^2 \sin L - \frac{1}{3}e^4 \sin^3 L - \frac{1}{5}e^6 \sin^5 L - \ldots\right] \quad \ldots \mathbf{5.21a}$$

From (5.21a), a simplified numerical formula (ignoring e^6 and higher powers) for the *WGS 84 Spheroid*, giving the *Meridional Parts* '*m*' correct to four decimal places is:

$$m = 7915.7045 \log_{10} \tan\left(45° + \frac{L°}{2}\right) - 23.01358 \sin L - 0.05135 \sin^3 L \quad \ldots \mathbf{5.21b}$$

b. **Calculation of the Rhumb Line Course.** The *Rhumb Line Course* may now be calculated from formula (5.22), where m_1 and m_2 are the *Meridional Parts* evaluated from formula (5.21b) for the start and end points of the *Rhumb Line*:

$$\tan Course = \frac{d.long}{m_1 \pm m_2} = \frac{d.long}{DMP} \quad \ldots \mathbf{5.22}$$

c. **Length of the Meridional Arc.** It is shown at Appendix 5 that the length 'ℓ' of the *Meridional Arc 'EM'* (see Fig 5-6 below) may be found from the formula:

$$\ell = a\left(\phi - \frac{e^2\phi}{4} - \frac{3e^2}{8}\sin 2\phi - \frac{3e^4}{64}\phi - \frac{3e^4}{32}\sin 2\phi + \frac{15e^4}{256}\sin 4\phi\right) \quad [\phi \text{ in radians}] \ldots \mathbf{5.24}$$

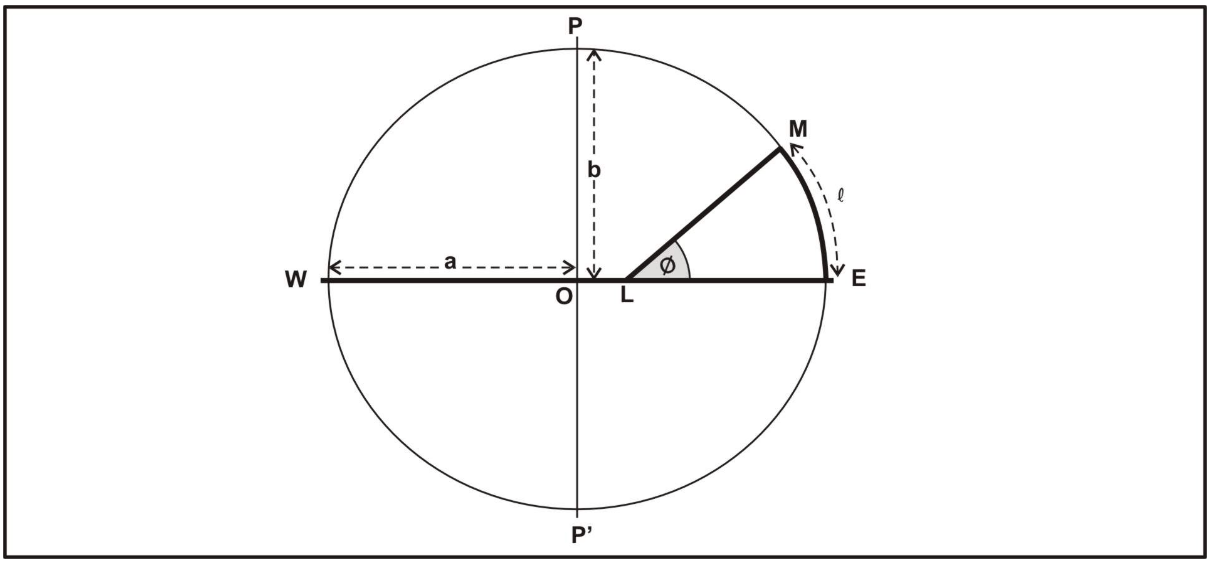


Fig 5-6. Length of the Spheroidal Meridional Arc

(0531) d. **Length of the Meridional Arc in WGS 84.** For the *WGS 84 Spheroid*, where values of a and e^2 from Table 3-1 (see Para 0322) are applied (taking care to select the correct units), formula 5.24 may be simplified with negligible loss of accuracy, as:

$$\ell = 60.007\phi - 8.660 \sin 2\phi + 0.009 \sin 4\phi \qquad [\phi \text{ in degrees}] \quad \ldots \mathbf{5.24a}$$

e. **Calculation of the Rhumb Line Distance.** The *Rhumb Line Distance* may now be calculated, using the *Course* calculated from formula (5.22) and the *Meridional Arcs* 'ℓ' calculated from formula (5.24) for the start and end points of the *Rhumb Line.*

$$\textit{Distance} = (\ell_1 \pm \ell_2) \sec \textit{Course} \quad \ldots \mathbf{5.23}$$

Example 5-5: Calculating Spheroidal Rhumb Line Course and Distance

Find *Rhumb Line Course* and *Distance* from F (40˚43'N, 74˚00'W) to T (55˚45'S, 37˚37'E) on the *WGS 84 Spheroid*.

- **Meridional Parts for the Spheroid:**

$$m = 7915.7045 \log_{10} \tan\left(45° + \frac{L°}{2}\right) - 23.01358 \sin L - 0.05135 \sin^3 L \quad \ldots \text{(formula 5.21b)}$$

Thus: $m_1\ (F) = 2664.094$
$m_2\ (T) = \underline{-4028.114}$
$DMP = -6692.208$ (*ie 6692.208 S*)

- **Calculation of the Rhumb Line Course:**

$$\tan \textit{Course} = \frac{d.long}{m_1 \pm m_2} = \frac{d.long}{DMP} \quad \ldots \text{(formula 5.22)}$$

d.long = 111˚37'E = 6697'E

Thus: $\tan \textit{Course} = 6697 \div -6692.208$
$\textit{Course} = -45°.021 = S45.021°E = 134.98°$ ... (formula 5.22)

- **Calculation of the Length of the Meridional Arcs:**

$$\ell = a\left(\phi - \frac{e^2\phi}{4} - \frac{3e^2}{8}\sin 2\phi - \frac{3e^4}{64}\phi - \frac{3e^4}{32}\sin 2\phi + \frac{15e^4}{256}\sin 4\phi\right) \quad \ldots \text{(formula 5.24)}$$

$a = 3443.918467$ *n. miles*
Lat F (ϕ) $= 40.716° = 0.71063989$ *radians*
Lat T (ϕ) $= 55.750° = 0.97302106$ *radians*
Thus: $\ell_1 = 2434.724$ *n. miles* $\quad \ell_2 = 3337.326$ *n. miles*

- **Calculation of the Rhumb Line Distance:**

$$\textit{Distance} = (\ell_1 \pm \ell_2) \sec \textit{Course} \quad \ldots \text{(formula 5.23)}$$

$$= 8165.83 \text{ n. miles} \quad \ldots \text{(formula 5.23)}$$

Note 5.2. *Lengths of the Meridional Arcs for WGS 84 by the simplified formula (5.24a) are:*
$\ell_1 = 2434.724$ *n. miles,* $\ell_2 = 3337.327$ *n. miles*

Note 5.3. *Full accuracy figures were used in the above calculations, but all intermediate and final answers shown (above) have been simplified to a lower numbers of decimal places.*

0532-0539. Spare.

SECTION 4 - SPHEROIDAL GEODESIC (GREAT CIRCLE) SAILING

0540. Spheroidal Geodesic (Great Circle) Terminology, Methods and Accuracies

a. **Geodesic - Definition and Properties.** The definition of a *Great Circle* was stated and the concept of a *Geodesic* was introduced at Para 0111c. A full explanation of a *Geodesic* is as follows.

- **Geodesic - Definition.** A *Geodesic* is the intersection of a *Spheroidal* surface and a plane which passes through the centre of the *Spheroid*.
- **Geodesic - Properties.** A *Geodesic* is the shortest distance between two points on the surface of a *Spheroid*.
- **Geodesic - Equivalence to Great Circle Properties.** The *Geodesic* (on a *Spheroid*) is the equivalent of a *Great Circle* on a *Sphere*; it properties and use are similar to those of a *Great Circle* (see details at Para 0115).

b. **Geodesic - Common Usage and Precise Terminology.** In everyday colloquial use, the terms *Great Circle* (in lieu of *Geodesic*) and *Small Circle* are usually applied to the Earth's *Oblate Spheroidal* shape. However, within this Section (Paras 0540-0541) and in Appendix 5, to avoid any possible mathematical confusion, the precise terminology is used. Elsewhere in this book, the colloquial usage of *Great Circle* is normally adopted to include the meaning of *Geodesic.*

c. **Methods for Calculating Geodesics.** There are a variety of solutions for computing the *Geodesic*; some of these use the *Geodetic Latitude* (see Para 0312) and some the *Parametric Latitude* (see Para 0313). Some of the formulae required are too complex for general use, but one of the most suitable formulae is the Andoyer-Lambert method using *Parametric Latitude,* which is fully explained at Para 0541 (opposite).

d. **Summary of the Andoyer-Lambert (Parametric Latitude) Method.** In this method, *Distance* and *Course* are pre-computed on a *Sphere* of radius equal to the semi-major axis of the *Spheroid* on which the positions are located. Corrections are then made to obtain the corresponding *Spheroidal* values.

e. **Accuracy of the Andoyer-Lambert (Parametric Latitude) Method.** The Andoyer-Lambert (*Parametric Latitude*) method is extremely accurate and has been adopted by the Royal Navy and the US Naval Oceanographic Office for navigational use. The magnitude of errors are as follows:

- **Distance Error.** This method has a maximum *Distance* error of 1 metre at 500 *n.miles* (0.00011 %) and 7 metres at 6000 *n.miles* (0.000063 %).
- **Course Error.** *Course* is correct to within 1 second of arc with this method.

0541. Spheroidal Geodesic - Parametric Latitude (Andoyer-Lambert) Method

a. **Sign Convention.** In this calculation, *Latitude* N, *Longitude* E, and *d.long* E are given a positive (+) value, while *Latitude* S, *Longitude* W, and *d.long* W are given a negative (-) value.

b. **Conversion of Geodetic to Parametric Latitudes.** *Geodetic Latitudes* (ϕ) are reduced to *Parametric Latitudes*(β) using '*a*' and '*b*' as the *Equatorial* and *Polar* radii respectively (see Para 0313) in formula (3.7):

$$\tan \beta = \frac{b}{a} \tan \phi \qquad \text{. . . (formula 3.7)}$$

c. **Initial Geodesic Course.** The *Geodesic Azimuth* (and thence initial *Course*) from *F* (*Parametric Latitude* β_1) to *T* (*Parametric Latitude* β_2) is found from formula (5.25):

$$\textit{tan Azimuth} = \frac{\sin d.long}{\cos \beta_1 \tan \beta_2 - \sin \beta_1 \cos d.long} \qquad \text{. . . } \mathbf{5.25}$$

d. **Spherical Distance.** The *Spherical Distance* (σ) is calculated in degrees from formula (2.9) and then converted into radians for use with the *Spheroidal Corrections* at Para 0541e (below):

$$\cos \sigma = \sin \beta_1 \sin \beta_2 + \cos \beta_1 \cos \beta_2 \cos d.long \qquad \text{. . . (formula 2.9)}$$

e. **Spheroidal Corrections.** The *Spheroidal Correction* s *M, N, U, V* are now calculated as follows:

$$M = (\sin \beta_1 + \sin \beta_2)^2 \qquad \text{. . . } \mathbf{5.25a}$$

$$N = (\sin \beta_1 - \sin \beta_2)^2 \qquad \text{. . . } \mathbf{5.25b}$$

$$U = \frac{\sigma - \sin \sigma}{1 + \cos \sigma} \quad [\sigma \textit{ in radians}] \qquad \text{. . . } \mathbf{5.25c}$$

$$V = \frac{\sigma + \sin \sigma}{1 - \cos \sigma} \quad [\sigma \textit{ in radians}] \qquad \text{. . . } \mathbf{5.25d}$$

f. **Further Small Spheroidal Corrections.** A further small *Spheroidal Correction* (in seconds) may be applied for extreme accuracy, but for practical purposes this may usually be ignored, and is not covered here.

g. **Geodesic Distance.** Finally, the *Geodesic Distance* is calculated from formula (5.26), where '*a*' is the *Equatorial* radius measured in *International Nautical Miles* and '*f* 'is the *Flattening* coefficient for the *Spheroid* in use (see Table 3-1 at Para 0322):

$$\textit{Geodesic Distance } [\textit{in radians}] = \sigma - \frac{f}{4}(MU + NV) \qquad \text{. . . } \mathbf{5.26a}$$

$$\textit{Geodesic Distance } [\textit{in nautical miles}] = a\left[\sigma - \frac{f}{4}(MU + NV)\right] \qquad \text{. . . } \mathbf{5.26} \quad [\sigma \textit{ in radians}]$$

h. **Example.** An illustration of this calculation is at Example 5-6 overleaf.

(0541 continued)

Example 5-6. Calculating Spheroidal Geodesic Initial Course and Distance

What is the *Geodesic* initial *Course* and *Distance* from point F (40˚43'N, 74˚00'W) to point T (55˚45'S, 37˚37'E) on the *WGS 84 Spheroid?* See Fig 5-7 (below).

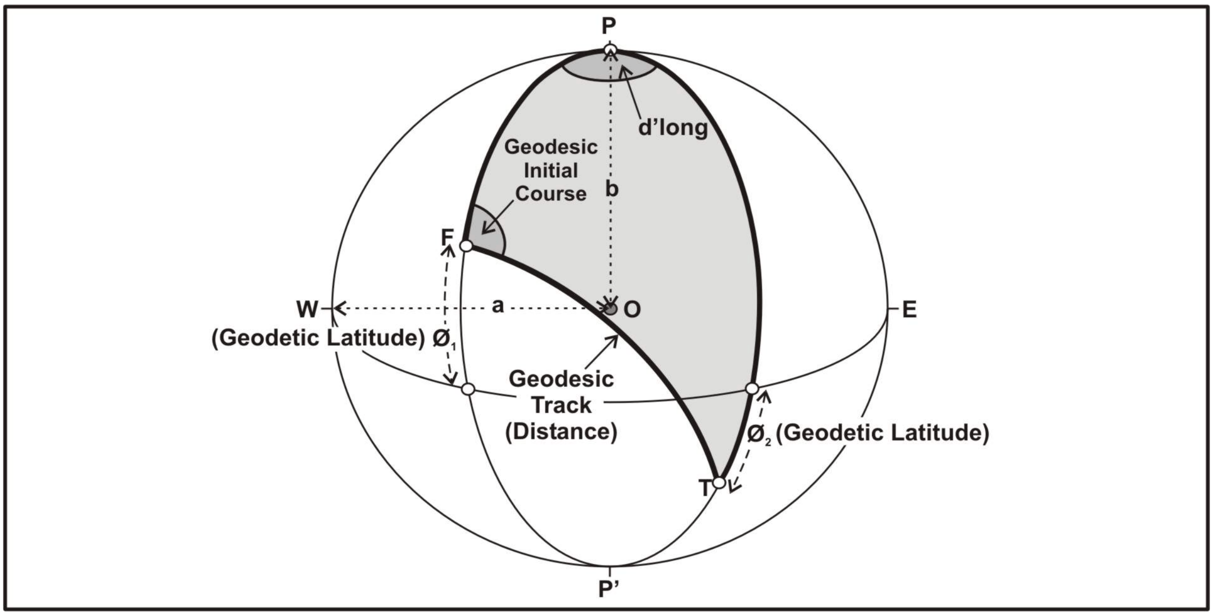


Fig 5-7. Spheroidal Geodesic Initial Course and Distance

- **Conversion of Geodetic to Parametric Latitudes:**

$a = 6{,}378{,}137m \div 1852 = 3443.918467$ *n. miles* (*see Table 3-1 at Para 0322*)
$b = 6{,}356{,}752.314m \div 1852 = 3432.37166$ *n. miles* (*see Table 3-1 at Para 0322*)

$$d.long = +111.61667^\circ$$

$$\phi_1 = +40.71667^\circ \qquad \phi_2 = -55.75^\circ$$

$$\tan\beta = \frac{b}{a}\tan\phi \qquad \text{. . . (formula 3.7)}$$

Thus: $\beta_1 = +40.62155172^\circ$ $\qquad \beta_2 = -55^\circ.66042739^\circ$

- **Initial Geodesic Course:**

$$tan\ Azimuth = \frac{sin\ d.long}{cos\ \beta_1\ tan\ \beta_2 - sin\ \beta_1 cos\ d.long} \qquad \text{. . . (formula 5.25)}$$

$$Azimuth = 133.140^\circ \text{ or } 313.140^\circ$$

By Inspection: $Course = 133.1^\circ$

- **Spherical Distance (σ):**

$$\cos\sigma = \sin\beta_1 \sin\beta_2 + \cos\beta_1 \cos\beta_2 \cos d.long \qquad \text{. . . (formula 2.9)}$$

$$\sigma = 134.0526724^\circ = 2.3396605\ radians$$

(0541 Example 5-6 continued)

- **Spheroidal Corrections:**

$$M = (\sin \beta_1 + \sin \beta_2)^2 \qquad \text{. . . (formula 5.25a)}$$
$$= 0.030502$$

$$N = (\sin \beta_1 - \sin \beta_2)^2 \qquad \text{. . . (formula 5.25b)}$$
$$= 2.180845$$

$$U = \frac{\sigma - \sin \sigma}{1 + \cos \sigma} \quad [\sigma \text{ in radians}] \qquad \text{. . . (formula 5.25c)}$$
$$= 5.320193$$

$$V = \frac{\sigma + \sin \sigma}{1 - \cos \sigma} \quad [\sigma \text{ in radians}] \qquad \text{. . . (formula 5.25d)}$$
$$= 1.804003$$

- **Geodesic Distance:**

$f = 0.0033528107$ (*see Table 3-1 at Para 0322*)

$$\text{Geodesic Distance } [\text{in nautical miles}] = a\left[\sigma - \frac{f}{4}(MU + NV)\right] \qquad \text{. . . (formula 5.26)}$$
$$[\sigma \text{ in radians}]$$

Geodesic Distance = *8045.775 n. miles*

Note 5.4. *Full accuracy figures were used in the above calculations, but all intermediate and final answers shown (above) have been simplified to a lower numbers of decimal places.*

0542-0549. Spare.

SECTION 5 - COMPARISON AND CHOICE OF METHODS

0550. Summary of Methods of Calculation Available (New)

a. **Mathematical Concepts and Calculations.** Chapters 1-5, supported by Appendices 1-5, have presented the reader with a detailed mathematical explanation of the shape of the Earth and the calculations necessary to create charts and plan *Sailings*. This provides the mariner with a thorough understanding of the fundamental 'tools of the trade', together with a working knowledge of their capabilities and limitations. It thus provides a firm foundation on which practical use of automated systems (eg *ECDIS / WECDIS* etc) may be carried out with confidence.

b. **Practical Solutions to Calculations.** In the current age of computerised navigation systems (eg *ECDIS / WECDIS*) and very capable stand-alone software (eg HM Nautical Almanac Office *NAVPAC* program [*UKHO* - DP 330]), in practice, it is unlikely that mariners will need to resort to lengthy hand-calculations, even with the assistance of programmable calculators or computer spreadsheets. However, care must be taken with computerised navigation systems (eg *ECDIS / WECDIS*) and stand-alone software that the user fully understands the parameters (ie *Spheroidal* / *Spherical* / *Flat Earth*) of the automatic calculation being performed. In the case of *Spheroidal* calculations, the user must also know which *Spheroid* / *Geodetic Datum* is being employed.

0551. Choice of Methods

a. **NAVPAC.** In the Royal Navy, the use of HM Nautical Almanac Offices's *NAVPAC* program (available from *UKHO* as DP 330) is regarded as the prime authoritative source for *Rhumb Line* and *Great Circle* calculations, as well as for astro navigation and other calculations. *NAVPAC* calculates *Rhumb Lines* on the *WGS 84 Spheriod / Geodetic Datum; Great Circle*s are calculated on a *Spherical* basis (ie not *Geodesics*), but a series of *Rhumb Lines* (with user-defined parameters) are also provided automatically for each *Great Circle* so that convenient *Rhumb Line* approximations may be selected and plotted for practical navigation (see Para 0208e and Para 0441b).

b. **ECDIS / WECDIS.** Most modern *ECDIS / WECDIS* provide a method of calculating *Rhumb Line* distance / courses and *Great Circle* distances on a variety of *Spheroids Datums* as well as on the *Sphere*. However, the facility provided in *NAVPAC* for automatic user-defined *Rhumb Line* approximations of *Great Circles* is not normally available; if it is not provided, this action will have to be carried out manually on the chart-display.

c. **Paper Chart Method.** The intended track may be plotted on a paper chart *(Mercator Projection* and/or *Gnonomic Projection* as appropriate) and the distance / course(s) measured.

d. **Manual Calculations by Hand.** In the event that none of the above methods are available, the necessary calculations (see Chapters 1 to 5 and Appendices 1 to 5) may have to be carried out by hand. In view of the mathematical complexity of some of the equations involved, the use of a computer spreadsheet is strongly recommended.

CHAPTER 6

CHARTS AND PUBLICATIONS - OVERVIEW

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CHAPTER 6

CHARTS AND PUBLICATIONS - OVERVIEW

0601. Scope of Chapter

Chapter 6 provides a brief summary of British Admiralty charts and publications. Further details of these products may be found at BR 45 Volume 7, and in *United Kingdom Hydrographic Office (UKHO*) publications 'The Mariners Handbook' (NP 100) and 'Catalogue of Admiralty Charts and Publications' (NP 131). This chapter replaces both Chapters 6 and 7 of the 1987 Edition of this book

0602-0609. Spare

SECTION 1 - CHARTING CONCEPTS AND POLICY

0610. UK Charting Policy

The policy followed by the *UKHO* is to chart all waters, ports and harbours in UK waters, UK Overseas Territories and certain Commonwealth and other areas on a *Scale* sufficient for the safe navigation of all vessels. Elsewhere overseas, Admiralty charts are schemed to enable ships to cross the oceans and proceed along the coasts of the world to reach the approaches to ports using the most appropriate *Scales*. In general, foreign ports are charted by *UKHO* at a *Scale* adequate for ships under *Pilotage* (but see Para 0611e). Major ports are charted on larger *Scales* commensurate with their importance or intricacy. Further details are at the *UKHO* publication 'The Mariners Handbook' (NP 100).

0611. Types of Charts

The following information is a brief overview only; for further details see BR 45 Volume 7 Chapter 1, 'The Mariners Handbook' (NP 100) Chapter 1 and 'Catalogue of Admiralty Charts and Publications' (NP 131).

a. **Standard Navigational Charts.** Standard navigational charts, which are usually *Mercator Projection* or *Transverse Mercator Projection* and available commercially from the *UKHO*, make up the vast bulk of a vessel's chart outfit. They are listed primarily in the 'Catalogue of Admiralty Charts and Publications' (NP 131) and (for RN / RFA vessels) in the 'Chart Correction Log and Folio Index' (NP 133B).

b. **'Fleet' and Other Protectively Marked Charts.** Certain charts and diagrams are available to authorised users only; they are not listed in NP 131 or made available for sale commercially. Some of these charts and diagrams are known as 'Fleet' charts and many are standard navigational charts overprinted with additional information; if so, they retain their normal chart number, prefixed by a letter. Their 'Protective Marking' status varies but, in some cases, where the additional data relates only to information (eg exercise area limits) now in the public domain, they have been downgraded to 'Unclassified' while still retaining their 'Fleet' status.

c. **Thematic Charts.** A variety of 'Thematic' charts are available from the *UKHO* including: Routeing Charts, Routeing Guides, *Gnomonic* Charts, Instructional Charts, Hydrographic Practice & Symbol Charts, UK Practice & Exercise (PEXA) Charts, Miscellaneous (UK & World Series Thematic) Charts, Astronomical Charts, Climatic Charts, Magnetic Variation Charts, Meteorological and Upper Air Charts, Territorial Sea Baseline Charts, Tidal Charts, *Bathymetric Charts,* and 'Plotting Diagrams and Sheets'. Details are listed in the 'Catalogue of Admiralty Charts & Publications' (NP 131).

(0611) d. **X Charts.** If a new regulation (eg new *Traffic Separation Scheme* [*TSS*]) affecting a chart is due to come into force on a particular date, *UKHO* will publish a *New Edition (NE)* of the chart well in advance showing the new information. At that time, the existing chart will be authorised to be prefixed 'X' and retained until the *NE* comes into force. In the interim, both charts should be corrected for *Notices to Mariners (NMs*).

e. **Foreign Charts.** *Foreign Charts* may be supplied by *UKHO* to RN / RFA vessels on request, if they are of a better *Scale* or more appropriate than the *UKHO* versions.

0612. Organisation of Charts

a. **Chart Folios.** Although charts may be supplied individually, they are also grouped together in 'folios' by geographical area. Chart outfits are normally supplied to RN / RFA vessels in folios, together with the associated *Navigational Publications (NPs)*.

- Folios 1-100 each provide all the available 'standard navigation charts' (see Para 0611a) for a particular geographical area.
- Folios in the '700 series' contain 'Fleet' charts (see Para 0611b).
- Folios in the '300 series' contain special diagrams, plotting sheets and 'Routeing' charts etc (see Para 0611c).

Chart folios are supplied with buckram covers of an appropriate colour and charts are arranged in numerical order within each folio. A list of unclassified chart folios is contained in 'Chart / Publication Outfits For HM Ships, RFAs and RMAS' (NP 104). A list of protectively marked folios, together with their contents, is listed in the 'Catalogue of Classified and Other Charts and Hydrographic Publications' (NP 111).

b. **Geographical Folio Coverage.** Approximate geographical limits of Folios 1-100 are in the 'Catalogue of Admiralty Charts & other Hydrographic Publications' (NP 131).

Note 6-1. *When deciding on the outfit, care must be taken to ensure sufficient folios are held, particularly if operating near the boundary of folios (eg the boundaries of folios 1, 7, 8, 9 and 16 are adjacent, in the southern North Sea / Thames Estuary / Dover Straits area).*

c. **Folio Labels.** When chart folios are supplied, a 'folio label' (H119 for standard folios / H82 for Fleet folios) is pasted on the cover showing the folio number, dates of issue, the correction state, and the name of ship / submarine to which it has been issued.

d. **Folio Lists.** When chart folios are supplied, in addition to the folio label (see Para 0612c above), a 'folio list' showing the numbers and titles of the charts contained in the folio, the *NP* numbers and titles of the appropriate volumes of *Admiralty Sailing Directions (Pilots)* and of the *Admiralty List of Lights and Fog Signals (ALLFS)*, and if appropriate, any other necessary *NP*. Duplicate folio lists are also supplied and kept in the 'Chart Correction Log and Folio Index' (NP 133B).

e. **Local Folios.** The number of 'spare' charts needed (eg for *Pilotage, Blind Pilotage*, exercises etc) may exceed the capacity of the folio cover (eg Folio 1, which includes naval base ports and exercise areas). It is thus appropriate to transfer some charts from the main folio to a 'local folio' for ease of storage and use; a note must be made in the 'Chart Correction Log and Folio Index' (NP 133B) and that the appropriate chart corrections must be properly logged and applied (see BR 45 Volume 7 Chapter 4).

f. **RN Backup Folios.** *UKHO* provides ten regional 'Backup' folios of paper charts for RN / RFA vessels corresponding to the ten regional *ARCS* areas.

0613. Legacy Chart Data

The compilation and production processes for British Admiralty charts has evolved from the days of the quill pen (see below) to [*Warship*] *Electronic Chart Display and Information Systems* (*WECDIS / ECDIS)* (see Para 0614). British Admiralty charts are produced by *UKHO,* which is an Agency of the British Ministry of Defence. This department was formed in 1795 because, it was said, more RN warships were being lost on uncharted or badly charted shoals than were being sunk by enemy action.

a. **Sounding Methods.**

- **Lead and Line Soundings: 1795-1935 / 1950.** Lead and line was the only method of sounding until *Echo Sounders* came into use in about 1935; the hand lead continued for inshore work into the 1950s. A sounding with lead and line covered only the few centimetres actually struck by the lead and objects less than a metre away from each cast remained undetected.
- **Vertical Echo Soundings: from 1935.** Vertical *Echo Sounders* only examine a narrow strip immediately under the hull of the ship, and even on a large-*Scale* harbour chart these strips can be as much as 60 metres apart.
- **Sidescan Sonar: from 1973.** Sidescan sonar was introduced in about 1973, allowing the detection of shoals and wrecks lying between sounding lines.
- **Examination of Dangers.** Until the early 1960s, *UKHO* did not examine in detail any object likely to be deeper than 66 feet (20 metres).

b. **Charted Soundings.** Although sidescan sonar has been employed extensively by *UKHO* since 1973, the large majority of charts in use are still based on older surveying methods. Ships can still find that in every part of the world there are areas which were surveyed using the hand lead only.

c. **Uncharted Dangers.** For the above reasons, it is still quite possible to find uncharted rocks, shoals and wrecks anywhere in the world. Rocky pinnacles rising to within 10 metres of the surface have been found in well-used waters (eg approaches to Holyhead [Wales] and Auckland [New Zealand]. The 18 metre ‘Walter Shoals’, surrounded by great oceanic depths on the route between Cape of Good Hope and Sunda Strait, were not discovered until 1962. It is estimated that there are some 20,000 wrecks or underwater obstructions in British coastal waters alone, but the exact position or the depth of water over many of them is unknown. For historical reasons, dangers to deep-draught ships still exist on continental shelves, even in well recognised shipping lanes (see CAUTION below). **Thus no chart is infallible.** Every chart is liable to be incomplete, either through imperfections in the surveys on which it is based, or through subsequent alterations to the topography and sea-bed.

CAUTION

UNCHARTED DANGERS. Deep-draught ships need to exercise care within the 200 metre depth contour, even in well recognised shipping lanes, because of the historic limit of 20 metres for the examination of dangers.

0614. Digital Chart Production Methods

UKHO methods for modern chart production are as follows. (See Para 0632 for details of *Vector* and *Raster* digital chart types [ie *Electronic Navigation Charts (ENCs), Raster Navigation Charts (RNC)* and 'non-*IHO*-compliant' *Vector* charts] and their legal status.)

a. **Digitisation of Paper Chart Data.** Over some years, navigational data previously stored on paper charts has been scanned digitally. The latest version of each *RNC* and paper navigational chart is now held digitally in *Raster* format in a *Raster* database.

b. **Input of New Data for Charts.** New data for charts is forwarded to the *UKHO* from a very wide range of sources in both analogue and digital formats; however, it can only be applied to *ENCs* which have been produced by *UKHO*. All new data (eg new foreign government charts, textual *NMs*, new surveys etc) is first assessed by the appropriate 'Nautical Chart Branch' or 'Specialist Branch' for safety-criticality before appropriate chart action is taken (*NM* textual correction / block correction for paper charts etc). Large *NM* updates and blocks for paper charts will be issued as *NEs* for *ENCs*. New data is transformed into the *UKHO* digital format for charts, as follows:

- **New Data in Digital Format.** On receipt of new digital data in the *UKHO*, it is validated and verified to ensure it is fit for purpose. Using the digital source as a backdrop to the *Raster* image of the existing chart, changes are captured in *Vector* format. Once the new information has been captured, it is then converted into *Raster* format and combined with the existing file.

- **New Data NOT in Digital Format.** Hard copy data is *Raster* scanned and used as a *Raster* backdrop to the existing chart, in order to view the existing chart and the new data together. The same editing process is then followed.

c. **Raster Database.** Once a chart has been revised, the digital *Raster* files are returned to the *Raster* database. The *Raster* database therefore holds the latest data for <u>all</u> *UKHO* charts. Several products are generated from the *Raster* database; these include paper charts, the (*RNC*) *Admiralty Raster Chart Service* (*ARCS*) version of the chart and inputs to the *ENC* database for the creation of updates and *NEs* of *ENCs.*

d. **Paper Chart Production.** The *Raster* database is used to export digital files to the computer-to-plate technology, which produces thin aluminium printing plates. These plates are used once, for a single multi-colour print run of the full sized paper chart using high quality paper and precision printing machines to ensure perfect colour registration.

e. **Notices to Mariners.** *Notices to Mariners (NM),* including their associated tracings and colour blocks for pasting onto charts, are produced similarly to paper charts.

0615. Correction and Upkeep of Paper Charts

To be safe and effective, navigational charts (whether in paper or electronic format) must be corrected up to date for safety-critical navigational information. The methods of promulgating this information are described in detail in 'The Mariners Handbook' (NP 100) Chapter 1; the detailed procedures for updating charts are at BR 45 Volume 7 Chapters 5 and 7 (paper charts) and at BR 45 Volume 8(1) Chapter 8 (charts in electronic format). The following information is a brief overview of the methods of promulgating safety-critical navigational information.

a. **Promulgation.** Safety-critical information is promulgated as follows:

- **Permanent Notices to Mariners (NMs).** Permanent *Notices to Mariners* (*NMs*) are issued weekly in hard and soft copy by *UKHO*. They are usually in text but may include a coloured 'block' to be pasted over part of the chart.

- **Temporary and Preliminary Notices to Mariners (T& Ps).** *Temporary and Preliminary Notices to Mariners* (*T&Ps*) are issued weekly in hard and soft copy by *UKHO*, together with ordinary *NMs*. They are normally plotted on paper charts in pencil and erased when no longer in force.
  - **Temporary Notices to Mariners.** *Temporary Notices to Mariners* are used when the information is only valid for a limited period.
  - **Preliminary Notices to Mariners.** *Preliminary Notices to Mariners* are used when early notification of changes needs to be promulgated (eg harbour development work about to take place, or notification that a *New Chart (NC) / New Edition (NE)* is about to be published).

- **Local Notices to Mariners.** Local *NMs* are issued by local authorities.

- **New Editions (NEs) / New Charts (NCs).** When substantial chart changes have accumulated *UKHO* will issue a *New Edition* (*NE*), or if the boundaries / number have changed will issue a *New Chart (NC)*. A *Preliminary Notice to Mariners* will normally precede the issue of a *NC / NE*.

- **Urgent New Edition (UNE).** An *NE* may be issued as an *Urgent New Edition* (*UNE*) but if so, may NOT include all available safety-critical data.

- **Radio Navigational Warnings.** Three types of *Radio Navigational Warning* are issued for urgent safety-critical navigational information: *NAVAREA Warnings, Coastal Warnings* and *Local Warning*s. *Radio Navigational Warnings* are normally plotted on the appropriate paper charts in pencil and erased when no longer in force.
  - **NAVAREA Warnings.** *NAVAREA Warnings* contain safety information for ocean-going mariners, divided into 16 areas (see Fig 6-1 overleaf). They are broadcast by 'SafetyNET' and appropriate *NAVTEX* stations.
  - **Coastal Warnings.** *Coastal Warnings* are promulgated by the appropriate National Coordinator for areas out to about 250 *n. miles* from the coastline. They are normally broadcast by the appropriate *NAVTEX* station but may also be broadcast on 'SafetyNET' or other means.
  - **Local Warnings.** *Local Warning*s supplement *Coastal Warnings* for detailed inshore or port information for ships visiting a particular harbour. They are usually issued by port, pilotage or coastguard authorities, often by VHF voice messages, which may be in English or the local language.

(0615a continued)

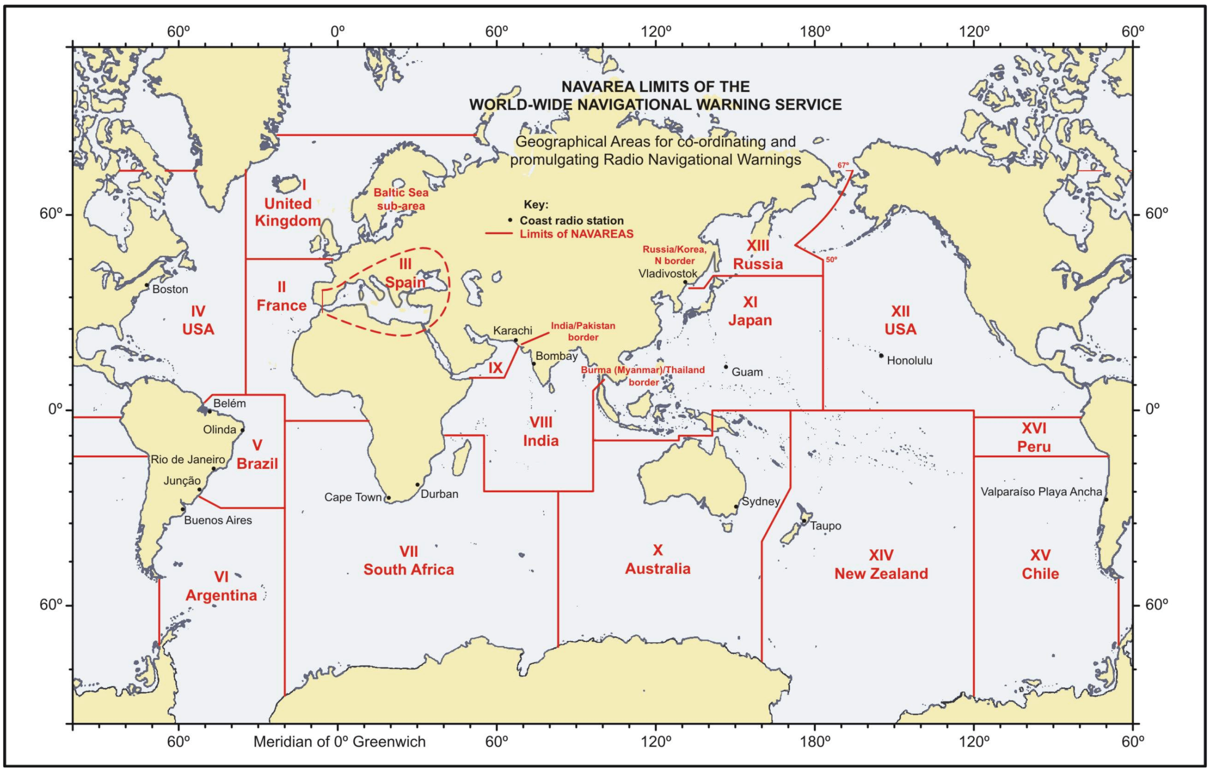

Fig 6-1. NAVAREA Limits of the World-wide Navigational Warning Service

(0615) b. **Digital Promulgation.** *NMs, T&Ps, NEs* and *NCs* are promulgated by cumulative weekly CD for digital charts (ie *RNCs* and *ENCs* etc). *Radio Navigation Warnings* (see Para 0615a) for digital charts are manually input to *RNCs / ENCs,* using digital methods.

0616. Reporting Hydrographic Information - Hydrographic Notes

a. **Requirement.** Subject to the provisions of international law concerning innocent passage through territorial seas or national laws where appropriate, mariners should endeavour to note where charts and navigational publications disagree with fact, and report any differences to the *UKHO*. In addition, statements confirming the accuracy of charted and published information which may be old is also of considerable value.

b. **Method.** Reports should be made to the "UKHO, Admiralty Way, Taunton, Somerset TA1 2DN, United Kingdom", either in manuscript, or by e-mail, or as a *Hydrographic Note* on Forms H.102 / H.102a. Current web, e-mail, FAX and phone contact details for *UKHO* are shown at the front of *Weekly NMs*. Copies of Forms H.102 / H.102a are included on *RNC / ENC* weekly CDs, in every *Weekly NM*, and in The Mariners Handbook (NP 100); they may also be obtained gratis from any Admiralty Chart Agent. Digital *Hydrographic Notes*, including instructions for completion are available in (NP 145) 'HMOG Forms', under 'H102, H102A and H102B'. These can also be downloaded from *UKHO* and e-mailed to *UKHO* when complete.

c. **Further Information.** For further information on all aspects of *Hydrographic Notes*, see 'The Mariners Handbook' (NP 100) Chapter 8.

0617-0619 Spare.

SECTION 2 - NAVIGATIONAL CHARTS

0620. Mercator Projection Charts

A list of *Mercator Projection* properties and uses is at Para 0414b and a full description of its construction is at Paras 0420-0425; an explanation of the mathematical basis for *Meridional Parts* and *Position Circles* is at Appendix 4. For the convenience of readers, an extract from Para 0414b (Uses of *Mercator Projection* charts) is repeated below:

(Extract from Para 0414b):

- **Uses.** The *Mercator Projection* is extensively used for small *Scale* nautical charts of 1:75,000 and smaller. However, for large *Scale* nautical charts (1:50,000 and greater), the *Transverse Mercator Projection* is generally used instead.

0621. Transverse Mercator Projection Charts

A list of *Transverse Mercator Projection* properties and uses is at Para 0414c and a full description of its construction is at Paras 0430-0431; an explanation of the mathematics for geographical / *Grid* conversions is at Appendix 4. For the convenience of readers, an extract from Para 0414c (Uses of *Transverse Mercator Projection* charts) is repeated below:

(Extract from Para 0414c):

- **Uses.** The *Transverse Mercator Projection* is used for:
  - Most *UKHO* large *Scale* charts of 1:50,000 or larger (ie covering a small area) as well as for land maps.
  - Most land maps (including UK Ordnance Survey maps / NATO military maps).
  - *Polar* charts and maps, although the *Polar Stereographic Projection* is more commonly used for this purpose.

0622. Gnomonic Projection Charts

A list of *Gnomonic Projection* properties and uses is at Para 0414e, a full description of its construction is at Paras 0440-0442 and an explanation of its mathematical basis is at Appendix 4. For the convenience of readers, an extract from Para 0414e (Uses of *Gnomonic Projection* charts) is repeated below:

(Extract from Para 0414e):

- **Uses.** The distortion of the *Gnomonic Projection Graticule*, which gives neither *Orthomorphic* nor *Equal Area* properties, makes it quite unsuitable for general *Navigation* purposes. Its usage is limited entirely to plotting *Great Circles* as straight lines, usually in order to obtain *Great Circle Waypoints*.

a. **Gnomonic Projection - Available Charts.** A series of small-*Scale Gnomonic Projection* charts covering the major oceanic areas is available for plotting *Great Circles* as straight lines. They are particularly useful for *Composite Tracks* (combinations of *Great Circles* and a *Safe Parallel*). They should NOT be used for *Navigation*.

b. **Gnomonic Projection Chart - Incorrect Use of Title for Large Scale Plans.** In a few older Admiralty charts and plans of *Scale* 1:50,000 and larger, '*Gnomonic Projection*' was printed under the chart title, when in fact, a modified form of *Polyconic Projection* (see Para 0414g) was actually used. Although it was strictly incorrect to call these charts'*Gnomonic*', on such large *Scale* plans, lines of sight and other *Great Circles* are correctly represented by straight lines; for all practical purposes straight lines can be used on them to plot all bearing and direction lines. Modern charts of this *Scale* are constructed on the *Transverse Mercator Projection* (see Para 0623 overleaf).

0623. Large Scale Harbour Plans

a. **Modern Harbour Plans.** Most modern large-*Scale* harbour plans are constructed on *Transverse Mercator Projection* and are graduated for *Latitude / Longitude*. Linear *Scales* of feet, metres and cables (1 cable = 0.1 *Sea Mile*, see Para 0113) are given on all plans, with heights and depths in metres. A modern harbour plan is at Fig 6-2 below.

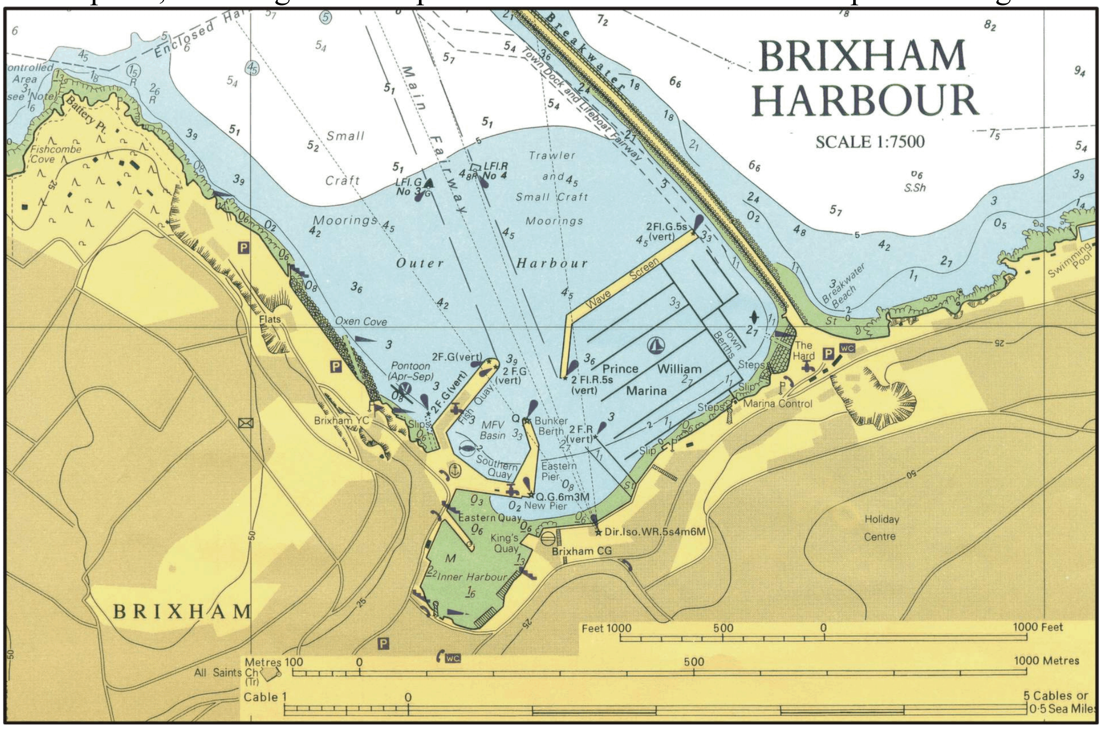

Fig 6-2. Modern Harbour Plan - Linear Scales (ft/m/c) with Depths / Heights (m)

b. **Older Harbour Plans - Constructing a Longitude Scale.** On older harbour plans, the *Longitude Scale* may not be given. This may be found from the following construction.

- From the zero on the *Latitude Scale*, draw a line making an angle with it equal to the *Latitude* of the plan (see example of 45˚at Fig 6-3 below).
- From each division on the *Latitude Scale*, draw a perpendicular to this line. Intersections of the perpendiculars with the line mark the *Longitude Scale*.

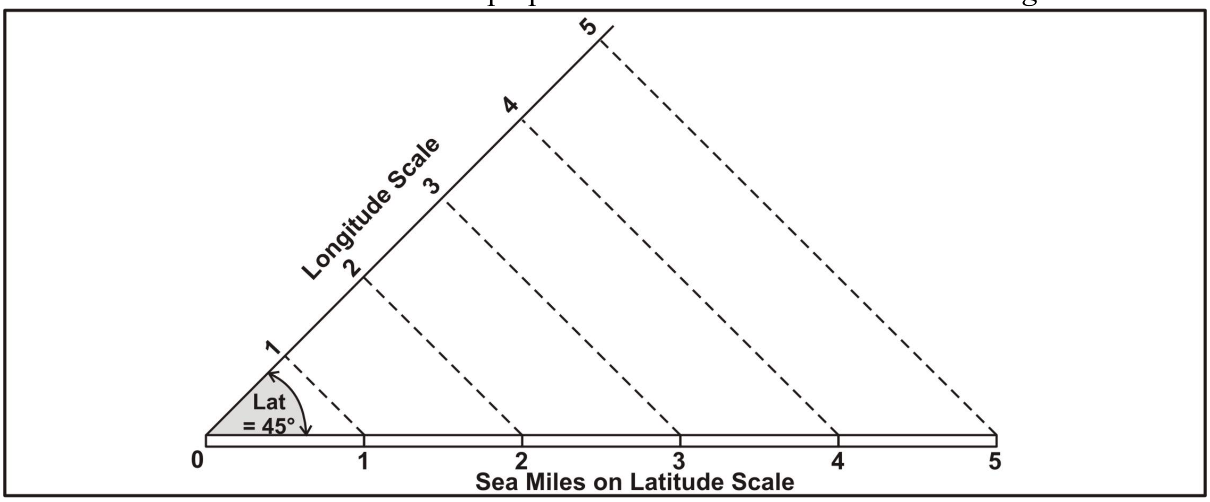

Fig 6-3. Older Harbour Plans - Constructing a Longitude Scale (eg 45˚ Latitude)

0624. Information Shown on Paper Charts

Information contained on Admiralty charts is summarised as follows:

a. **Chart Number and Thumb Label.** The chart number is shown outside the bottom right-hand and top left-hand corners of the chart. If the chart is also in the International Series, this number is also shown, prefixed 'INT'. A small label, known as the 'thumb label', is printed on the reverse of each chart; it shows the number, title, printing date and printing batch number of the chart, and provides space for notation of the folio number.

b. **Chart Title.** The chart title is shown in the most convenient place so that no essential navigational information is obscured by it. The chart title is also shown in the thumb-label on the reverse of the chart.

c. **Source Data Diagram.** A *Source Data* diagram (see Fig 6-4 below) is normally shown on each chart. These diagrams indicate the source, date and *Scale* of the survey in each part of the chart.

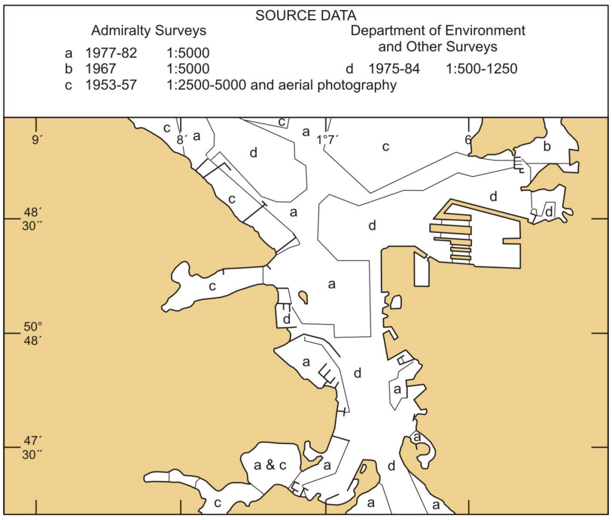

Fig 6-4. Source Data Diagram

d. **Source Data Legends.** On some (usually older) charts, in lieu of the *Source Data*diagram, the source may be shown under the chart title.

e. **Satellite Derived Positions.** The *Datum Shift* (see Para 0323) is included on older charts adjacent to the chart title, indicating the amount by which a position obtained from a satellite navigation system should be moved to agree with the chart. Charts published in *WGS 84 Datum* indicate this in magenta, outside the bottom right and top left-hand corners, in lieu of the 'Depths in Metres' marking.

f. **New Charts (NCs) - Publication Date.** When a *New Chart* (*NC*) is published, the date of publication is shown outside its bottom margin, in the middle eg: "Published at Taunton, 30th March 2006" (Chart 2692).

g. **New Editions (NEs) / Urgent New Editions (UNEs) - Publication Date.** When a chart is revised, a *New Edition* (*NE*) or *Urgent New Edition* (*UNE*) is published. The edition number and date of publication is shown at the bottom of the chart eg:

"Edition Number 6 Edition Date 6th Mar 2003" (Chart 2675)

Older charts (pre-2000) state: "New Editions 27th July 1990, 1st July 1999" (Chart 3154)

(0624) h. **Notices to Mariners (Small Corrections).** *Notices to Mariners* (*NMs*) which have been applied to the chart are listed, by year and *NM* number, at the bottom left-hand corner of the chart eg:

"**Notices to Mariners 2005** - 6183- **2006** - 2652 - 4546" (eg Chart 34)

Charts issued as *NEs* prior to May 2000 may show the legend '*Small Corrections*' in lieu of '*Notices to Mariners'*. Admiralty charts corrected for Australian or New Zealand *NMs* list the applicable British Admiralty (BA) and AUS or NZ *Small Corrections* separately, prefixed accordingly.

i. **Large Corrections and Bracketed Corrections (Obsolete).** Until 1972, *Large Corrections* were sometimes issued instead of *New Editions (NEs)*. Until 1986, *Bracketed Corrections* were used to give information which was useful to mariners but not essential. These notations may still be found on some older charts.

j. **Date of Printing.** The date of printing is shown in the format YY / MM / DD in the thumb-label on the reverse of the chart eg: "01 / 05 /20" (Chart 30).

k. **Chart Dimensions.** Chart dimensions in millemetres (eg 630.0 x 980.0 mm) are shown in parentheses outside the lower right-hand border of the chart; some fathoms charts use inches. The dimensions are for the inner border of the chart (neat lines) and exclude chart borders. In the case of charts on the *Gnomonic Projection*, dimensions are quoted for the north and south borders, and on the *Transverse Mercator Projection* for all four borders. These dimensions may be used to check for chart distortion.

m. **Corner Co-ordinates.** Co-ordinates expressing the *Latitude* and *Longitude* of the limits of Admiralty charts published after 1972 as *New Charts* (*NCs*) or *NEs* are shown at the upper right and lower left corners of the chart.

n. **Scale of the Chart.** The natural *Scale* is shown beneath the title; on *Mercator Projection* charts this is for a stated *Latitude*. A *Scale* of metres is shown in the side margins of some charts of *Scale* larger than 1:100,000 to facilitate the plotting of ranges from radar displays graduated in this way. Large *Scale* plans have an additional *Scale*, normally showing feet, metres and cables.

o. **Colours Used on Metric Charts.** Shallow water on metric charts is distinguished by a light blue tint and a darker blue tint between the coastline and appropriate depth contours; these tints includes all isolated patches within the depth range and the darker blue tint indicates shallower water. Drying (intertidal) areas are shown in a green tint. Dry land is shown in a yellow tint. Magenta is used for the emphasis of certain details, notably lights and radio aids and to distinguish numerous other features superimposed on the basic hydrography. Example of modern metric charts are at Fig 6-2 (facing previous page) and Figs 12-1 & 12-2a/b/c (all at Para 1214).

p. **Depth Soundings - Position.** On all charts, the position of a depth sounding is the centre of the space occupied by the sounding figure(s).

q. **Depth Soundings - Units.** The unit in use for depths is stated in bold lettering below the title of the chart. It is also shown on older charts, in magenta, outside the bottom right and top left-hand corners of metric charts. Charts published in *WGS 84 Datum* indicate this in magenta, outside the bottom right and top left-hand corners, in lieu of the 'Depths in Metres' marking (see Para 0624e).

(0624) r. **Depth Soundings - Chart Datum.** Depths on charts are given below *Chart Datum* (see definition at Para 1062c).

- **UKHO as Charting Authority.** On metric charts for which the *UKHO* is the charting authority, *Chart Datum* is the approximate level of *Lowest Astronomical Tide* (*LAT*) - see definition at Para 1062e.
- **Charts Based on Foreign Charts.** For charts based on *Foreign Charts* in tidal waters, *Chart Datums* are LW levels which range from *Mean Low Water* (*MLW*) to the lowest possible LW; in non-tidal waters (eg the Baltic) *Chart Datum* is usually *Mean Sea Level (MSL)* - see definition at Para 1062e.
- **Chart Datum Information Legend / Panel.** A brief description of the level of *Chart Datum* is given under the title of all metric charts. Large and medium *Scale* charts contain a panel giving the heights above *Chart Datum* of either *Mean High Water Springs* (*MHWS*) and *Mean Low Water Neaps (MLWN)*, or *Mean Higher High Water* (*MHHW*) and *Mean Lower Low Water* (*MLLW*), whichever is appropriate (see definitions at Para 1062e).

s. **Depth Soundings - Metric Charts.** On metric charts, soundings are generally shown in metres and decimetres in depths of less than 21 metres; elsewhere in whole metres only. Where navigation of deep-draught vessels is a factor and where the survey data are sufficiently precise, soundings between 21 and 31 metres may be expressed in metres and half-metres.

t. **Depth Soundings - Fathom Charts.** On fathom charts, soundings are generally shown in fathoms and feet in depths of less than 11 fathoms, and in fathoms elsewhere. In areas used by deep-draught vessels where the depth data are sufficiently precise, charts show depths between 11 and 15 fathoms in fathoms and feet. Some older charts show fractional parts of fathoms in shallow areas and a few older charts express all soundings in feet.

u. **Depth Contours.** On charts, all soundings less than and equal to certain depths are enclosed by appropriate metre or fathom contour lines.

v. **Heights / Elevations.** Heights and *Elevations*, except *Vertical Clearance* and (drying) heights (see Paras 0624w/x), are given in metres or feet above a *Vertical Datum*, usually *Mean High Water Springs* (*MHWS*), or *Mean Higher High Water* (*MHHW*) or, where tidal *Range* is negligible, *Mean Sea Level* (*MSL*). In most instances the position of the height is that of the dot alongside the figure (eg **.135)**. Heights which are displaced from the feature to which they refer (eg a small islet), or which qualify the description of a feature (eg a chimney) are placed in parentheses.

w. **Vertical Clearance Heights.** Since 2004, *Vertical Clearance* of bridges and overhead power cables have been quoted above *Highest Astronomical Tide* (*HAT*) where there is an appreciable tidal *Range* (instead of *MHWS*), and above *MSL* where the *Range* is negligible. The *Vertical Datum* used for clearance heights is stated on the chart.

x. **Drying Heights.** Underlined figures on rocks and banks which uncover give the drying heights above *Chart Datum* in metres and decimetres, or in feet, as appropriate.

y. **Compass Roses.** *Compass Roses* are printed at intervals on paper charts and show true bearings from 0˚-360˚ (and usually local *Magnetic Variation* and its annual change).

(0624) z. **Tidal Stream Information.** *Tidal Stream* information may be shown either in a table, or (when insufficient data for constructing tables is available) by *Tidal Stream* arrows and notes giving the times of slack water and the rate of the *Tidal Stream*s. All information about *Tidal Stream*s is given in a convenient place on the chart and referred to by a diamond symbol (ie ◊) at the position for which the information is given.

aa. **Reference to Adjoining Chart or Large Scale Plan.** References to adjoining charts are shown in magenta on many charts. Larger *Scale* charts or plans are indicated on many charts by a magenta outline with the chart number shown in magenta.

ab. **Cautionary Notes.** Any 'Cautionary Notes' are printed prominently, usually under the chart title. 'Cautionary Notes' should ALWAYS be read before using the chart.

ac. **Abbreviations and Symbols.** Standard abbreviations and symbols used on Admiralty charts are shown in 'Chart Booklet' 5011, 'Thematic Charts' D6067 (Use of Symbols and Abbreviations) and D6695 (Borders, Grids, Graduations and Scales).

ad. **Crown Copyright.** All Admiralty charts are 'UK Crown Copyright' and this is indicated adjacent to the date of publication (see Para 0624f).

0625. Information Shown on ENCs

Much of the information on paper charts (see Para 0624) is presented on *ENCs* but often in a different form (see Para 0632). In particular, *ENCs* do not have *Source Data* diagrams but instead have *Category of Zone of Confidence* (*CATZOC*) data fields with 5 assessed categories (A1, A2, B, C, and D) and 1 category (U) which is not assessed. Details of *CATZOC* parameters are at 'The Mariners Handbook' (NP 100) Chapter 2.

0626. Selection and Use of Charts

a. **Using Charts.** Each Admiralty chart, or series of charts, is designed for a particular purpose. Large *Scale* charts are intended for entering harbours or anchorages or for navigating close to potential hazards. Medium *Scale* charts are intended for coastal navigation, while small *Scale* charts are intended for offshore navigation. Always use the largest *Scale* chart appropriate to the purpose.

- **Coastal Passage.** For passage along a coast, use the continuous series of medium *Scale* charts provided for that purpose.
- **Use of Larger Scale Charts on Coastal Passage.** Only transfer to the larger *Scale* chart where it depicts potential hazards close to the intended route more clearly. There is usually no need to transfer for short distances to a larger *Scale* chart intended for entering an adjacent port or anchorage. Although the larger *Scale* chart depicts information in more detail, the next smaller *Scale* shows all the dangers, traffic separation scheme, navigational aids etc that are appropriate to the purpose for which the smaller *Scale* chart is designed.
- **Sea-Bed.** The sea-bed is likely to correspond to the adjacent land features, even when the chart gives no hint of bottom irregularities. Off an area where sharp hillocks and rocky, off-lying islands abound, the sea-bed is likely to be equally uneven and old surveys must be even more suspect than off a coast where the visible land is flat and regular. There may be uncharted dangers on or near the rim of a saucer-like plateau surrounding a coral group.

(0626) b. **Distinguishing a Well-Surveyed Chart.** As stated at Para 0613c, no chart is infallible; every chart is liable to be incomplete in some way. The date of survey and sounding methods used (see Paras 0613, 0624c/d and 0625) will give a good indication of the chart's reliability. It should be carefully inspected to check the following:

- The survey should be reasonably modern (check *Source Data / CATZOC*).
- The survey authority should be reliable (check *Source Data / CATZOC*).
- The survey *Scale* should be adequate and will usually be larger *Scale* than that of the chart itself (check *Source Data/ CATZOC*).
- The character and completeness of the original survey material and of subsequent updates should be reliable (check *Source Data / CATZOC*).
- Soundings should be close together, regularly spaced with no blank spaces.
- Lead-line surveys should be treated with **particular caution**, particularly if the chart is to be used for pilotage (check *Source Data / CATZOC*). Vertical *Echo Sounder* surveys should also be treated with caution.
- Depth and height contour lines should be continuous, not broken.
- Topographical detail should be good.
- All the coastline should be completed, with no pecked portions indicating lack of information.
- Any suspicious inconsistencies of any sort (eg errors in *Latitude* and *Longitude* for geographical position) should be carefully assessed.

c. **Distortion of Paper Charts.** Paper charts are liable to slight distortion at various stages in the process of reproduction but the effect is seldom sufficient to affect navigation. Any distortion may be observed by checking the dimensions (see Para 0624k). If there is distortion, bearings of objects, however accurate, may not plot correctly, particularly if those objects are at a distance as displayed on the chart. The larger the *Scale* of the chart, the effects of distortion lessen proportionately.

d. **To Identify a Particular Copy of a Chart.** When ordering a chart from *UKHO* or elsewhere it is normally only necessary to state the chart number. However, to identify a particular copy of a chart, the following parameters are needed:

- Chart number.
- Chart title.
- Date of publication (as a *NC*)
- Date of the last *NE*
- Number (or date) of the last *NM* (or pre-2000: *Small Correction*).

0627-0629. Spare

SECTION 3 - DIGITAL NAVIGATION SYSTEMS AND ELECTRONIC CHARTS

0630. Digital Navigation Systems - International Standards

A wide variety of electronic navigation systems for use with charts in electronic format are available commercially. However, not all such systems are '*IMO*-compliant' and not all such charts conform to *International Hydrographic Organisation* (*IHO*) specifications for safe *Navigation*. Thus a wide variation in quality may be experienced between different electronic navigation systems and data, with obvious implications for safe *Navigation*. Detailed guidance on this subject is at BR 45 Volume 8, but this Section provides a brief summary.

0631. Digital Navigation Systems - ECS, ECDIS and WECDIS Equipments

a. **Electronic Chart System (ECS).** An *Electronic Chart System (ECS)* is a generic term for equipment that displays charts in electronic format, which do NOT satisfy *IMO SOLAS* requirements, and may NOT be used as a substitute for up-to-date official charts. However, an *ECS* may legally be used alongside paper charts and can improve navigational safety and situational awareness provided care is exercised in its use.

b. **Electronic Chart Display and Information System (ECDIS).** An *Electronic Chart Display and Information System (ECDIS)* is a navigational information system which complies with the *IMO* performance standards and which, with adequate back-up arrangements, can be accepted as complying with carriage requirements of Chapter V Regulation19 of the 2002 *IMO SOLAS* Convention. The presence of a second *ECDIS* system, robust *Uninterruptible Power Supply (UPS)* and/or the immediate availability of adequate, corrected paper charts covering the present area (although not necessarily the best *Scale* available) are deemed 'adequate back-up' by the *IMO* and RN.

c. **Warship Electronic Chart Display and Information System (WECDIS).** A *Warship Electronic Chart Display and Information System (WECDIS)* is an *ECDIS* with extra functionality to make it more effective for warfare purposes (see Fig 6-5 below).

Fig 6-5. A WECDIS Terminal fitted in an RN Warship (operating ECPINS software)

(0631) d. **Management of ECDIS and WECDIS - BR 45 Volume 8.** The effective management of an *ECDIS / WECDIS* being used for *Navigation* is a major task. All *RNCs / ENCs* held also need to be systematically updated and the updates recorded, particularly when duplicate / triplicate *ECDIS / WECDIS* are fitted. In the RN, *Standard Operating Procedures* (*SOPs*) have been devised for these purposes. Detailed *ECDIS / WECDIS* operating guidance and *SOPs* are at BR 45 Volume 8.

0632. Digital Navigation Systems - Electronic Chart Data

Wide variations exist in the quality and reliability of charts in electronic format available commercially. The existing types of chart in electronic format and their legal status are:

a. **Legal Status of ENCs and RNCs.** *SOLAS* Chapter V (2002) Regulations 2 and 19 state that paper charts and *ENCs* are legally classified as *'Nautical Charts'*, but *RNCs* are not because they are neither a 'map' nor a 'database', but are only a digital facsimile of the original paper chart. However, *ECDIS / WECDIS* using *RNCs* fulfills the carriage requirement when no *ENC* is available, provided it is used with an up-to-date outfit of paper charts.

b. **Official Chart Data.** Official chart data is issued from National Hydrographic Offices or equivalent, and should conform to *IHO* specifications. Official chart data includes: 'S-57' format *ENCs*, 'Official Paper Charts', *ARCS* and *Australian Hydrographic Office* (*AHO*) '*Seafarer*' *RNCs*. See Para 0615b for updating of official chart data.

c. **Raster Navigation Charts (RNC).** *RNCs*, known colloquially as '*Raster'* charts, are produced by scanning a paper chart. Official *RNCs* conform to *IHO* specifications, but as they are only a digital facsimile of the original paper chart, the image has no 'intelligence' and cannot be 'interrogated' other than by visual means.

d. **Raster Navigation Chart (RNC) Formats.** There are two *Raster* chart formats: *HCRF* and *BSB*. The *UKHO* produces *ARCS* charts in their proprietary *Hydrographic Chart Raster Format (HCRF)*. The *AHO* also uses *HCRF* to produce *AHO 'Seafarer' Raster* charts. The *US National Geospatial & Chart Agency (NGA)* produce *Raster* charts in *BSB* format, which is not compatible with *HCRF*.

e. **Vector Charts - General.** *Vector* charts comprise a digital database of chart data which can be input to an *ECDIS / WECDIS.* Certain *Vector* charts may, or may not, meet *IHO* standards and users should take particular care to establish the origin and reliability of any *Vector* charts used with an *ECDIS / WECDIS.*

f. **Electronic Navigation Charts (ENC).** *Electronic Navigation Charts (ENC)* are *Vector* charts which conform to the following legal (*IHO* specification) conditions:

- *ENC*s are compliant with *IHO* 'S-57 Edition 3.1' (see Note 6-2 below), for content, structure and format.
- *ENC*s are issued for use with *ECDIS / WECDIS* on the authority of government authorised Hydrographic Offices.

Note 6-2. *The IHO (Standard) "S-57 Edition 3.1" database format ensures that ENCs contain all the information necessary for safe Navigation, and may contain supplementary information in addition to that contained on the paper chart (eg Sailing Directions, tidal information, etc) which may assist safe Navigation.*

(0632) g. **Non-Compliant Vector Charts.** *Vector* charts which are NOT compliant with the *IHO* "*S-57* Edition 3.1" standard (eg *US NGA 'DNCs'*) have been produced by a number of organisations. **These charts are NOT legally considered to be *ENCs.***

h. **ENC Chart Symbols.** The S-57 chart symbol specification complies with the *IHO*'s S.52 Presentation Library, and allow a user-choice of *Traditional Symbols* and *Simplified Symbols* to be displayed on *ENCs. Traditional Symbols* are closely matched to paper chart symbols, although variations do occur. *Simplified Symbols* are usually in a more generic style and one *Simplified Symbol* may represent two features (eg 2 buoys) which have different functions. *Simplified Symbols* may vary from manufacturer to manufacturer (see manufacturers' system-specific guidance for details).

i. **Capabilities of ENCs.** *ENCs* are 'intelligent charts' and have information that can be 'interrogated' electronically by the user (eg depths - which can be used to generate automatic alarms based on crossing danger depths). The information displayed can also be adjusted by the user, by removing certain parts of the chart file from the display. The 'look and feel' of *ENCs* is quite different to paper charts / *RNC*s and users may need some time to familiarise themselves with this (see Fig 6-6 below).

j. **Additional Military Layers (AML).** *Additional Military Layers (AMLs)* are layers of extra information for military use which may be displayed over the navigational chart information provided by *ENCs* or *RNCs*. *AMLs* will NOT function correctly in a commercial *ECDIS* and should ONLY be used on *WECDIS* or other systems specifically designed to exploit them. ***AMLs* are NOT to be used as a primary aid to *Navigation*.**

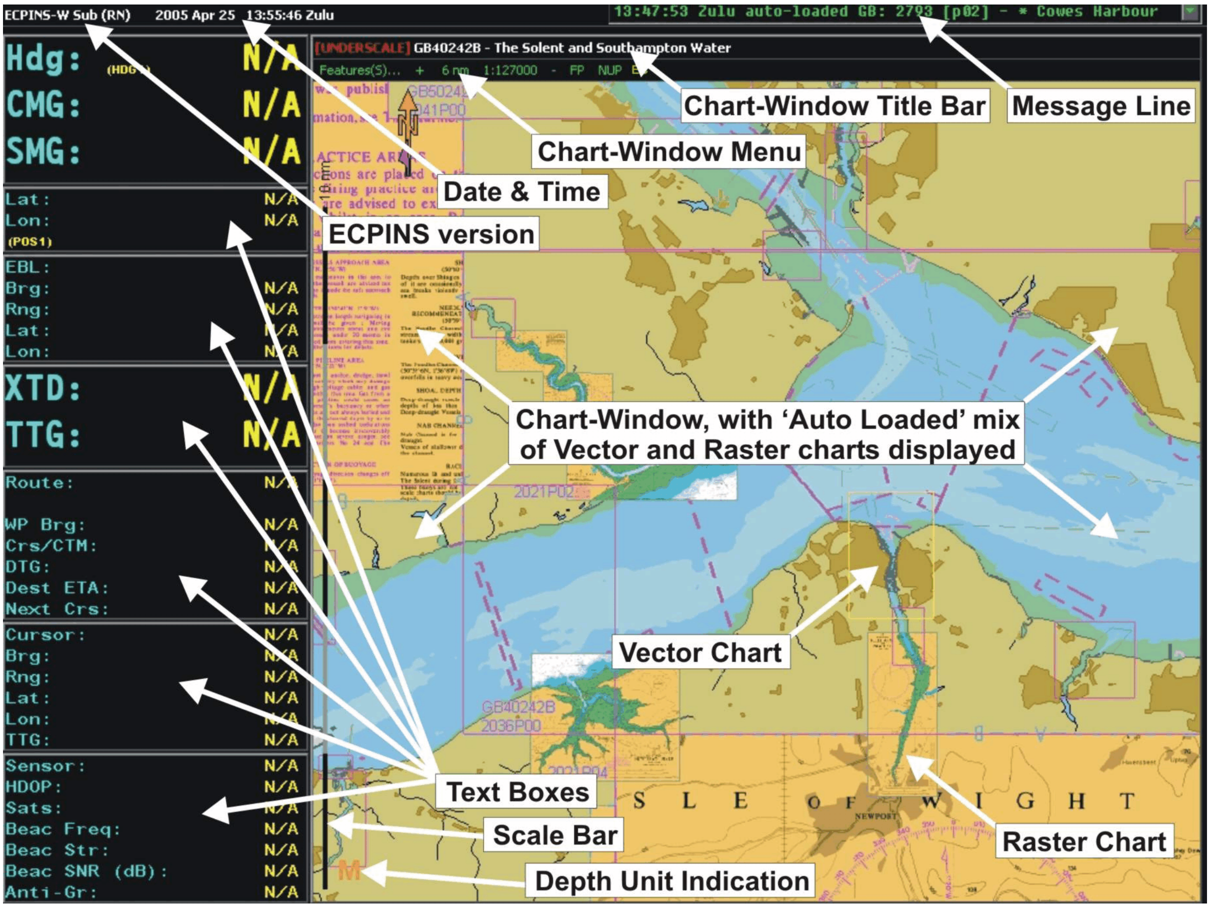

Fig 6-6. A WECDIS display fitted in an RN Warship, operating ECPINS software, showing mix of Vector and Raster charts, plus other data at the left and top of the screen.

0633. RNC Raster and ENC Vector Charts - Advantages & Disadvantages

Table 6-1 (below) summarises the main advantages / disadvantages of *RNC (Raster) / ENC (Vector)* chart formats.

Table 6-1. Summary of RNC Raster / ENC Vector Chart Advantages & Disadvantages

RNC (RASTER) CHARTS		ENC (VECTOR) CHARTS	
ADVANTAGES	**DISADVANTAGES**	**ADVANTAGES**	**DISADVANTAGES**
Direct copy of existing paper chart. Contents are as comprehensive, accurate and reliable as paper charts.	Chart displays CANNOT be customised, although other information may be overlaid.	Chart information is stored in layers in a database. Users CAN customise chart displays.	Creation of ENC Vector chart database is a very complex process.
Much easier to ensure the quality and integrity of Raster data during production. Raster charts are relatively cheap to produce.	Raster file sizes are generally large and have large memory / processing requirements.	ENC Vector file sizes are generally smaller than those for Raster charts and are easier to process for display.	More difficult to ensure the quality and integrity of ENC Vector data during production. ENC Vector charts are very expensive to produce.
Same familiar colours and symbols as paper charts.	Colour palette used is limited to that for paper charts.	Larger colour palette may be used for ENC Vector charts than is used for paper / Raster charts	Symbols and colours are often different to those on paper / Raster charts
Users will not require additional training to be able to interpret the charts			Users will require additional training to ensure that ENC Vector charts are used safely
Inadvertent omission of significant navigational info from the chart display information IS NOT possible	Chart features CANNOT be interrogated or alarmed. Chart boundaries are visible.	Chart features CAN be interrogated and some features CAN be alarmed. Chart boundaries are seamless.	Inadvertent omission of significant navigational info from the chart display information IS possible.
System display parameters are relatively simple to set.	May become cluttered when zoomed or overlaid with other information.	Zooming automatically filters features shown to avoid confusion on display	Careful management of system parameters needed to ensure correct feature filtering is set
Information from other nautical publications may be displayed in the same Raster format.		Information from other nautical publications may be displayed in Raster format.	
'Official' Raster charts are available virtually worldwide			Worldwide coverage in ENC Vector charts will not be available for many years
	Charts can ONLY be displayed North-up	Charts can be displayed OTHER than North-up (eg Head-up)	
It is possible to use SOME of the ECDIS / WECDIS functionality with Raster charts.		COMPLETE ECDIS / WECDIS functionality available with ENC Vector charts.	

0634-0639. Spare

SECTION 4 - NAVIGATIONAL AND DIGITAL PUBLICATIONS

0640. UKHO Navigational and Digital Publications (NPs / DP)

The following is a list of useful *UKHO* 'Unclassified' *Navigational Publications* (*NPs*) and *Admiralty Digital Publications* (*ADP*s). BR 45 Volume 7 contains amplifying information on each *NP / ADP* **and covers 'Protectively Marked' (Classified) publications**.

NP 1-72	Admiralty Sailing Directions (Pilots)
NP 1(S)-72(S)	Supplements to Sailing Directions
NP 74 - 84	Admiralty List of Light and Fog Signals (ALLFS):Volumes A - L
NP 100	Mariner's Handbook
NP 109	NW Europe Catalogue
NP 129	Wallet With Hydrographic Forms
NP 131	Catalogue of Admiralty Charts and Publications
NP 133B	Chart Correction Log and Folio Index
NP 136	Ocean Passages for the World
NP 139	Echo Sounding Correction Tables
NP 145	Hydrographic & Meteorology Operational Guidance (HMOG)
NP 164	Dover: Times of High Water
NP 167	Tidal Streams in the Approaches to HM Naval Bases
NP 201-204	Admiralty Tide Tables
NPs 209-266, 337 & 628-636	Admiralty Tidal Stream Atlases
NP 234A / NP 234B	Cumulative List of Admiralty Notices to Mariners
NP 281	Admiralty List of Radio Signals (ALRS) Vol 1 (Parts 1 & 2) - Coast Radio Stations
NP 282	ALRS Vol 2: Radio Aids to Navigation, Satellite Navigation Systems, Legal Time Radio Time Signals and Electronic Position Fixing Systems
NP 283	ALRS Vol 3 (Parts 1 & 2):Maritime Safety and Information Services
NP 284	ALRS Vol 4: Meteorological Observation Stations.
NP 285	ALRS Vol 5: Global Maritime Distress and Safety System
NP 286	ALRS Vol 6 (Parts 1 to 5) - Pilot Services, VTS and Port Operations
NP 303	Sight Reduction Tables For Air Navigation
NP 314	Nautical Almanac
NP 320	Nories Nautical Tables
NP 323	Star Identifier
NP 350	Admiralty Distance Tables
NP 400	Sight Forms
NP 401	Sight Reduction Tables for Marine Navigation (6 Volumes)
NP 441	Guide to Completing Bridge Weather Log
NPs 713 / 715	Folio Covers
NP 718	Polythene Chart Cover
NP 720	Conversation Table - Metres to Feet / Fathoms
NP 727	Wallet Containing Ships' Boats' Charts
NP 735	IALA Buoyage System
ADP Licence	Admiralty TotalTide / Digital List of Lights / Digital List of Radio Signals

0641. Navigational & Meteorological Stores, 'RNS' Forms and Other Stationary Products

Details of current RN / RFA Navigational & Meteorological Stores, 'RNS' Forms and other stationary products are contained in BR 45 Volume 7, Appendix 2.

CHAPTER 7

CHARTWORK

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CHAPTER 7

CHARTWORK

0701. Scope of Chapter

Chartwork procedures, whether 'traditional' with a paper chart or 'electronic' with a digital system, must be clear to all who use them. Thus standard symbols should be used for all forms of chartwork, including the planning and execution of all types of *Navigation*. This chapter replaces Chapter 8 of the 1987 Edition of this book

- **Section 1.** Section 1 covers 'traditional' chartwork, using a 2B pencil on a paper chart. It also provides the foundation principles for digital chartwork.
- **Section 2.** Section 2 provides an overview of the differences to be applied to 'traditional' paper-chartwork procedures, when carrying out similar functions with *WECDIS / ECDIS / ECS* equipments and software.

0702-0709. Spare

SECTION 1 - PAPER CHARTWORK PROCEDURES

0710. Positions and Position Lines

Fig 7-1 (below) sets out the standard symbols used in the Royal Navy to display positions and *Position Lines* on paper charts. Amplifying comments are at Para 0710a-b (overleaf).

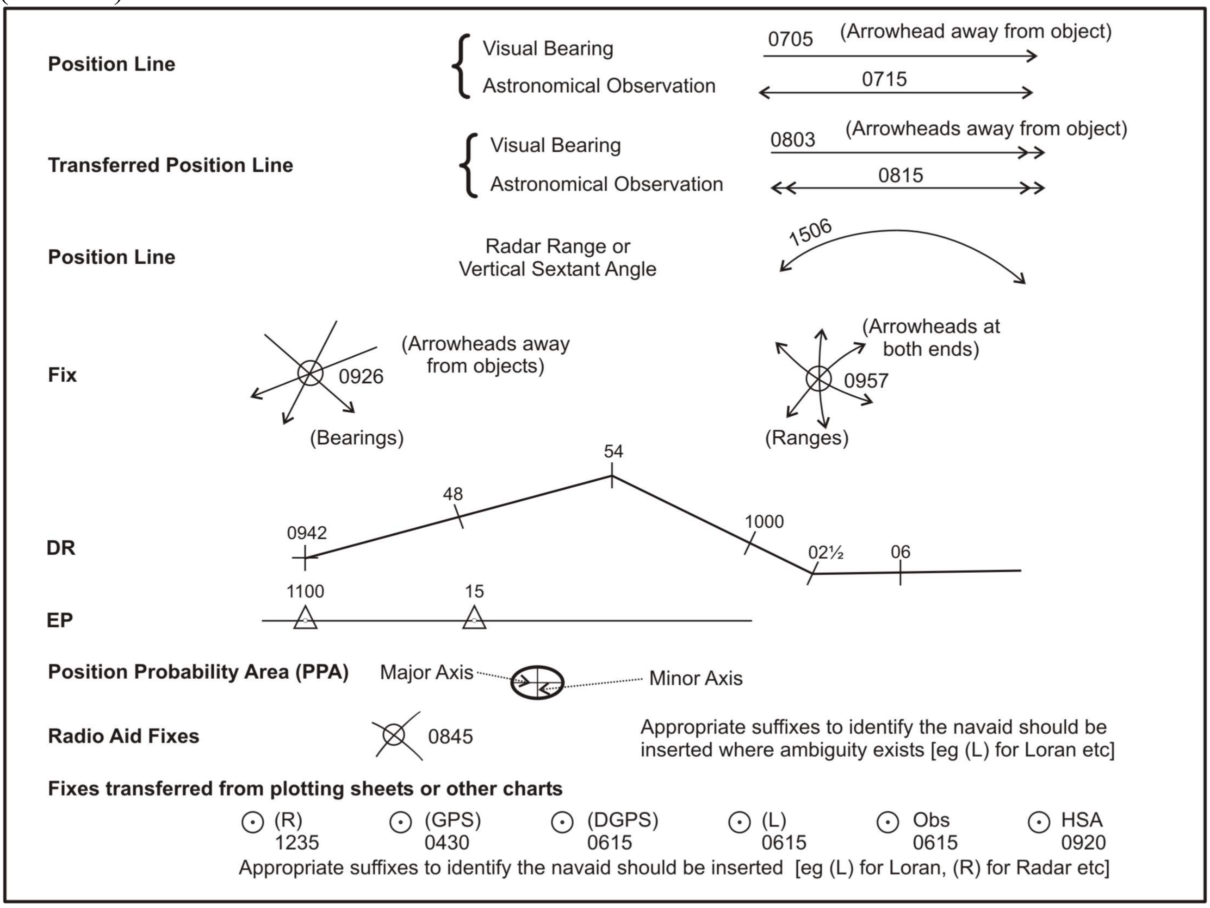

Fig 7-1. Positions & Position Lines (Paper Charts) - Symbols in Use in the Royal Navy

(0710) a. **Arrowheads on Position Lines.** See illustrations at Fig 7-1 (previous page).

- **Terrestrial Bearing Line.** A *Position Line* obtained from a bearing of a terrestrial object is shown with a single open arrow at the outer end.
- **Terrestrial Transferred Position Line.** A terrestrial *Transferred Position Line* is shown with a double open arrow at the outer end.
- **Astronomical Position Line or Terrestrial Range.** A *Position Line* obtained from an astronomical observation or from the range of a terrestrial object is shown with a single open arrow at both ends.
- **Astronomical Transferred Position Line.** An astronomical *Transferred Position Line* is shown with a double open arrow at both ends.

b. **Positions.** See illustrations at Fig 7-1 (previous page).

- **Fix.** A *Fix* is shown on the chart as a dot surrounded by a circle with the time alongside, with its *Position Lines* (if appropriate) passing through the position of the *Fix*. An appropriate suffix may be added to the *Fix* to indicate the navaid used if other than by visual bearings.
- **Dead Reckoning (DR).** A *Dead Reckoning* (*DR*) position is shown on the chart as a small line drawn across the course being steered, with the time alongside. A small cross may be used to originate the *DR* if a *Fix* or *Estimated Position* (*EP*) is not available.
- **Estimated Position (EP).** An *Estimated Position* (*EP*) is shown on the chart as a dot surrounded by a small triangle, with the time alongside. The estimated *Ground Track* of the ship should pass through the dot.
- **Position Probability Area (PPA).** A *Position Probability Area* (*PPA*) may be shown on the chart as an ellipse with a major and a minor axis.

0711. Defining Position and Plotting Bearings

a. **Defining a Position.** A position may be defined by its *Latitude* and *Longitude* (provided the *Spheroid* and *Datum* used are also known or specified - see Chapter 3), or as a range and bearing from a specific object. It may be plotted on the chart using a parallel rule and dividers, and with the chart's *Latitude* and *Longitude Scale*s.

b. **Method of Transferring a Position.** More serious mistakes are made when transferring a position between two charts than any other single error. When transferring a position from one chart to another, it is essential to use two methods as a cross-check on each. Normally, *Latitude* / *Longitude* is used as one method, and bearing / distance from a distinguishing feature common to both charts (eg point of land or lighthouse etc) as another. A *Fix* should be taken as soon as possible thereafter.

c. **Position Line.** A *Position Line* represents a line on the Earth's surface on which the observer is believed to lie; it may be based on observation or detection of some terrestrial or astronomical information. In its simplest form, it may be the visual bearing of a conspicuous landmark. In the context of Astro-Navigation, the term *'Position Line'* is frequently used as an abbreviation for *'Astronomical Position Line'*.

(0711) d. **Plotting a Terrestrial Position Line.** In Fig 7-2 (below) a lighthouse *L* bears 065˚ at 1030. A line drawn in the direction 065˚ passing through *L* is the *Position Line*. It is only necessary to draw the *Position Line* in the vicinity of the ship's position ('Q'), the arrowhead being placed at the outer end to indicate the direction in which the observer must lie from the observed object. Always use the nearest *Compass Rose* (Para 0624y).

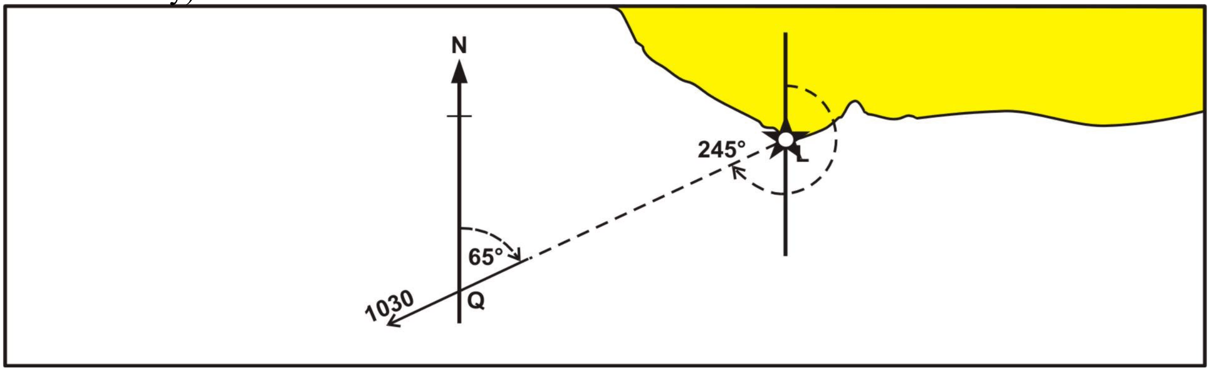

Fig 7-2. Plotting a Terrestrial Position Line

e. **The Fix**. If two or more *Position Lines* can be obtained at the same time, the position of the ship is at their point of intersection and this position is known as a '*Fix'*. *Position Lines* obtained from different sources may be combined in a *Fix*.

f. **The Observed Position.** If a *Fix* is obtained by astronomical observations, it is known as an *Observed Position* and is marked with the suffix 'Obs' (see Fig 7-1).

0712. Calculating Positions

a. **DR (Dead Reckoning).** *DR* (*Dead Reckoning*) is the expression used to describe the position obtained from the true course steered and the ship's speed through the water, and from no other factors. This is normally plotted either manually or automatically (via *WECDIS / ECDIS* or plotting tables), using compass and (speed) log inputs. In the absence of a speed log input, ship's speed may be calculated from engine rpm, making due allowance for the state of the ship's bottom, weather effects etc.

b. **EP (Estimated Position).** *EP* (*Estimated Position*) is the most accurate position that can be obtained by calculation and estimation alone. It is derived from the *DR* position (course and speed steered) adjusted for the effects of *Leeway, Tidal Stream, Currents* and *Surface Drift*. An *EP* symbol may also be used to update a *DR / EP* if only one *Position Line* is available (ie one *Position Line* does NOT comprise a *Fix*).

c. **Position Probability Area (PPA).** The *Position Probability Area* (*PPA*) is derived from a combination of appropriate *Position Lines* obtained from available navigational aids (including *Speed Log* and *Gyro*), after applying the relevant statistical error correction to each *Position Line* in turn (see Paras 1622-1623). It may be shown on the chart in the form of an ellipse with a major and a minor axis (see Fig 7-1).

d. **Most Probable Position (MPP).** Within the *PPA*, the *Most Probable Position* (*MPP*) may be determined (see detailed procedures at Para 1623), which, dependent on the quality of the inputs, may be considered as a *Fix*, an *EP* or a *DR*.

(0712) e. **Leeway.** *Leeway* is the effect of wind in moving a vessel bodily to leeward at right angles to the course steered (effects of the wind in altering ship's speed through the water are subsumed within the *DR*). *Leeway* is complex and depends on:

- **Own Ship's Speed.** The higher own ship's speed, the less is the *Leeway*.
- **Wind Speed and Own Ship's Course.** The greater the component of wind speed at right angles to the course, the greater is the *Leeway*.
- **Windage and Draught.** The greater the ratio of the area above the waterline to that below (ie the draught), the greater is the *Leeway*.
- **Depth of Water.** The shallower the depth of water in relation to the draught, the less *Leeway* is made.

f. **Estimation of Leeway.** *Leeway* may exceed 2 knots in storms, particularly if the ship is at slow speed; it can only be judged by experience, assisted by any records of previous experience. Throughout history many ships have gone aground through losing more ground to *Leeward* than expected; **it is wise to allow a liberal margin for safety when passing dangers to *Leeward*, even with modern ships.**

- **Apparent Mitigation of Leeway.** When steering manually in heavy weather, an inexperienced helmsman may steer a course 2˚ or 3˚ to windward of that ordered, as most ships tend to 'fly' into wind, especially when running with a quartering wind and sea. This may compensate for the effect of *Leeway*, and may be gauged by comparing the course ordered with that recorded on automatic plotters over a period, or by close observation.
- **Leeway Vector and Leeway Angle.** As RN warships frequently proceed at different speeds, it is usual to quantify *Leeway* in terms of a *Leeway Vector* (eg 120˚ ½ knot). In merchant ships, which normally proceed at a set service speed, *Leeway* is normally quantified in terms of *Leeway Angle* - the angular difference between the ship's course and its track through the water.

g. **Terminology of Wind and Current / Tidal Stream Directions.**

- **Wind Direction.** Wind direction is stated as the direction from which the wind blows.
- **Current / Tidal Stream Direction.** The direction of a *Current* or *Tidal Stream* is the direction toward which the water is moving.

h. **Tidal Streams.** *'Tidal Streams'* are the **periodic horizontal** movements of the water accompanying the vertical rising and falling of *Tides* (see Para 1040). *Tidal Stream* data is provided by *UKHO* on Admiralty charts, in *ENC* databases, *Admiralty Sailing Directions* ('*Pilots*') , Admiralty Tide Tables, *Tidal Stream Atlases* and in the comprehensive '*TotalTide*®' software (see Paras 1040-1042 and 1051).

i. **Differences in Predicted / Actual Tidal Streams.** *Tidal Stream* data must always be used with caution, particularly at *Springs* and around the calculated time of change-over from ebb to flood and vice-versa. **It will often be found that the actual *Tidal Stream* experienced is different from that predicted.**

(0712) j. **Currents.** An ocean *Current* is a non-tidal movement of water, which may flow at all depths in the oceans and may have both horizontal and vertical components; a *Surface Current* can only have a horizontal component (see Para 1120). *Current* data is provided at Chapter 11 (Paras 1120-1125) of this book, and by *UKHO* on Admiralty charts, in *Admiralty Sailing Directions* ('*Pilots*'), on Routeing charts, in Ocean Passages for the World and in The Mariner's Handbook (see Para 0640).

k. **Estimating Surface Drift.** When wind blows over the sea surface the frictional effect tends to cause the surface water to move with the wind; this is known as *Surface Drift* (or *Surface Drift Current*). Sometimes there may be no reliable data on ocean *Currents* in a particular area, or the wind itself may be in a different to that normally prevailing and thus affecting the usual ocean *Current.* It may therefore be necessary to to make an estimate for *Surface Drift* (or *Surface Drift Current*), which may or may not be in addition to that made for *Currents*. In practice it is often difficult to distinguish between the effects of *Surface Drift* and *Leeway*. *Surface Drift* can only be estimated from experience and with a knowledge of the meteorological conditions in the area through which the ship is passing; however, the following outline parameters may be helpful.

- **Maximum Drift Speeds.** The speeds of *Surface Drifts* are variable although it has been postulated that at a maximum they can amount to 1/40th of the wind strength. This figure should be treated with caution, as it also depends on the length of time and the *Fetch* involved (the extent of open water over which the over which the wind has been blowing).

- **Build up of Surface Drift.** The build-up of *Surface Drift* in response to wind is slow and a steady state takes some time to become established. With light winds the slight *Current* resulting may take only about 6 hours to develop, but with strong winds about 48 hours is needed for the *Current* to reach its full speed. Hurricane force winds may give rise to a *Surface Drift* in excess of 2 knots, but it is rare for such winds to persist for more than a few hours without a change in direction. The piling up of water caused by a storm near a coastline may lead to particularly strong *Surface Drift* parallel to that coast.

- **Surface Drift Direction.** The effect of the rotation of the Earth (Coriolis force) is to deflect water movement to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This produces a direction of the *Surface Drift* inclined at some 20˚ to 45˚ to the right of the wind direction in the Northern Hemisphere and to the left in the Southern Hemisphere.

- **Example.** If, for example, the wind has been blowing steadily from the north-east (045˚) at 20 knots for several days, the rate and direction of the *Surface Drift* in the Northern Hemisphere may be expected to be of the order of ½ knot in a direction between 225˚+20˚= 245˚ and 225˚+45˚= 270˚.

m. **Set and Drift - Definitions.** See Para 0713c (overleaf).

0713. Plotting the Track

a. **Plotting the Estimated Position (EP).** Plotting the *EP* from an initial *Fix* is carried out in two steps (see Fig 7-3).

- **Step 1.** Plot the *DR* position (course steered / speed through the water).
- **Step 2.** Plot the EP from the *DR* position by plotting the effect of:
  - *Leeway*
  - *Tidal Stream*
  - *Current*
  - *Surface Drift*

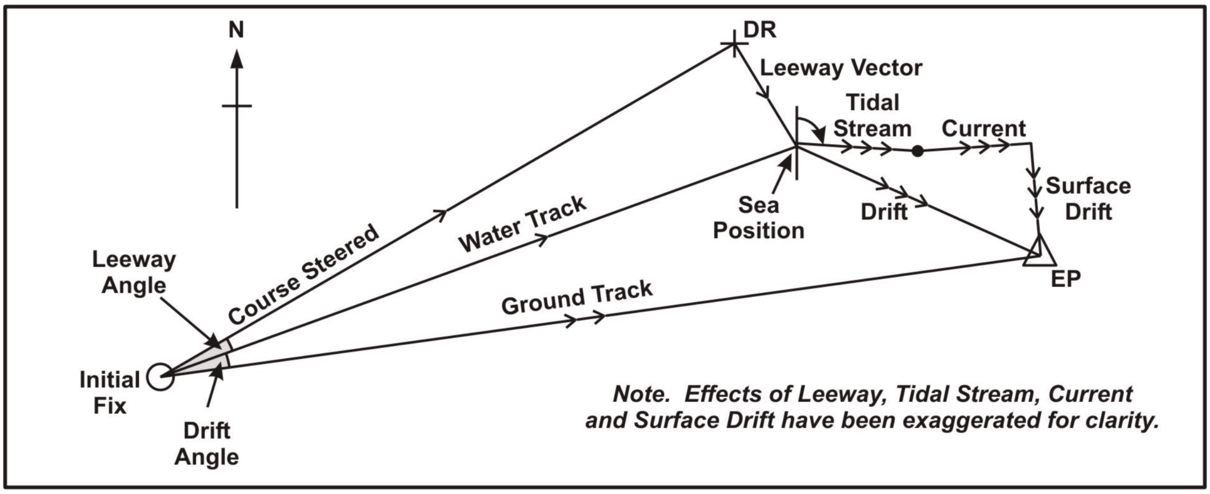

Fig 7-3. Plotting the Estimated Position (EP) - (*Diagram exaggerated for clarity*)

b. **Terms and Definitions.** Fig 7-3 also displays the terms and symbols used in the calculation of the *EP*. Those not already defined are at Table 7-1 (below).

Table 7-1. Terms, Symbols and Definitions used for Construction of an EP

Term or Symbol	Definition and/or Use
Single Arrowhead	Course steered, Water Track (Track through the water), Leeway Vector.
Double Arrowhead	Ship's Ground Track (Track over the Ground / Course made good).
Treble Arrowhead	Tidal Stream, Current, Surface Drift and Drift
Track	The path followed or to be followed, between one position and another. It may be the Ground Track, Water Track, Relative Track or a True Track.
Track Angle	The direction of a Track.
Track Made Good	The mean Ground Track actually achieved over a given period.
Set	The resultant direction towards which Current, Tidal Stream and Surface Drift flow.
Drift	The distance covered in a given time due solely to the movement of Current, Tidal Stream and Surface Drift.
Drift Angle	The angular difference between the Ground Track and Water Track.
Sea Position	The point at the termination of the Water Track.

(0713) c. **Set and Drift - Definitions.** *Set* and *Drift* result from the combined effects of *Tidal Stream, Current* and any *Surface Drift*. *Set* and *Drift* are defined as direction and distance respectively (eg 103˚ 3.5'). *Drift* may also be expressed as *Drift Rate* in knots (eg if the time over which the drift of 3.5' has been determined is 2 hours, the *Drift Rate* would be 1.75 knots [ie *Drift* 3'.5 or *Drift Rate* 1.75 knots]).

d. **Practical Examples of Calculations.** Four practical examples of *Tidal Stream* calculations follow; the same method of solution applies to problems associated with *Leeway, Current* or *Surface Drift*. The fifth (final) example involves all four factors.

Example 7-1. Find the course to steer, allowing for Tidal Stream.
Find the course to steer at 12 knots, to make good 090˚ if the *Tidal Stream* is 040˚ at 3 knots?

- Lay off the *Ground Track* (course to make good - *AB* in Fig 7-4 below). From *A* lay off the direction of the *Tidal Stream* AC. Choosing a suitable *Scale*, mark along *AC* the distance *AD* that the *Tidal Stream* runs in any convenient interval. In Fig 7-4 an interval of 1 hour has been allowed: thus, *AD* will be 3 miles.

- With centre *D* and radius equal to the distance the ship runs in the same interval (ie 12 miles), and on the same *Scale*, cut *AB* at *E*. Then *DE* (101˚) is the course to steer. *AE* (13.7 miles) is the *Ground Track* distance in an 090˚ direction in 1 hour.

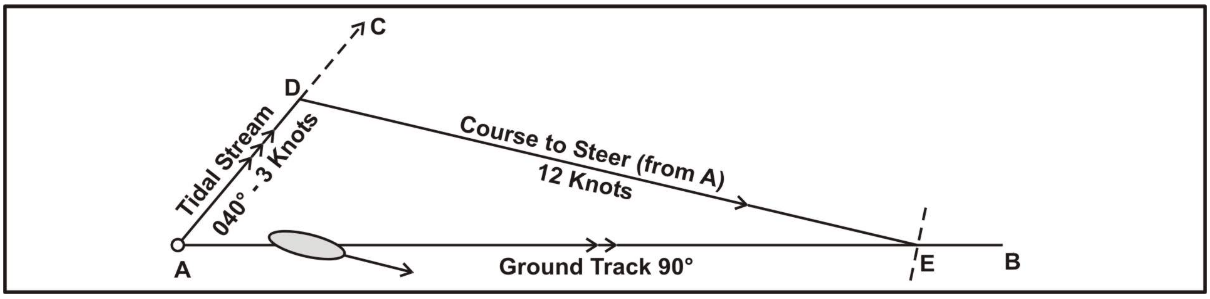

Fig 7-4. Find the Course to Steer, Allowing for Tidal Stream (*Not to Scale*)

Example 7-2. Find the course to steer and speed required, to make an ETA.
Find the course to steer and speed required, to proceed from *A* to position *B* (15 *n.miles*) in 1½ hours, allowing for a *Tidal Stream* setting 150˚ at 3 knots?

- Join *AB*, as shown in Fig 7-5 (below). Measure the *Ground Track* and distance required in 1½ hours (eg 090˚ 15'); thus, the *Ground Speed* required is 10 knots. Mark a position *D* along *AB* using a convenient time interval depending on the *Scale* of the chart (eg1 hour; in this case *AD* will be 10 miles).

- From A lay off the *Tidal Stream AC* for the same interval (ie 150˚ 3'). Join *CD*. *CD* will give the course (073˚) to steer and the speed (8.9 knots) required.

Fig 7-5. Find the Course to Steer and Speed Required, to make an ETA (*Not to Scale*)

(0713d continued)

Example 7-3. Find the time to pass an object at a given distance, allowing for Tidal Stream. A ship at *A* (see Fig 7-6 below) steers to clear a lighthouse '*L*' by 2 miles, allowing for a *Tidal Stream* setting 345˚. Show the method to calculate the time when the lighthouse *L* be abeam.

- From *L* draw the arc of a circle, radius 2'. From the ship's present position draw a tangent to the arc. This is the *Ground Track* required, *AD*.
- Find the course to steer *BC* by the method at Example 7-1. The light is abeam when 90˚ from the course steered (ie when the ship is at *E*, not at *D* which is the *Closest Point of Approach* [CPA] - 2 miles). The time will be that taken to cover the distance *AE* at a speed represented by *AC*, which is the *Ground Speed*.

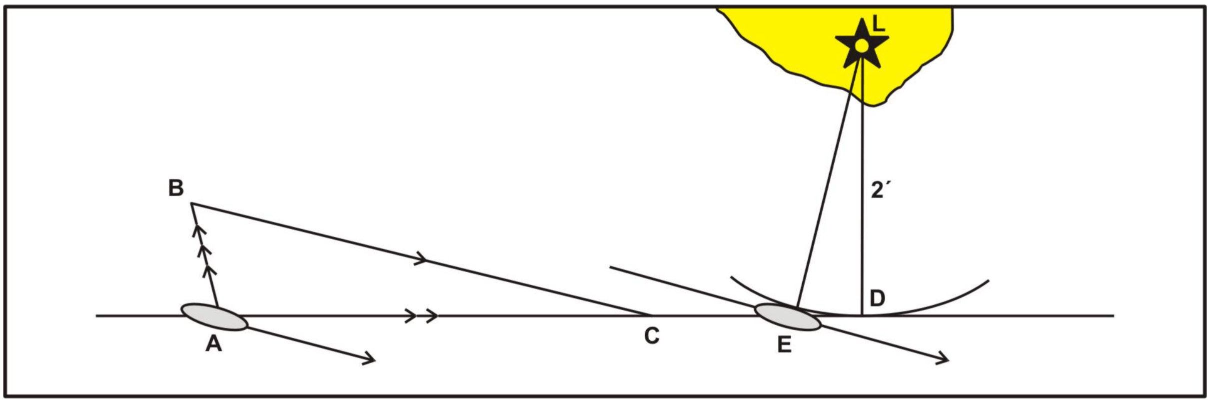

Fig 7-6. To find the time to pass an object at a given distance, allowing for Tidal Stream
(*Not to Scale - Diagram exaggerated for clarity*)

Example 7-4. Find the direction and rate of the Tidal Stream between two Fixes. A ship is at *A* at 0100 (see Fig 7-7 below), and steers 110˚ at 10 knots. At 0300 it is *Fixed* at *B*. What is the direction and rate of the *Tidal Stream* from 0100 to 0300?

- Plot the ship's course 110˚ for a distance of 20' from *A*.
- The difference between the *DR* position *C* and the *Fix B* at 0300 gives the direction of the *Tidal Stream CB* (025˚) and the distance it has displaced the ship in 2 hours (7.6 miles). From this data the *Tidal Stream* may is as 025˚ at 3.8 knots.

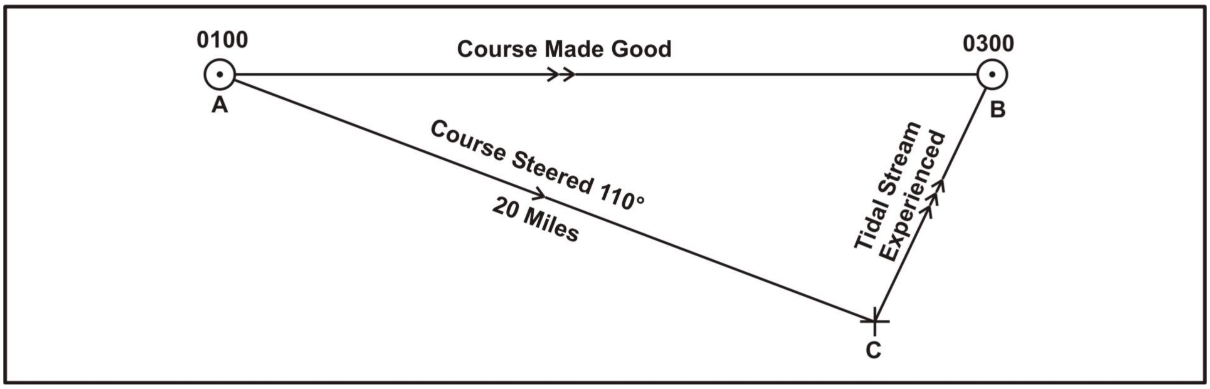

Fig 7-7. To find the direction and rate of the Tidal Stream between two Fixes
(*Not to Scale - Diagram exaggerated for clarity*)

(0713d continued)

Example 7-5. Find the EP allowing for Leeway, Tidal Stream, Current and Surface Drift.

The ship is *Fixed* at 0700, on an ordered course of 090˚, speed 15 knots. At the end of 1 hour, course steered recorded by automatic plotting is 090½˚, speed through the water from the log is 14.7 knots (allowing for calibration errors). Estimated *Tidal Stream* (from tables) is 295˚ 1.5 knots. Estimated *Current* (from routeing charts) is 060˚ 0.75 knots.

The wind has been blowing steadily in the area from the south at about 20 knots over the past 2-3 days. *Leeway* as deduced from the data in the Navigational Data Book is ¾ knot. Plot the *EP* after 1 hour, and calculate the estimated *Ground Track* / *Ground Speed* (course / speed made good), and the *Set / Drift* from the combined effects of *Tidal Stream, Current* and *Surface Drift*. The ship is in the Northern Hemisphere. From a study of the area and the data available it is estimated that *Surface Drift* will be in addition to the predicted *Current*.

- The *Leeway Vector* will be perpendicular to the course steered; thus, in this case it will be 000½˚ ¾ knot. (The *Leeway Angle* is 3˚.) Estimated *Surface Drift* will be 020˚ to 045˚ ½ knot; allow for 030˚.
- Plot the *DR* position *B* at 0800 from the course steered 090½˚ at the speed through the water 14.7 knots (*AB* in Fig 7-8 below). (Hint: to achieve 090½˚, lay the parallel rule through 091˚ and 270˚ on the compass rose).
- Plot the *Leeway BC*, 000½˚ 0.75 knot (Hint: lay parallel rule through 000˚and 181˚ on the compass rose).
- Plot the *Tidal Stream CD*, 295˚ 1.5 knots.
- Plot the *Current DE*, 060˚ 0.75 knot.
- Plot the *Surface Drift EF*, 030˚ 0.5 knot.
- *F* is the *EP* at 0800, *AF* is the estimated *Ground Track* / *Ground Speed* (course and speed made good over the ground) and *CF* is the estimated *Set* and *Drift*.
- The estimated *Ground Track* / *Ground Speed* is 082˚ 14.4 knots; the *Set* and *Drift* are estimated to be 343˚ 1.5 miles, rate 1.5 knots.

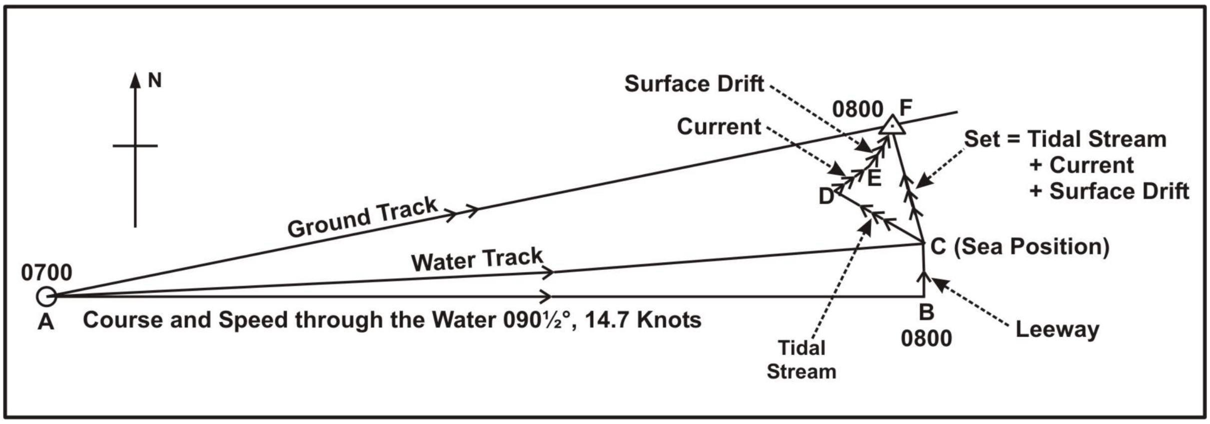

Fig 7-8. To find the direction and rate of the Tidal Stream between two Fixes
(*Not to Scale - Diagram exaggerated for clarity*)

0714. Allowance for Turning Circles in Chartwork

During chartwork for *Pilotage* and some *Coastal Navigation*, it is necessary to plot the '*Wheel-Over*' position as well as the *Waypoint* where tracks meet. To do this, knowledge of the ship's *Turning Circle* is necessary. Use of *Turning Circle*s and the *Manoeuvring Data* trials necessary to establish it are covered in detail in BR 45 Volume 6 (Shiphandling) Chapter 1, but a brief summary is repeated here. *Manoeuvring Data* trials are conducted for all classes of RN warships and the results are contained in the Navigation Data Book for each ship.

a. **Calculation, Monitoring and Control of Turns.** The path followed by the Bridge of most RN warships turning with headway closely approximates that taken by the *Pivot Point*, and this makes the Bridge an ideal position from which to plan, monitor and control turns. As the *Manoeuvring Data* supplied is intended for use from the Bridge position of RN warships, this allows turns to be planned and executed with precision.

b. **Transfer**. The *Transfer* for a specific alteration of course is **the lateral distance moved in a direction at right angles to the original course.** In Fig 7-9 (opposite), distance *QC* is the *Transfer* for an alteration of 60˚. When calculating the *Wheel-Over* position for a specific alteration of course, ***Transfer* is the first measurement that should be plotted** (as shown by the line *QC* in the 60˚ turn example at Fig 7-9).

c. **Advance.** The *Advance* for a specific alteration of course is **the distance moved in the direction of the original course** from the *Wheel-Over* position to the point where the ship steadies on her new course. In Fig 7-9 (opposite), the distance *AQ* is the *Advance* for an alteration of 60˚; in this example, should be measured back from position Q (obtained from plotting the *Transfer*) to obtain the *Wheel-Over* position at *A*.

d. **Distance to New Course (DNC).** *DNC* is the distance along the original course from the *Wheel-Over* position to the point of intersection between the new and old courses. In Fig 7-9 (opposite), distance *AL* is the *DNC* for an turn of 60˚. *DNC* is of little use for *Pilotage* and is misleading for turns over 120˚; it is thus little used.

e. **Intermediate Course and Intermediate Distance**. The *Intermediate Course* and *Intermediate Distance* are the direction and distance respectively between the *Wheel-Over* position on the original course and the point where the ship steadies on the new course. *Intermediate Course* and *Intermediate Distance* for a 120˚ alteration are shown Fig 7-9 by the angle RAE and the distance AE. Plotting the *Intermediate Course* and *Distance* does NOT give the position at which the ship steadies on the new course.

f. **Times of Turning.** The times taken for alterations of course under different speeds and rudder angles are also given in the *Manoeuvring Data* for RN warships. They are normally provided in minutes and seconds and are useful for certain manoeuvres.

g. **Tactical Diameter.** The *Tactical Diameter* (shown by distance PF in Fig 7-9) is the distance moved at 90˚ to the original course after turning through 180˚. *Tactical Diameter* does increase slightly with speed, but only by a relatively small amount.

h. **Final Diameter (Steady Turning Diameter).** The *Final Diameter* (also known as the *Steady Turning Diameter*) is the diameter of the *Turning Circle* when the ship is turning at a steady rate. It is shown by distance TU in Fig 7-9 (opposite).

(0714 continued)

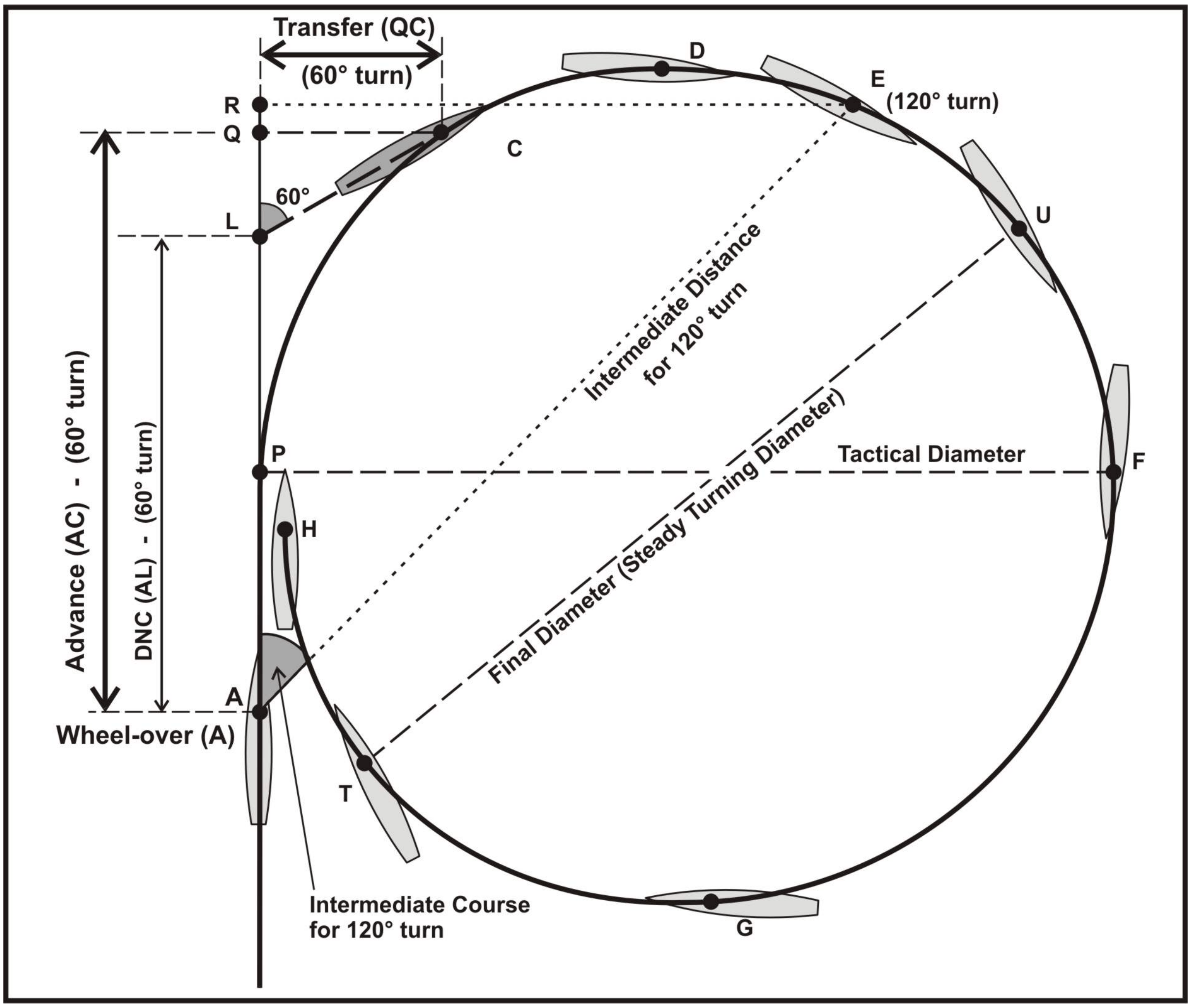

Fig 7-9. A Turning Circle showing, Transfer, Advance, Distance to New Course, Intermediate Course, Intermediate Distance, Tactical Diameter and Final Diameter

(0714) i. **Allowance for Loss of Speed in Turns.** Ships inevitably lose speed in turns, and it is sometimes necessary to adjust the *DR* time for this. Automated systems (eg *WECDIS / ECDIS / ECS* etc) have log speed and/or *GPS* inputs, and thus will normally allow for this automatically. *Manoeuvring Data* for RN warships includes an '*Acceleration Distance*' in yards per knot (also known as '*Speed Factor*'). To make the adjustment to a manual chart *DR*, multiply the *Acceleration Distance* by the speed in knots lost in the turn; convert this distance into time at the original speed and add this 'time correction' to the *DR* time at which the ship is calculated (from *Advance / Transfer* and '*Duration of Turn*' figures) to steady on the new course. A table of such 'time corrections' may easily be constructed for turns at various speed and rudder angles. This technique may also be applied to speed changes on the same course. In practice, application of this manual 'time correction' is rarely necessary, but the calculation is available if such precision is needed.

Example 7-6. A ship with an *Acceleration Distance* of 60 yards/knot alters course at 15 knots and loses 3 knots in the turn. On steadying, the ship's manual *DR* at 15 knots will be in error by a time equivalent to a distance at 15 knots of 3 x 60 = 180 yards. At 15 knots this distance equates to approximately 21½ seconds, which should be added to the manual *DR* time.

(0714) j. **Warships Manoeuvring in Groups.** When warships are manoeuvring together in groups, it can be difficult to apply *Manoeuvring Data* manually to the many bold alterations of course and speed occurring in quick succession, in order to maintain an accurate *DR*. Possible solutions to achieve an accurate *DR* are:

- Use automatic plotting facilities (eg *WECDIS / ECDIS* etc) to maintain the *DR* position and to provide frequent / continuous *Fixing*.
- Establish the Guide's position on the chart, *DR* it on (at what will normally be a track with fewer alterations of course and speed than warship consorts), and plot own ship at intervals from the Guide. *Fix* frequently to update the *DR*.

k. **Manoeuvring Data in Merchant Ships.** *Manoeuvring Data* in merchant ships is usually sparse, and often confined to the *IMO* minimum requirement (currently one hard turn port and one hard turn starboard at full rudder and one crash stop, all at maximum manoeuvring speed). In addition, due to different upper deck configurations and cargo loadings, the *Pivot Point* may be far displaced from the Bridge, thus making it difficult to plan and execute turns with the precision normally expected of an RN warship.

0715. Chartwork Planning Symbols

At the planning stage, the following symbols should be used for chartwork on paper charts (see Fig 7-10 opposite). *Blind Pilotage* symbols are at Para 1316. Symbols used with *WECDIS / ECDIS* equipments, although based on the paper chart equivalent, may vary widely depending on the system facilities available. See details at Para 0720.

a. **Planned Track.** Draw the planned track boldly, writing the course along the track with the course to steer in brackets alongside and the speed in a box, north orientated, underneath. The figures for course and speed should be sufficiently far away from the track to permit the necessary chartwork. See Fig 7-10a (opposite).

b. **Tidal Stream.** Indicate the expected *Tidal Stream*, showing the direction by a triple arrowhead, the rate in a box, and the time at which it is effective. See Fig 7-10b.

c. **Currents.** Indicate the expected *Current* (or *Surface Drift*), showing the direction by a single arrowhead on a 'wavy' line and the rate in a box. See Fig 7-10c (opposite).

d. **Wind.** Show the expected wind with direction and speed in a black arrow-headed box (do not confuse with a magenta 'Direction of Buoyage' symbol). See Fig 7-10d.

e. **Dangers.** Emphasise dangers near the track by outlining them boldly in pencil (or coloured ink for often-used charts). The *Limiting Danger Line* (*LDL*) depth ('*No Go Line*') should ALWAYS be drawn to show the navigable channel; this will vary with the *Height of Tide* (*HOT*) and should NOT be inked-in. See Fig 7-10e (opposite).

f. **Clearing Bearings.** Draw *Clearing Bearings* boldly, using solid arrowheads pointing to the object. 'NLT ...' (Not Less Than ...) or 'NMT ...' (Not More Than ...) should be written alongside. *Clearing Bearings* should be drawn sufficiently clear of the danger so that the ship is still safe even if the Bridge is on the *Clearing Bearing* line but turning away from danger. Allow for the Bridge being on the line with the stem or stern on the dangerous side of it, whichever is the greater distance. See Fig 7-10f (opposite).

g. **Distance to Run.** Indicate the distance to run to the destination, rendezvous, etc. Numbers should be upright. See Fig 7-10g(opposite).

(0715 continued)

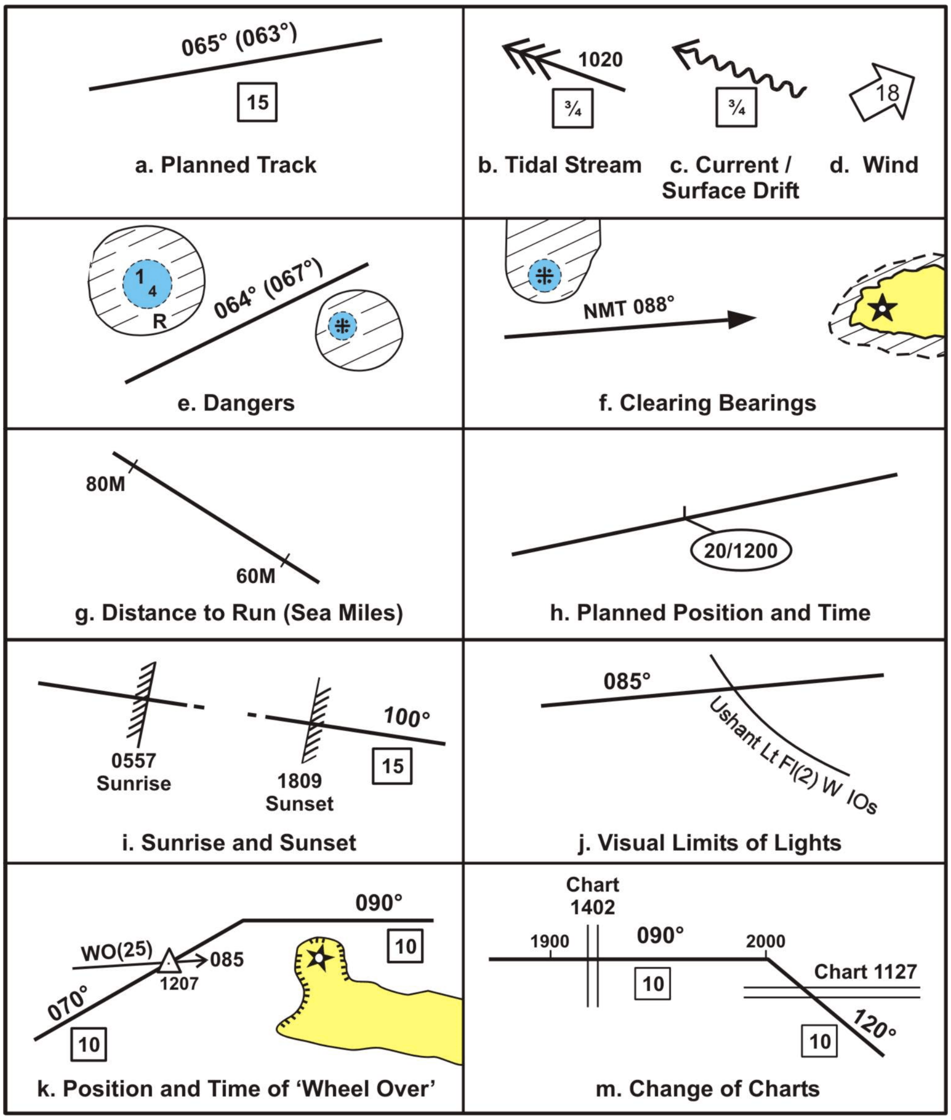

Fig 7-10. Chartwork Planning Symbols

(0715) h. **Planned Position and Time.** Indicate the time it is intended to be at particular positions at regular intervals, using 'bubbles' close to but clear of the track (ie '*Bubble Times*'). Ocean passages are normally marked every 12 hours and coastal passages more frequently, every 2 or 4 hours. See Fig 7-10h (above).

i. **SunRise and SunSet.** Indicate the times of *SunRise* (*SR*) and *SunSet* (*SS*)at the expected positions of the ship at those times. See Fig 7-10i (above).

j. **Visual Limits of Lights.** Indicate the arcs of the visual limits of lights that may be raised or dipped – the rising/dipping range. See Fig 7-10j (above).

k. **Position and Time of 'Wheel-Over'.** Show position and *DR / EP* time of '*Wheel-Over*' for alterations of course as a 4-figure time. The amount of wheel can be stipulated if this differs from standard. See Fig 7-10k (above).

m. **Change of Chart.** The positions of changes of chart should be indicated by double parallel lines, either vertical or horizontal. See Fig 7-10m (above).

0716. Execution of Chartwork on Passage

*Extracts of Paras 0716b/c are repeated at Paras 1231f/h and 1313a.

a. **Fixing.** Methods of *Fixing* the ship are at Chapter 8. With a sound plan, regular visual / radar *Fixing* has traditionally been the foundation of all *Coastal Navigation*, but this is now being supplanted by continuous *GPS / DGPS Fixing* with automatic *WECDIS / ECDIS* position displays. Although *WECDIS / ECDIS* give the Officer of the Watch (OOW) more time for other tasks (such as lookout), it is essential to make independent position checks at frequent intervals (see Para 0721).

b*. **Fixing and Comparison with DRs / EPs.** A *DR* (or *WECDIS / ECDIS* equivalent) from the last *Fix* should always be maintained ahead of the ship and an *EP* should be derived from all available information (ie *Leeway, Tidal Stream, Current*). Use of a *DR* alone may be acceptable when these factors are insignificant, **otherwise, an *EP* should ALWAYS be generated.** As soon as a new *Fix* is obtained, it should be compared with the *DR / EP* to ensure that there has been no mistake, to estimate the strength and direction of any *Tidal Stream* or *Current* since the last *Fix*, and to assess any actions needed. Generate a new *DR / EP* after an alteration of course.

c*. **Paper Charts - Frequency of Fixing.** The frequency of *Fixing* on paper charts should, in principle, depend on the distance from navigational hazards and the time the ship would take to run into danger before the next *Fix*; it is thus at least partly speed-dependent. The decision on the frequency of *Fixing* is ultimately for the CO and is usually specified in Captain's Standing Orders, although may be delegated to the NO or OOW. Useful 'Rules of Thumb' for *Fixing* on paper charts are:

- **Ocean Navigation - Fixing.** The frequency of *Fixes* will depend on the availability of position data (ie *GPS, LORAN-C / eLORAN*, astronomical observations, long range radar *Fixes* etc) and the distance from danger.

- ***Coastal Navigation - Fixing.** A useful *Coastal Navigation Fixing* 'Rule of Thumb' at 12-15 knots is as follows, although the actual *Fixing* interval chosen MUST be selected according to the circumstances prevailing:
  - *Fix* every 5 miles (approx) when navigating <u>well offshore</u> on a 1:150,000 coastal chart.
  - *Fix* every 2½ miles (approx) when coasting <u>closer inshore</u> on a 1:75,000 chart.
  - *Fix* every 1 mile (approx) when <u>approaching a port</u> using a 1:20,000 chart.
  - 6 minute intervals between *Fixes* are convenient for converting distance to *Ground Speed*, as 6 minutes is one-tenth of an hour (ie 1.35 miles in 6 minutes equals13.5 knots *Ground Speed* [speed made good]).

- **Fix Timing.** Every *Fix* should be timed to coincide with the *DR / EP* time, in order to check for unexpected position differences (see Para 0716b above).

- ***Pilotage.** In *Pilotage*, check *Fixes* should be plotted at least once per leg, and at intervals of not more than 6 minutes on long legs.

d. **WECDIS / ECDIS - Frequency of Check Fixing.** See Para 0721.

(0716) e. **Speed Calculations.** *Ground Speed* (speed made good) may be calculated from the distance run between *Fixes*, or estimated from predicted *Leeway, Tidal Stream, Current* and *Surface Drift*. Actual or estimated *Ground Speed* should always be used when projecting the *EP* ahead. *Ground Speed* is liable to fluctuate when any sea is running, also when the strength or direction of the *Tidal Stream* is changing.

f. **Chart Precautions.** Keep only one chart on the chart table, to avoid the error of measuring distances off the *Scale* of a chart underneath the one in use.

g. **Time Taken to Fix Manually.** The time taken to note the bearings and the time, plot the *Fix* on the chart, check the *DR* and lay off further *DR / EP*, verify time to '*Wheel-Over*' (if applicable), and return to lookout should not be more than 2 minutes at the most. A practised OOW should be able to complete the task within 60 seconds. If it is essential to reduce the *Fixing* time further, an assistant or a team should be used. Using an assistant, the time can be reduced to less than 30 seconds.

h. **Navigational Records.** Full details of the navigational records required in the RN / RFA are at BR 45 Volume 4. A summary is repeated here and amplified at Para 1238.

- **Navigational Record Book.** When any *Navigation* is being 'Executed', *Fixes* and alterations of course and speed, together with other information (see details at Para 1238) are entered in the *Navigational Record Book* (RNS 3034). The officer 'Executing' the *Navigation* keeps this record, but the NO has overall responsibility for supervising the OOWs to enter accurate and complete information. Care should always be taken to record times accurately as this is usually the single most difficult element when reconstructing the ship's track. The book's format is designed for entries to be made easily (see Fig 7-11) and the following symbols may be employed:

Ø	Transit
|←	Left-hand edge (of land, etc)
→|	Right-hand edge (of land, etc)
⊥ Port (5c)	Abeam to port (5 cables)
⊥ Stbd (1'.2)	Abeam to starboard (1.2 miles)

DATE	*15th June*		WATCH *Forenoon*	OOW *AGD*
TIME	**CO**	**SP**	**REMARKS**	
0900	*090*	*15*	*Fidra Ø Bass Lt. 089½°*	
			Elie Ness Lt. 047°	
			|← *E WEMYSS 344°*	
0915	*090*		*North Carr Lt. 335°*	
			Balcombe Tr. 279°	
			Fidra 243°	

Fig 7-11. The Navigational Record Book

- **The Ship's Log.** The NO has charge of the *Ship's Log* (RNS 0322) and is responsible for seeing that it is properly compiled in accordance with the instructions given inside the front cover. In particular, it should contain an extract of positional information from the *Navigation Record Book*.

0717-0719 Spare.

SECTION 2 - DIGITAL CHARTWORK PROCEDURES

0720. WECDIS / ECDIS Chartwork

Standard symbols to be used for chartwork on paper charts were specified at Paras 0710 and 0715, with *Blind Pilotage* paper chart symbols at Para 1316. Symbols used with *WECDIS / ECDIS* equipments, although based on the paper chart equivalents, may vary widely, depending on the system facilities available. One significant difference with most *WECDIS / ECDIS* equipments is that text labels are written horizontally and cannot normally be rotated parallel with a bearing line (eg *Clearing Lines* [ie *Clearing Range / Clearing Bearing*] or a 'Route' [Track] line). Paras 0720a-e with Figs 7-12 to 7-15 provide **some illustrative examples** of RN *WECDIS* chartwork, to provide an indication of current RN *WECDIS* symbology. A summary of *WECDIS / ECDIS / ECS* and *Electronic Chart* capabilities and limitations is at Paras 0630-0633. Full details are at BR 45 Volume 8.

a. **RN and RFA WECDIS / ECDIS Equipments.** In the RN / RFA, 'ECPINS' software (created by OSL Ltd) is installed on *WECDIS* terminals. This software provides comprehensive facilities to display automatic and manual chartwork symbols which are reasonably close to those specified for paper charts. A few older *ECDIS* systems are also fitted to some RFAs, and these have significantly less functionality and capability in symbology than *WECDIS* equipments.

b. **WECDIS - Safety Depth Contour and Interpolated Depth Contours.** On (*Vector*) *ENCs*, *WECDIS* is capable of higlighting a selected 'Safety Depth Contour' and drawing mathematically created intermediate interpolated depth contours, to assist with manual creation of the *Limiting Danger Line* (*LDL*). In addition *WECDIS* can display prominent 'Shallow Danger Marks' to highlight isolated dangers inside 'Safety Depth Contour'. See Fig 7-12 (below)

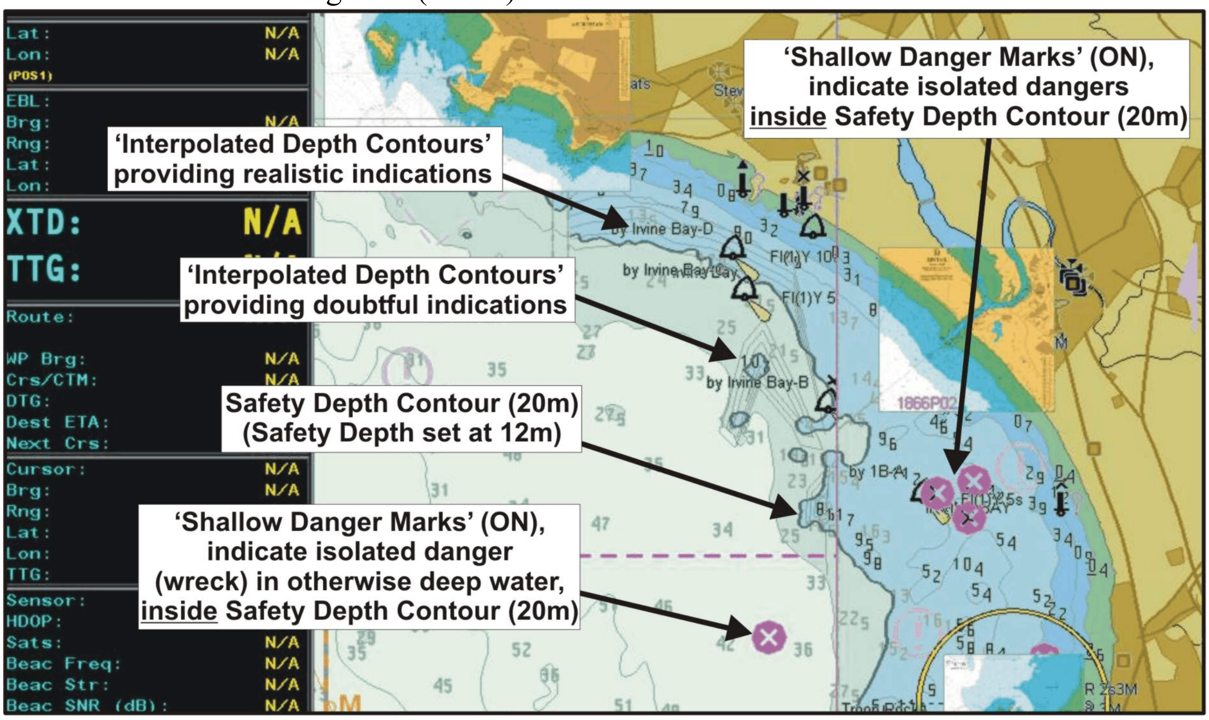

Fig 7-12. Example WECDIS Symbology - 'Safety Depth Contour', 'Interpolated Depth Contours' and 'Highlight Shallow Dangers'

(0720) c. **WECDIS - Route Display Options.** *WECDIS* can display the 'Active Route' (Track) and 'Track Labels'(with predicted *Set* and *Drift* plus 'Course to Steer'), the 'Active Leg' within that 'Route', *Wheel-Overs* and a *Wheel-Over* line, a 'Cross Track Distance' (XTD) 'Alarm Corridor' (which alarms if 'Own Ship' strays outside it), '*Waypoints*' and '*Waypoint* Labels', as well as 'Own Ship' itself and an 'Anti-Grounding Cone'(AGC). All items are user-selectable for display. See Fig 7-13 (below).

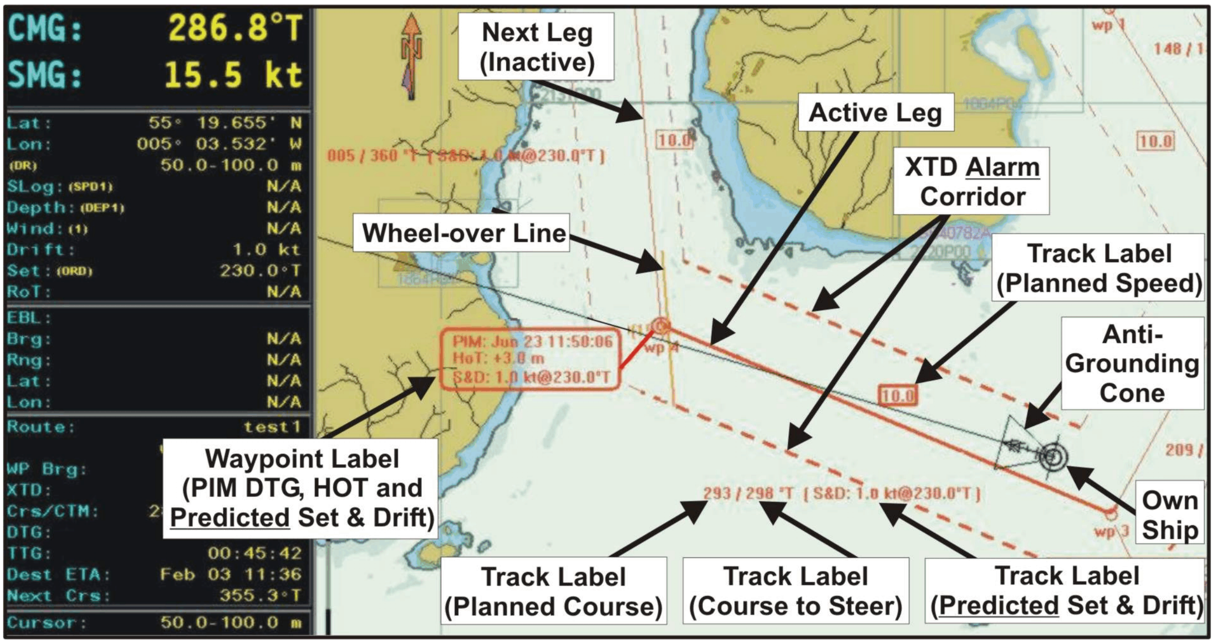

Fig 7-13. Example WECDIS Symbology - 'Route Display Options'

d. **WECDIS - Anti-Grounding Cone.** *WECDIS* provides an 'Anti-Grounding Cone' with user-selectable parameters, and which alarms against nominated dangers and certain features (see Fig 7-14a below). The 'Anti-Grounding Cone' (AGC) bends around *Waypoints* provided 'Own Ship' is inside the 'XTD Warning Corridor' (see Fig 7-14b below).

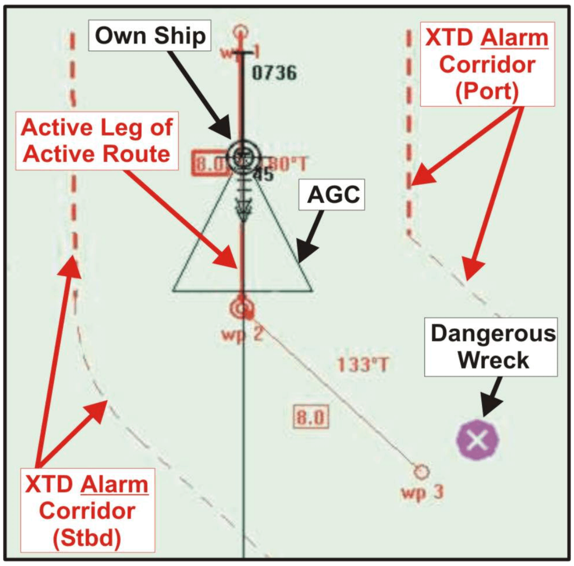

Fig 7-14a. Anti-Grounding Cone (AGC) and XTD Alarm Corridor

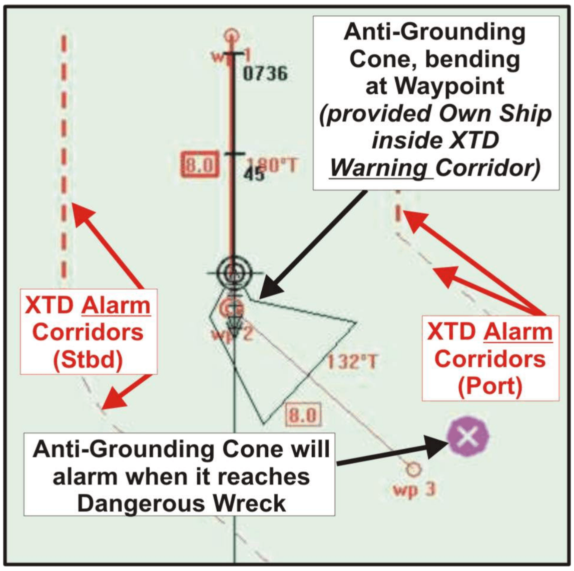

Fig 7-14b. AGC bending at Waypoint if Own Ship is inside XTD Warning Corridor (*Warning Corridor not shown in diagram*)

(0720) e. **WECDIS - Radar Image Overlay and 'Ghost Ship'.** A '*Radar Image Overlay*' (*RIO*) can be superimposed over the *WECDIS* chart display for checking that the radar coastline and other features (shown in **GREEN** at Fig 7-15 below) are in alignment with the chart (ie *GPS* has not slipped). *WECDIS* can also display a 'Ghost Ship' positioned at a user-selectable distance ahead of 'Own Ship'. *Para 0720e is repeated at Para 1528.*

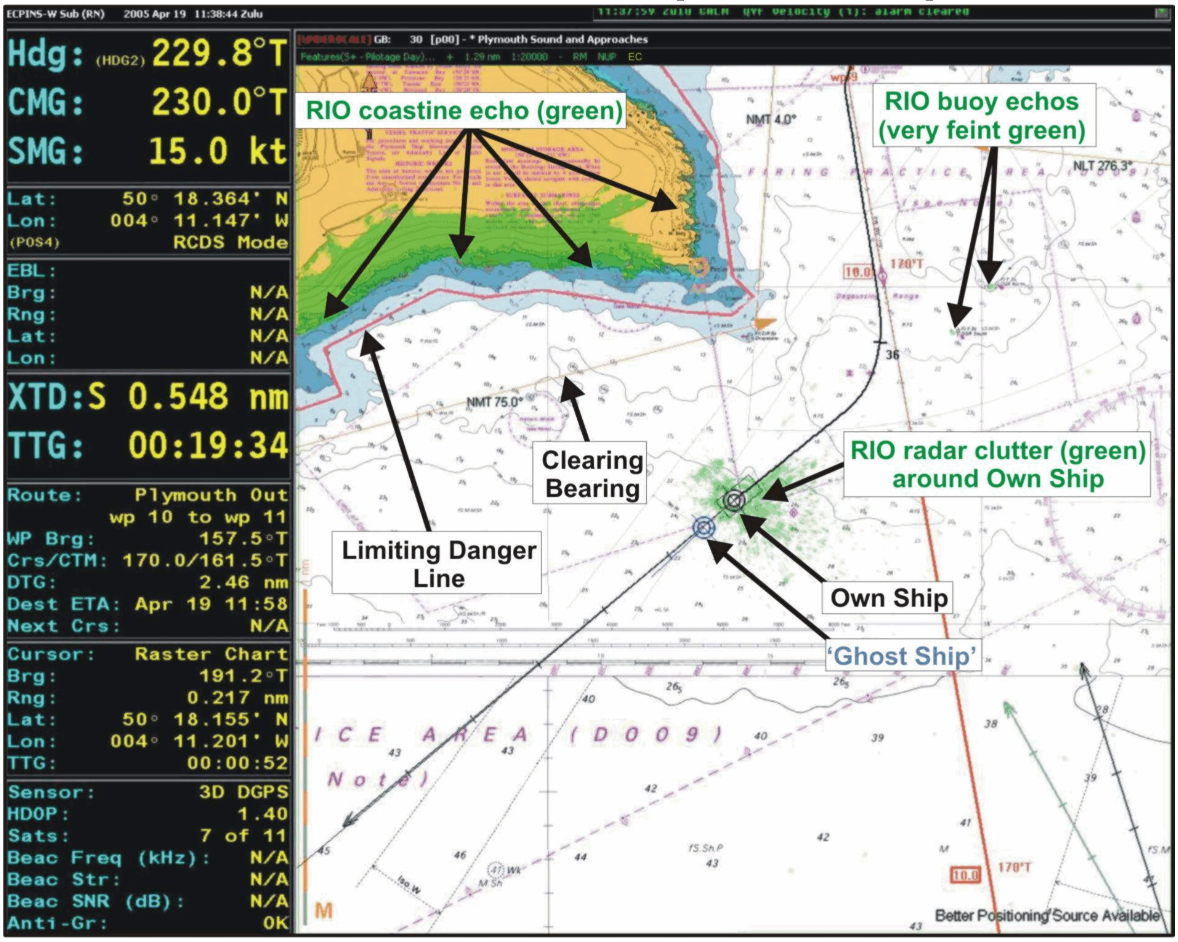

Fig 7-15. Example WECDIS Symbology - Radar Image Overlay (RIO) & 'Ghost Ship' (*RIO shown as a Green Image*)

0721. WECDIS / ECDIS Check-Fixing Intervals (**Extracts repeated at Paras 1231g/1313a*)

*For the purposes of check-*Fixing* in *WECDIS / ECDIS*, in the RN, *'Coastal Navigation'* is deemed (depending on circumstances and the size of vessel involved) to be *Navigation* at distances between about 2 *n. miles* and 15 *n. miles* from the *Limiting Danger Line* (*LDL*); *'Ocean Navigation*' is deemed to be *Navigation* at distances greater than 15 miles from the *LDL*. RN *WECDIS / ECDIS* check-*Fixing* intervals are as follows. See BR 45 Volume 8(1) for details.

a **Ocean Navigation.** In *Ocean Navigation*, the interval between manual check *Fixes* should not be greater than 30 minutes.

b*. **Coastal Navigation.** In *Coastal Navigation*, the interval between manual check *Fixes* should not be greater than 30 minutes, but a *RIO* coastline alignment check should be carried out (subject to emission policy) at not more than 15 minute intervals between *Fixes*.

c*. **Pilotage.** In *Pilotage*, check *Fixes* should be plotted at least once per leg and at intervals of not more than 6 minutes on long legs.

CHAPTER 8

VISUAL FIXING

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CHAPTER 8

VISUAL FIXING

0801. Scope of Chapter

This chapter builds on the principles established at Chapter 7 (Chartwork) and covers all aspects of *Fixing* a vessel's position by visual means. It replaces Chapter 9 of the 1987 Edition of this book. Radar *Fixing* is at Chapters 12, 13 and 15, as appropriate to the context.

0802. Azimuth Circles for Visual Bearings

A variety of bearing taking equipments (of greater or lesser ease of use) are found at sea, but most rely on some sort of prismatic *Azimuth Circle* fitted on a compass repeater.

a. **Admiralty Pattern Azimuth Circle.** The 'standard' Admiralty Pattern *Azimuth Circle* (see Fig 8-1 below) is easy to use and is accurate in all conditions. It uses a curved prism; thus if the object is seen through the ' V ' sight, the correct bearing is read, even if the *Azimuth Circle* itself is not aligned to the object (see Fig 8-2 below). A line is engraved on the face of the prism to assist reading of the bearing. Reflections of high altitude objects on the black mirror are sighted through the ' V ' sight; for this, the bubble level is used to keep the *Azimuth Circle* horizontal. The alignment of the *Lubbers Line* to ship's head should be checked frequently and adjusted if necessary (see Para 1230f).

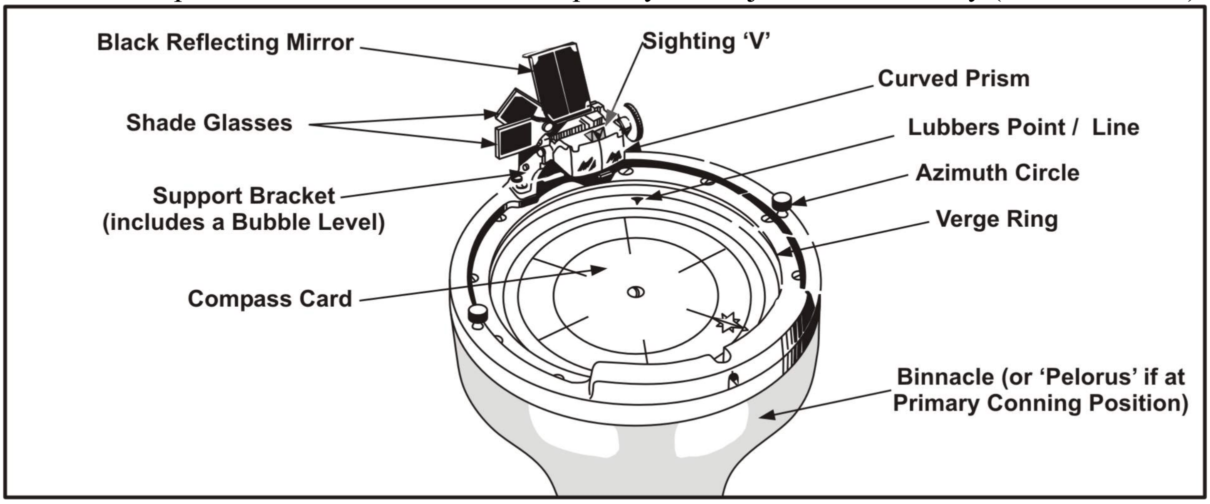

Fig 8-1. The 'Standard' Admiralty Pattern Azimuth Circle

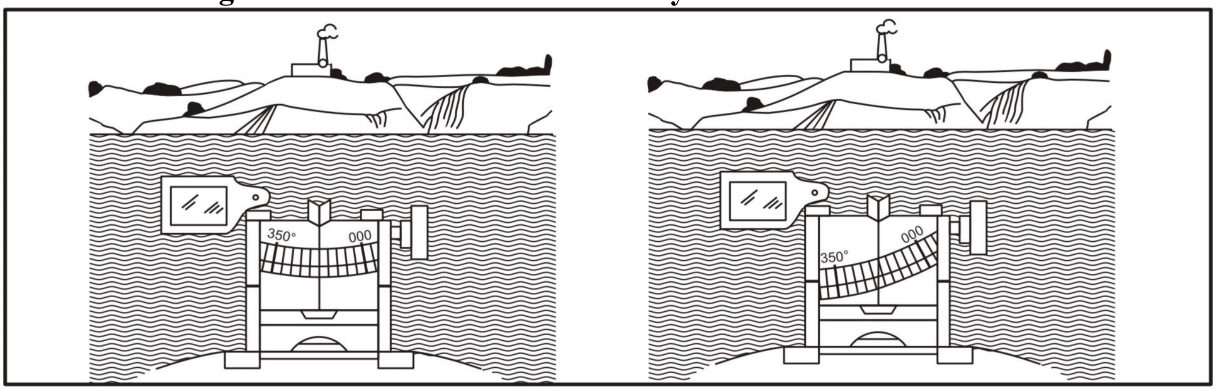

Fig 8-2. The Admiralty Azimuth Circle (does not need to be aligned to the object)

b. **Other Azimuth Circles.** Other designs of *Azimuth Circle* are often awkward to use and do not produce sufficiently accurate results, particularly in difficult conditions.

0803. Methods of Obtaining a Position Line

Although *Fixing* by *DGPS / GPS* is the norm, it may be necessary on occasion to use other means. A single *Position Line* (as opposed to a *Fix*) may be obtained from:

- **Compass Bearing (also radar bearings).** See details at Para 0803a.
- **Relative Bearing.** See details at Para 0803b.
- **Transit of Two Fixed Objects.** See details at Para 0803c.
- **Horizontal Angle (by compass or HSA).** See details at Para 0803d.
- **Radar Range.** See details at Para 0803e.
- **Range by Distance Meter (height of object known).** See Para 0803f.
- **Range by Rangefinder.** See details at Para 0803g.
- **Range by VSA / HSA (height of object known).** See Paras 0803h-i.
- **Soundings.** See details at Para 0803j.
- **Astronomical Observation.** See details at Para 0803k.
- **Sonar Range.** See details at Para 0803m.
- **Rising or Dipping Range (height of object known).** See Para 0803n.
- **Horizon Method of Rangefinding.** See Para 0803p.

a. **Compass Bearing (also Radar Bearings).** Bearings should be taken visually; radar bearings are significantly less accurate (see Para 1522) and should NOT normally be used as *Position Lines* unless no practical alternative exists. When a visual bearing of the edge of an object is taken, it is best to use a vertical edge if possible; the *Height of Tide* (*HOT*) must be taken into account if the bearing of a sloping edge of land is used, as the charted edge is the *MHWS / MHHW* line (or *Mean Sea Level* in areas where there are no tides). Symbols used for recording bearings of edges are at Para 0716h.

b. **Relative Bearing.** A *Position Line* may be calculated from a *Relative Bearing* (see Para 0126) when added-to / subtracted-from the True bearing of ship's head; if ship's head is not recorded at the instant of taking the *Relative Bearing,* inaccuracies may occur. At Fig 8-3 (below), an object is observed at Green 60; ship's head is 030˚, thus the True bearing of the object is 090˚.

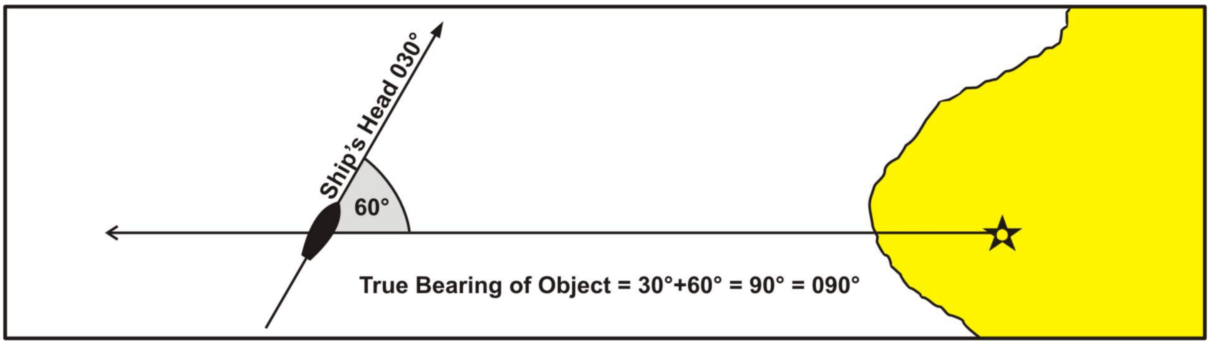

Fig 8-3. Position Line by Relative Bearing and Ship's Head

c. **Transit of Two Fixed Objects.** If two fixed objects are seen in transit (in line), then the observer must be on an extension of line joining them (see Fig 8-4 opposite). For good 'sensitivity' of relative movement between the objects, ideally, the distance between the observer and the nearer object should be not more than 3 times the distance between the objects. However, less sensitive transits may still be used with care. Transits are used in *Fixes*, for checking compass errors (see Para 0811), and for assessing ship movement when *Turning at Rest* even if the objects are not charted. The symbol ∅ is used for a transit, but the symbol ≠ is used by the *UKHO* to identify transits on Admiralty charts (see *UKHO* Chart 5011 [Symbols & Abbreviations]).

(0803c continued)

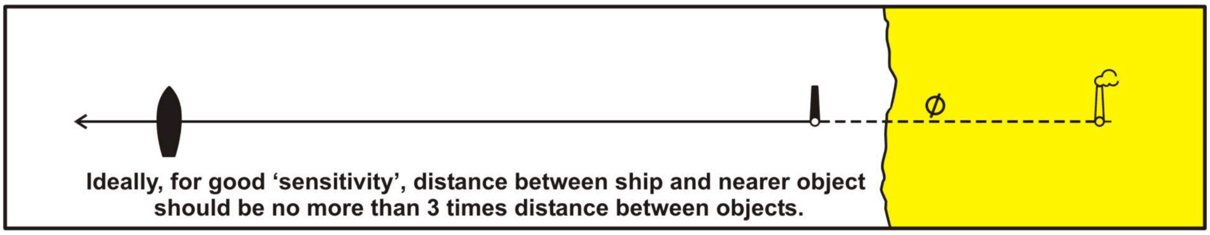

Fig 8-4. Position Line by a Transit of Fixed (Charted) Objects

(0803) d. **Horizontal Angle (Compass or HSA).** All angles subtended by a chord in the same segment of a circle are equal; it follows that if a horizontal angle between two objects is measured by sextant or by compass, the observer must lie on the arc of a circle containing the angle observed and both objects. In Fig 8-5 (below), the *Horizontal Sextant Angle* (*HSA*) between objects *A* and *C* is 80˚. The observer is thus on a *Position Line* which is the arc of the circle *ABC* along which the angle between *A* and *C* is always 80˚. An *HSA* may be used as a '*Danger Angle*' to clear a danger (see Fig 8-5 [below] and Para 1233 / Example 12-3). Detailed procedures for *HSAs* are at Para 0808.

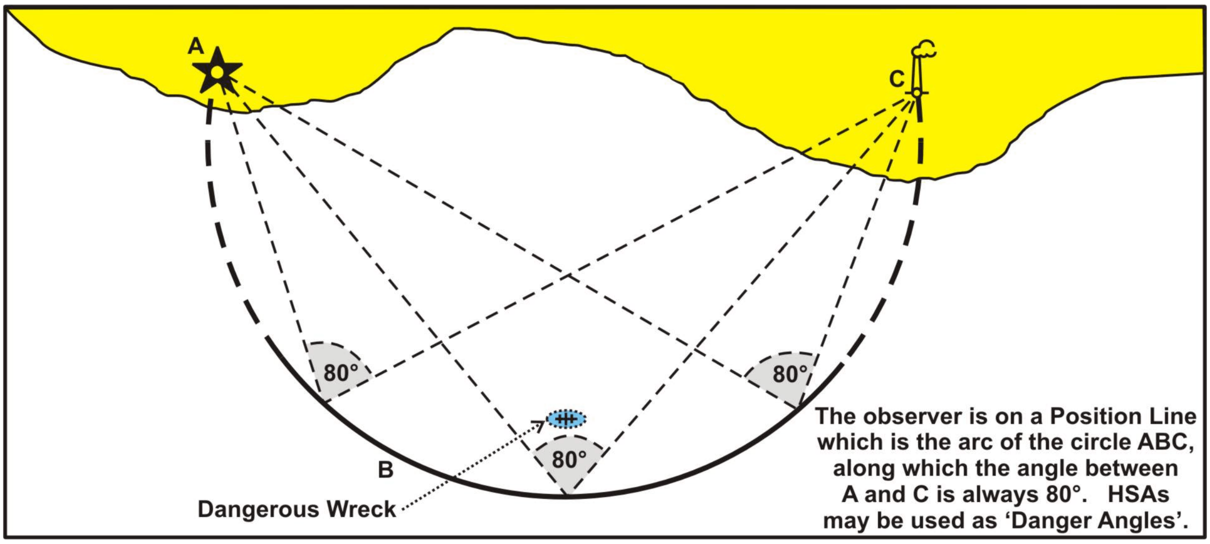

Fig 8-5. Position Line by Horizontal Sextant Angle (*also showing 'Danger Angle' use*)

e. **Radar Range.** Radar may be used to obtain a *Position Line* as a circular arc of a circle at both short and long ranges off the land. See details at Paras 1232 and 0710.

f. **Range by Distance Meter (Height of Object Known).** '*Distance Meters'* operate on the principle of the *Vertical Sextant Angle* (*VSA* - see Para 0803h overleaf) with the height of the object set onto the instrument and the ranges obtained read directly from it. *Distance Meters* of this type are normally for short range use. The operation of *Distance Meters* in RN service is covered in BR 45 Volume 3.

g. **Range by Rangefinder.** '*Optical Rangefinders'* use parallax principles to establish the distance of an object whose height is not known; none are currently in RN service. '*Laser Rangefinders'* operate by generating a narrow and intense beam of coherant infra-red radiation. Concerns over eye-safety, attenuation when used in wet weather and from enclosed Bridges have prevented their introduction into RN service.

(0803) h. **Range by VSA / HSA - Base of the Object Visible to the Observer.** A *Position Line* may be obtained from a *Vertical Sextant Angle* (*VSA*) of an object whose base is visible to the observer, by multiplying the height of the object by the *Cot* of the observed angle *BAC* (see formula (8.1) / Fig 8-6). The width may be used with formula (8.1a).

$$Distance = Visible\ Height\ of\ Object \times Cot\ (Observed\ Angle) \quad \mathbf{...\ 8.1\ (1987\ Ed\ ...\ 9.1)}$$

$$Distance = \frac{Width\ of\ Object \times 360}{2\pi \times Observed\ Angle\ (in\ degrees)} \quad \mathbf{...\ 8.1a}$$

- **Position Line.** The *Position Line* will be the circumference of a circle, centred on the object (B), which has this distance as its radius.
- **Danger Angle.** A *VSA* may be used as a *Horizontal* or *Vertical Danger Angle* to clear a danger (see Fig 8-6 [below] and Para 1233d / Example 12-2).
- **Charted Height / Elevation and HOT Correction.** The charted height or *Elevation* of an object is usually above *MHWS* or *MHHW* (see Para 0624v) so it must be corrected for *HOT*. If this correction is not made, the calculated distance will be less than the actual distance (thus erring on side of safety).
- **Charted Elevation and Height of Structure.** The charted *Elevation* of a lighthouse is taken from *MHWS* or *MHHW* to the centre of the lens and NOT to the top of the structure. Heights of light structures (base to top) are also tabulated in the *Admiralty List of Lights and Fog Signals* (*ALLFS*).
- **Norie's Nautical Tables.** Norie's Nautical Tables solve the triangle for ranges between 1 cable and 7 miles and heights between 7 metres (23 feet) and 600 metres (1969 feet).
- **Shoreline Errors.** Angle (α) in Fig 8-6 is *BDE* but, provided that shoreline-distance (*DC*) is greater than *Elevation H* (*BE*) and *Elevation H* is greater than shoreline-object distance (*CE)*, no appreciable error is introduced if angle *BAC* (α') is used instead of *BDE* (α) (see Notes 8-1 / 8-2 below).

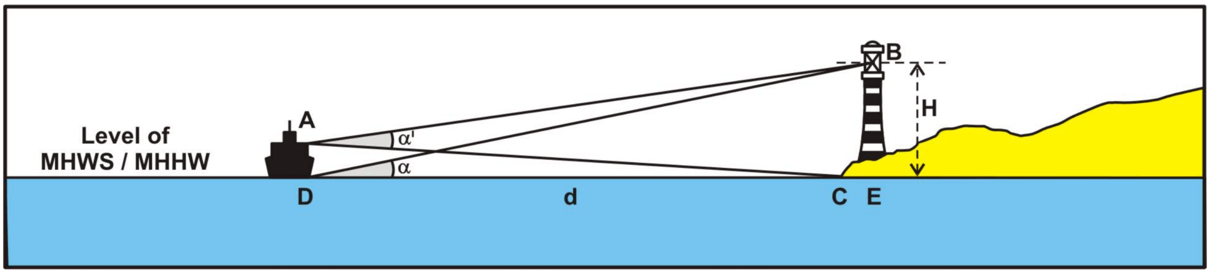

Fig 8-6. Vertical Sextant Angle (VSA) - Base of the Object Visible to the Observer

Notes:
8-1. Shoreline Errors - Object at Shoreline. *If the object observed (B) is vertically over the shore edge (C), and object-distance (DE) is greater than Elevation H (BE), the error in position will be less than the observer's height of eye (AD).*
8-2. Shoreline Errors - Object set back from Shoreline. *If the object is NOT vertically over the shore edge (C) (as in Fig 8-6), provided the shoreline-distance (DC) is greater than Elevation H, and Elevation H is greater than the shoreline-object distance (CE), the error in position is less than 3 times the observer's height of eye AD.*

(0803h continued)

Example 8-1. Vertical Sextant Angle (VSA) Calculation

A *VSA* of a lighthouse, charted *Elevation* 40 metres above *MHWS*, is 0°46'.2. *HOT* is 2.12m below *MHWS*. The *Index Error* of the sextant is + 1'.2. What is the range of the light?

Observed VSA	=	*0°46'.2*	
Index Error	=	*+1'.2*	
Corrected VSA	=	*0°47'.4*	*(0.79°)*
Charted Elevation	=	*40.00m*	
		+ 2.12	
Corrected Elevation	=	*42.12m (0'.02274 n. miles)*	. . . (formula 8.1)

Range = *Corrected Elevation* x *cot Corrected angle*
= *0.02274 cot 0.79°*
= *1.65 n. miles (Norie's Nautical Tables also give 1.65 n. miles).*

(0803) i. **Range by VSA - Base of the Object Beyond the Observer's Horizon.** A *Position Line* may also be obtained from the observation of the *Vertical Sextant Angle* (*VSA*) of an object (eg distant mountain peak) where the base is <u>out of sight beyond</u> the observer's horizon, but the calculation is more complex, and some approximations are necessary. See Appendix 6 Para 2b for details.

j. **Soundings.** Soundings are frequently of value in establishing a *Position Line*. In areas where a particular depth contour on the chart is sharply defined and reasonably straight, or in approaches to the land where there is a steady decrease in depth, a *Position Line* may be obtained. Good examples of this are in the south-western approaches to the British Isles, where the depth decreases rapidly from 2000 to 200 metres in a distance of some 10 to 20 miles, or in the southern approaches to Beachy Head, where the depth shoals from 50 to 30 metres in about 1½ miles. In the latter case, it will be necessary to allow for the *HOT* and to ensure that the *Echo Sounder* is reading accurately. The procedures for calibrating the *Echo Sounder* are at BR 45 Volume 3 and are summarised at Paras 1807 and 1827n.

k. **Astronomical Observation.** An *Astronomical Position Line* is a small element of the circumference of a *Small Circle* centred on the *Geographic Position* of a heavenly body with a radius equivalent to '*90° - Altitude*'(corrected for errors), converted into *n.miles.* Although an *Astronomical Position Line* is circular, for practical purposes it may be treated as a straight line, except in the case of very high altitudes. An *Astronomical Position Line* may be obtained from the observation of one of the heavenly bodies - the sun, moon, stars or planets. The term *'Astronomical Position Line'* is often abbreviated to '*Position Line*'. Full details of astronomical observation procedures and methods of making the necessary calculations are at BR 45 Volume 2.

m. **Sonar Range.** Provided sonar equipment is fitted, it is possible to obtain the range of an underwater object (eg sonar conspicuous rock or *Coral* outcrop) which can be used for navigational purposes as a *Position Line* in the form of a circular arc. However, it is often impossible to determine precisely from which part of the sea-bed the sonar range is being obtained, **and so <u>particular care</u> should be taken with this technique.**

(0803) n. **Rising or Dipping Range.** The 'Rising or Dipping Range' is the range at which an object is observed to rise above / fall below the horizon. It is approximate, is substantially affected by *Atmospheric Refraction* and **should be treated with caution**.

- **Distance of the Sea Horizon - Formulae.** The theoretical geometric distance of the sea horizon for a height of h metres is a range between $1.92\sqrt{h}$ to $1.93\sqrt{h}$ *Sea Miles,* or $1.93\sqrt{h}$ to $1.92\sqrt{h}$ *n. miles* (depending on *Latitude*), thus in practice these units are interchangeable in the calculation and are identical at approximately 45˚ *Latitude*. However, the effect of 'standard' *Atmospheric Refraction* (see Para 1515) is to increase the geometric distance by about 8%. Formulae for the approximate distance of the sea horizon are:

	Distance	$= 2.08\sqrt{h}$ *(metres)*	[approx]	**. . . 8.2 (1987 Ed . . . 9.2)**
or:	*Distance*	$= 1.15\sqrt{h}$ *(feet)*	[approx]	**. . . 8.3 (1987 Ed . . . 9.3)**

- **Distance of Sea Horizon - Nories Nautical Tables and ALLFS.** These distances may also be found in Norie's Nautical Tables or the *Geographical Range* table in the *ALLFS* - [see Para 0932]. As these tables make different allowances for *Atmospheric Refraction* the distances obtained will be different (see Note 8-3 below) .

- **Distance of Sea Horizon + Distance of the Object Beyond the Horizon.** The distance of sea horizon and the distance of the object beyond the horizon added together give the distance of the ship from the object (see Fig 8-7)

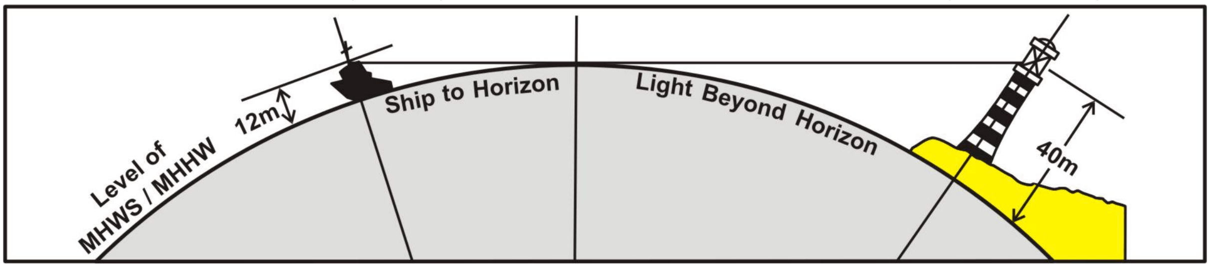

Fig 8-7. Position Line from a Dipping Range

Note 8-3. *Norie's Tables uses the formula: distance of the sea horizon $d = 2.095\sqrt{h}$ where h is the Elevation in metres. ALLFS uses the formula: $d = 2.03\sqrt{h}$, where h is the Elevation in metres, or $d = 1.12\sqrt{h}$, where h is the Elevation in feet.*

Example 8-2. Rising or Dipping Range Calculation

A shore light, *Elevation* 40 metres, is observed from the Bridge to dip below the horizon. Height of eye is 12 metres. What is the range of the light?

From . . . formula (8.2), the following ranges are obtained:

Range of sea horizon for height of eye of 12 metres	7.21	Sea Miles
Range of sea horizon for *Elevation* of 40 metres	13.16	Sea Miles
Range of light	**20.37**	**Sea Miles.**

From Norie's Nautical Tables the following ranges are obtained:

Range of sea horizon for height of eye of 12 metres	7.3	Sea Miles
Range of sea horizon for *Elevation* of 40 metres	13.3	Sea Miles
Range of light	**20.6**	**Sea Miles.**

The range given in the *Geographical Range* table in *ALLFS* is **19.9 Sea Miles.**

(0803) p. **Horizon Method of Rangefinding.** When the height of the object is unknown or negligible, the range may be found if one's own height of eye is known. The (theoretically negative) *VSA* between the waterline of the object and the sea horizon is measured (see Fig 8-7a below). This angle, corrected for *Index Error* and *Dip* ('Dip of the Sea Horizon' - see the Nautical Almanac or Norie's Nautical Tables) is angle θ. It can be shown that angle θ, *h* (height of eye) and *d* (distance in n miles) are connected by the formulae:

$$\theta = 0.5658\frac{h}{d} + 0.42\,d \qquad \textit{(h in feet, d in n.miles)} \qquad \textbf{. . . 8.3a}$$

$$\theta = 1.8563\frac{h}{d} + 0.42\,d \qquad \textit{(h in metres, d in n.miles)} \qquad \textbf{. . . 8.3b}$$

These formulae may be solved for *d*, or *d* may be found from the 'Dip of the Shore Horizon' table in Norie's Nautical Tables. This method should give the range to within about 3% at short ranges. The greater the height of eye, the greater should be the accuracy, since the base line is the vertical line from the sea to the observer. However, any values obtained are approximate, especially if *Abnormal Refraction* is suspected.

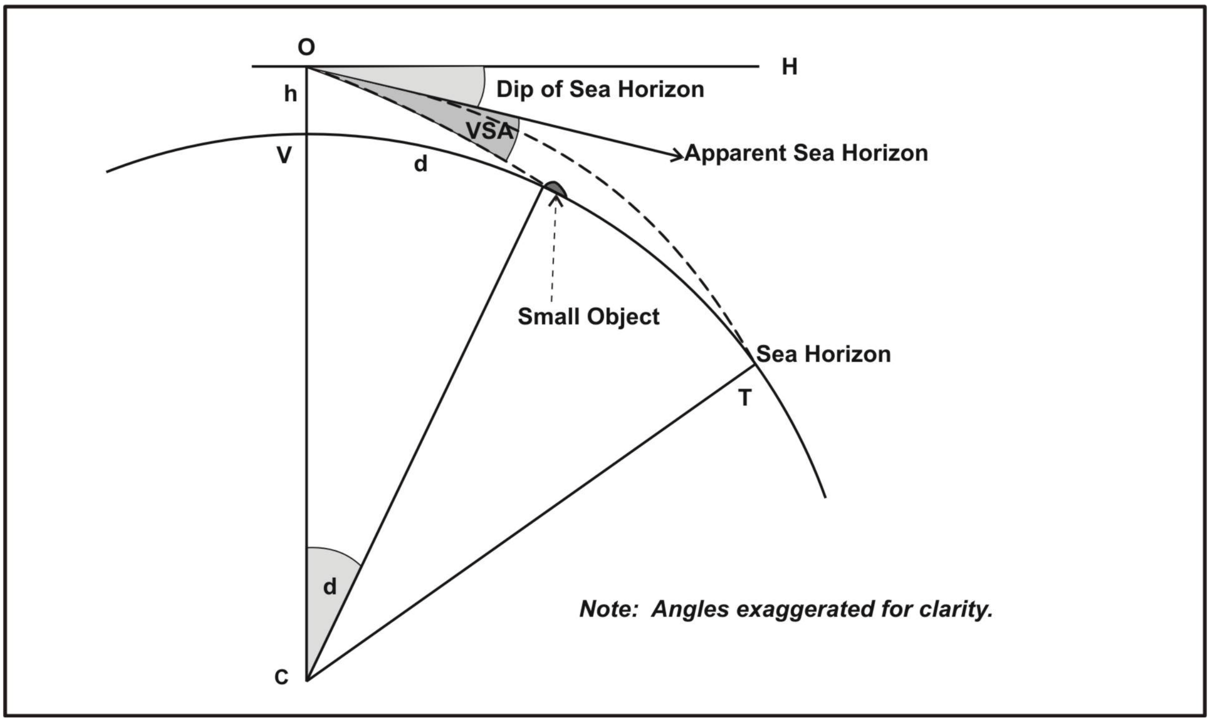

Fig 8-7a. Horizon Method of Rangefinding.

Example 8-3. The angle between the base of an object and the horizon is observed to be 0˚ 12.1´ *Index Error* is + 1´.2. Height of eye is 49 feet. What is the range of the object?

Observed angle	0˚ 12´.1
Index Error	+ 1´.2
Corrected angle	0˚ 13´.3
(-) *Dip* of sea horizon	6´.8
θ	0˚ 20´.1

$$20.1' = \frac{27.7242}{d} + 0.42\,d \qquad \text{. . . (formula 8.3a)}$$

$$d = 1.42 \text{ n. miles} \quad \textit{(or 1.43 from Nories)}$$

0804. Transferred Position Lines

A *Position Line* may be '*Run-on*' along the ship's track as a '*Transferred Position Line*', allowing for wind and the estimated *Tidal Stream / Current* if known. This procedure has many uses, predominately in a *Running Fix*, but may also assist in clearing some danger or making a *Wheel-Over* into an ill-defined anchorage or port. It is a standard procedure in astro navigation when plotting *Astronomical Position Lines* (see BR 45 Volume 2 Chapter 3). The technique is illustrated at Examples 8-4 to 8-6 (below and opposite).

Example 8-4. A ship on course 090˚ speed 8 knots observes a lighthouse bearing 035˚ at 1600. There is no *Tidal Stream* or wind. Plot the *Transferred Position Line* for 1630. See Fig 8-8.

- **Position Line.** Draw a line in a 215˚ direction from the light. This is the *Position Line* at 1600. The ship's actual position *A* at 1600 is unknown, although it must be somewhere on the *Position Line* from the light through *A*.
- **Transferred Position Line.** Assume that the ship may be at *A* (somewhere on *Position Line* from the light) and project *AB* in a direction equivalent to a 30 minute run, in this case 090˚ 4 miles. Construct a *Transferred Position Line*, with a double arrowhead at the outer end, parallel to *Position Line* through *A* and passing through *B*. The ship should be on the *Transferred Position Line* at 1630.

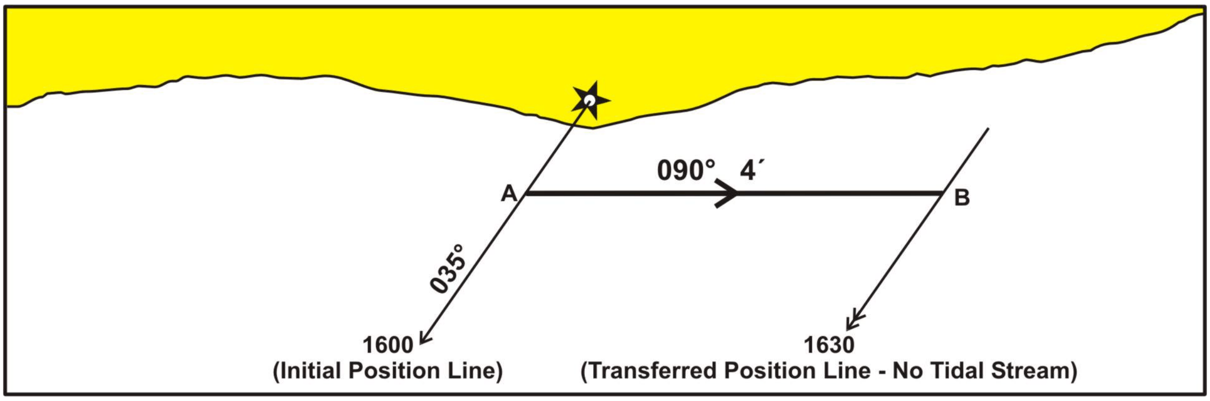

Fig 8-8. Plotting a Transferred Position Line (No wind or Tidal Stream / Current)

Example 8-5. *Plot the Transferred Position Line* from position A for 1630 as in Example 8-4 (above), but with a *Tidal Stream* of 132˚ 1.7 knots. See Fig 8-9.

- **Transferred Position Line.** Proceed as in Example 8-4, but add a *Tidal Stream* vector (*BK*) of 132˚ 0.85' before plotting the *Transferred Position Line*.

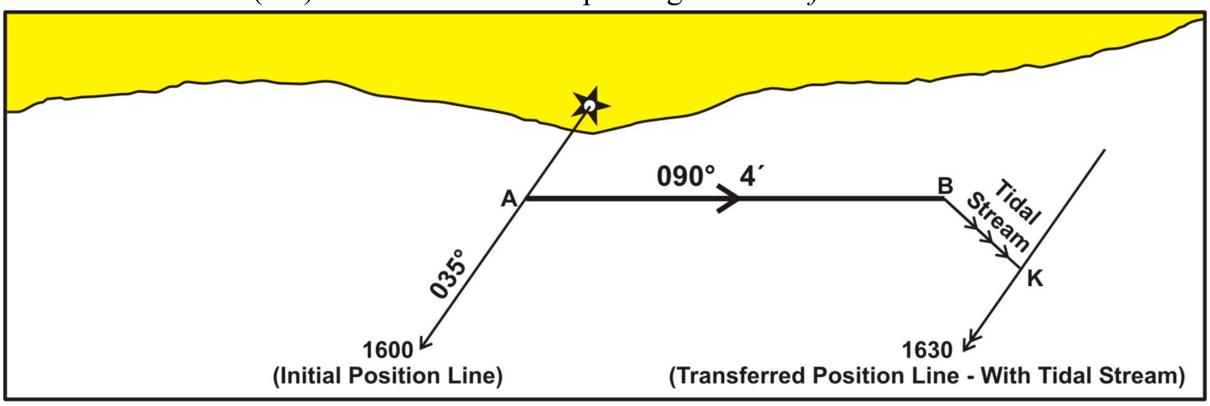

Fig 8-9. Plotting a Transferred Position Line (With Tidal Stream)

Example 8-6. A ship on course 180˚ requires to turn onto an anchorage approach 080˚ in an ill-defined bay without a suitable *Wheel-Over* mark. Using lighthouse '*L*', plot the transferred *Wheel-Over* bearing and *Transferred Position Line* '*CK*' for the approach to anchorage '*K*'. Assume that there is no wind or *Tidal Stream*. See Fig 8-10 (below).

- **Position Line.** Observe lighthouse *L* when it bears 080˚ (parallel to the anchorage approach course '*CK*') and note the time.
- **Transferred Position Line.** Plot *Wheel-Over* position *B* using ship' turning data. Transfer the 080˚ *Position Line AL* through *B*, calculating the time the ship will reach the transferred *Wheel-Over* line.
- **Tidal Stream.** In this example (for simplicity), wind and *Tidal Stream* was assumed to be zero. In practice, this will rarely be the case and the time of the transferred *Wheel-Over* will need to take wind and *Tidal Stream* into account, using the procedures at Example 8-5 (previous page).

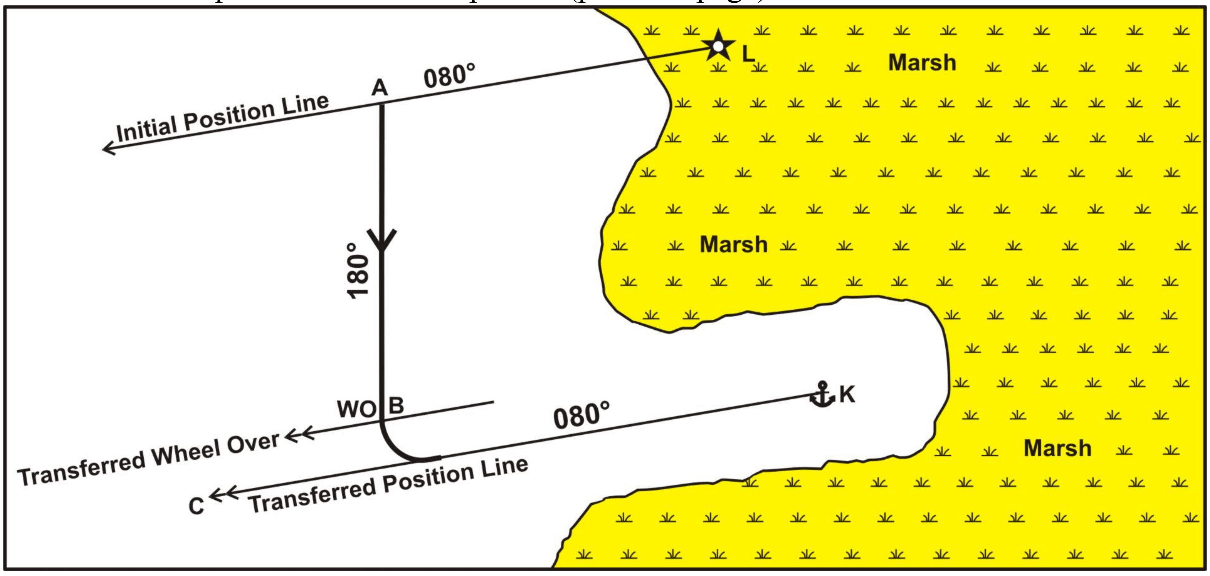

Fig 8-10. Plotting a Transferred Wheel-Over Line (No wind or Tidal Stream / Current)

0805. Fixing Techniques - Summary of Methods

A *Fix* is the position obtained by the intersection of two or more *Position Lines*. Unless the *Position Lines* are obtained at the same time, one or more of them must be transferred (see Paras 0804 and 0806). Methods of obtaining a *Fix* are:

- *DGPS / GPS* (or equivalent).
- *LORAN-C* and *eLORAN.*
- Cross Bearings - Theory and Practice.
- Bearing and Range.
- Transit and *HSA.*
- Bearing and *HSA*.
- Multiple *HSAs*.
- *Running Fix*.
- *Range Lattices* - Radar / Visual
- Astronomical Observations.
- Bearing / Range and Depth Contour Sounding.
- Bottom Contour Soundings.

a. **DGPS / GPS.** *DGPS / GPS* receivers are fitted in almost every vessel and *WECDIS / ECDIS* fits with automatic *DGPS / GPS* inputs are increasingly common (see Chapter 9 for *DGPS / GPS* theory). However, 'Traditional' *Fixing* methods are still important as a back-up method and to check automated systems.

- **DGPS / GPS Denial or Degradation.** *DGPS / GPS* signals are extremely weak; they may easily be jammed or seduced ('*GPS Denial*'). Graceful degradation of positional accuracy can occur which may be difficult to spot unless regular independent checks are made (see Note 8-4 opposite).

- **Check Fix Requirement and DR/EP Comparison.** It is essential to carry out independent regular check *Fixes* to check *DGPS / GPS* inputs, using another navaid; if this is not possible (ie outside radar range of land / no *LORAN* coverage / no 2nd *GPS* receiver / without astronomical observations), then comparison of the *DGPS / GPS* position against *DR/EP* is required.

- **Fix and Check Fix Intervals.** Intervals for *Fix*es and check-*Fixes* are at Paras 0716 / 0721 (with extracts repeated at Paras 1231f/g/h).

- **GPS Lattice.** Ships not fitted with *WECDIS / ECDIS* may need to construct a chart expansion with a *GPS Lattice* superimposed (*Latitude / Longitude Grid*) to allow rapid, accurate plotting of *GPS* positions and course alterations (eg for minefield swept channel transits etc). If the original chart is not based on *WGS 84*, a (*Geodetic*) *Datum Shift* must be applied.

- **Construction of GPS Lattice.** The quickest way of constructing a *GPS Lattice* chart expansion is by photocopying the navigational chart with a particular expansion factor set, taking care to include a suitable *Grid* and distance *Scale* on the area being photocopied. This *Grid* and distance *Scale* may then be subdivided into smaller divisions on the photocopy to assist rapid, accurate plotting of *Fixes*. **Care must be taken to ensure a uniform *Grid* and distance *Scale* expansion.** Alternatively, a *Mercator* chart expansion may be constructed using *UKHO* chart (Diagram) 5004, or from first principles by using *Meridional Parts* (see Para 0424).

Note 8-4. GPS Denial / Degradation. *'GPS Assisted' accidents, where over-reliance has been made on erroneous GPS inputs, are becoming increasingly common (see also Para 0916).*

- *In one notable case, a modern cruise liner with 1,509 persons on board, fitted with GPS and an automatic navigation system, grounded 17 n. miles from its presumed position. The GPS aerial lead had failed and GPS had automatically switched to DR mode for 34 hours until the grounding; aural and visual alarms were missed. With wind and a ½ kn Current taking effect, no independent position checks nor EP comparison with the GPS position were made in the entire 34 hour period.*

- *In another case, a large modern vessel equipped with 2 GPS receivers and ECDIS, grounded 15 n. miles from its presumed position. The GPS input to ECDIS had lost its signal some 30 hours earlier and had switched to DR mode; the alarm had been cancelled but no action taken. The 2nd GPS was operating correctly but was not used for comparison. No effective check Fixes were taken on the coastline, which had been in range for most of the preceding 30 hours prior to grounding.*

b. **LORAN-C / eLORAN.** *LORAN-C* is not as accurate as *GPS* but is valuable as a position source and for cross-checking *GPS* data. *eLORAN* is under development (2008) and has achieved accuracies similar to *GPS* during trials (see Para 0918).

c. **Cross Bearings in Theory.** When accurate bearings are obtained from two different known objects at the same time, the ship's position must be at the point of intersection of the two lines of bearing. To identify any possible errors, a third (check) bearing should always be taken at the same time and should pass through the point of intersection of the other two bearings. See Fig 8-11 (below).

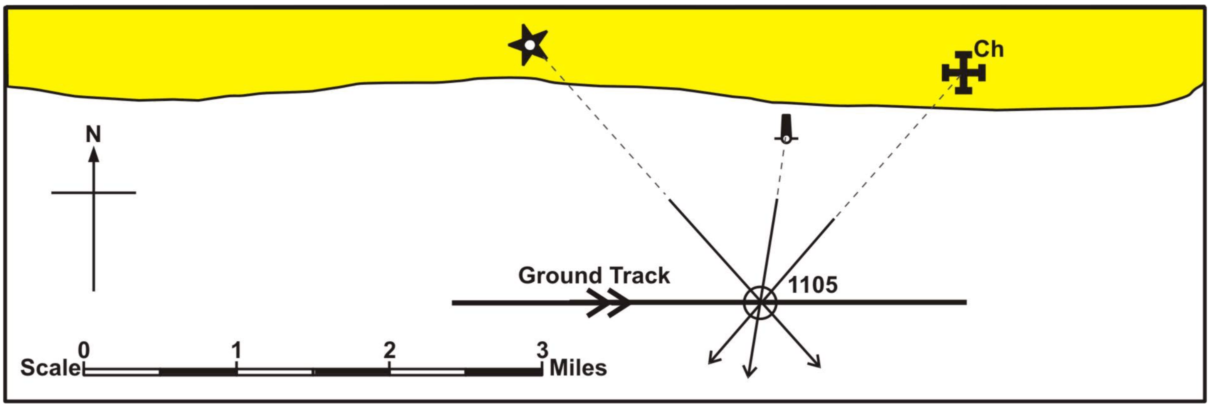

Fig 8-11. Fixing by Cross Bearings

d. **Cross Bearings In Practice - The Cocked Hat.** When 3 bearings are taken, the resulting *Position Lines* may NOT meet at a point but as a triangle known as a *Cocked Hat* (see Fig 8-12 overleaf). The cause of a *Cocked Hat* may be any of the following:

(1) Excessive time interval between observations.

(2) Error in identifying the object(s).

(3) Error in plotting the lines of bearing.

(4) Inaccuracy of observation due to compass / repeater / human error.

(5) Inaccuracy of the survey or the chart.

(6) Compass error unknown or incorrectly applied.

(0805d continued)

A detailed treatment of bearing errors is at Chapter 16 and Appendix 8, but in summary:

- **Errors (1), (2) and (3).** If the *Cocked Hat* is large, removal of errors (1), (2) and (3) may usually be attempted, or the *Fix* re-taken avoiding them. Error (1) may be removed by applying a '*Run-on*' / '*Run-back*'(see Example 8-7).
- **Error 4.** Error (4) should never be greater than ¼˚-½˚ with modern compass repeaters and may generally be disregarded.
- **Error 5.** Error (5) may be assessed by checking the chart *Source Diagram* (Paras 0624c/d) or *Category of Zone of Confidence* (*CATZOC*) (Para 0625).
- **Error 6.** Methods of eliminating error (6) are at Para 0811.

Example 8-7. A ship is on course 180 at 24 knots (see Fig 8-12 below). Bearings for a *Fix* were taken at 1059½, 1059¾ and 1100. On plotting the '1100 *Fix*', no allowance was initially made for the 15 second intervals (each equivalent to 1 cable's run) between each bearing, which caused a large *Cocked Hat*. On re-plotting with an appropriate '*Run-on*' for the earlier bearings, the 1100 *Position Lines* and *Transferred Position Lines* create a good *Fix*.

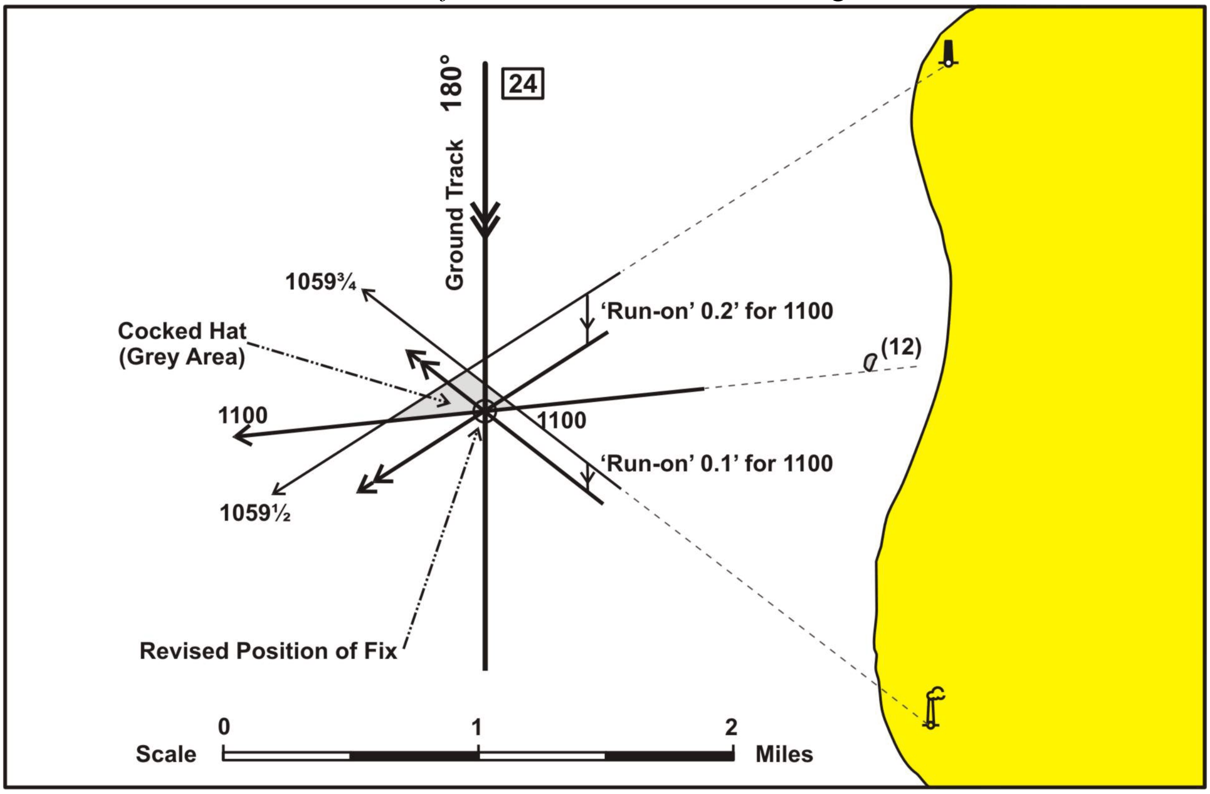

Fig 8-12. Example 8-7: The Cocked Hat and Resolution of 'Run' Errors

(0805) e. **Bearing and Range.** A visual bearing may be combined with a range to obtain two *Position Lines* and thus a *Fix*. To identify any possible errors, a third (check) bearing or range should always be taken at the same time (as for cross bearings). See Para 1232.

f. **Transit and HSA.** A transit may be combined with a range from an *HSA* between the nearer object of the transit and a third object. The position is the intersection of the transit and the arc of the circle obtained from the *HSA* (see Para 0803d / Fig 8-5). No compass is required for this method of *Fixing*.

(0805) g. **Bearing and HSA.** If passing a small island (or similar feature) where the compass bearings of the two edges give too small an angle of cut and a ranging aid (eg radar) is not available, a bearing of one edge may be combined with a range from the *HSA* between the edges. From the *Radian Rule* (see Para 0127), if the width of the island is measured on the chart, the range of the ship may be calculated (see Example 8-8).

Example 8-8. The *HSA* between the edges of an island 0.7 *n. miles* wide was 6˚ 00.0'. At the same time, the left-hand edge of the island bore 085˚. What was the range (R) of the island?

$$1° = \frac{2\pi}{360} \text{ radians} \qquad \text{thus:} \qquad 6° = \frac{6 \times 2\pi}{360} \text{ radians}$$

$$\text{Range (R)} = \frac{360 \times 0.7}{6 \times 2\pi} \text{ n.miles} = 6.68 \text{ n.miles}$$

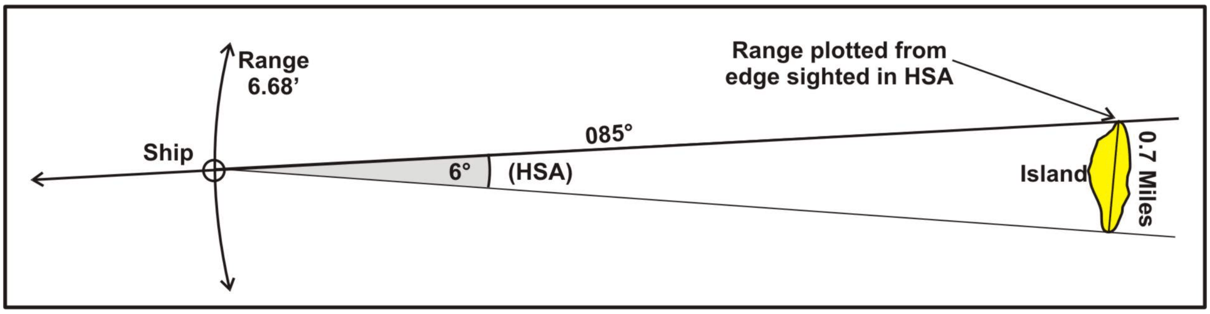

Fig 8-13. Example 8-8: Fixing by a Bearing and an HSA

h. **Multiple HSAs.** Multiple *HSAs* may be used to *Fix*. Details are at Para 0808.

i. **Running Fix.** A *Running Fix* involves transferring one *Position Line* to cross another (see Para 0804). Details of *Running Fix* techniques are at Para 0806.

j. **Range Lattices - Radar / Visual.** The intersection of range arcs (radar, *VSA* etc) provides a *Fix*. Radar / *VSA Range Lattices* of arcs cutting each other at angles of between 30˚ and 90˚ from two radar conspicuous objects may be constructed on the chart. The radar should be calibrated in *n. miles* and radar *Index Error* known to ensure accurate *Fixing*. Other radar *Fixing* techniques are at Para 1232.

k. **Astronomical Observations.** Calculation of an *Observed Position* (*Fix*) from astronomical observations is described in detail in BR 45 Volume 2.

m. **Bearing / Range and Depth Contour Sounding.** On transiting shoaling water, a *Position Line* may be obtained by an *Echo Sounder* reading when crossing (as nearly as possible at right angles) significant, substantially changing, clearly defined, reliable depth contours (see Fig 8-14 overleaf). The *Echo Sounder* reading may be combined with any other *Position Line* (eg visual bearing or radar / *VSA* range) taken simultaneously with the sounding. Before plotting the depth contour sounding *Position Line*, an allowance must be made for:

- The *HOT*.
- The draught of the *Echo Sounder* (usually ship's draught) if it is NOT set to read depths below the waterline.

(0805m continued)

In Fig 8-14 (below), at 1000, St Anthony's Head bore 342½˚ at the same time as depth 47m (below the keel) was recorded on the *Echo Sounder*. Draught was 6.1m, *HOT* 3.1m. The sounding *Position Line* was drawn along the 50 metre depth contour (47 + 6.1 - 3.1 = 50m) and where it intersected the bearing of 342½˚, was the position at 1000.

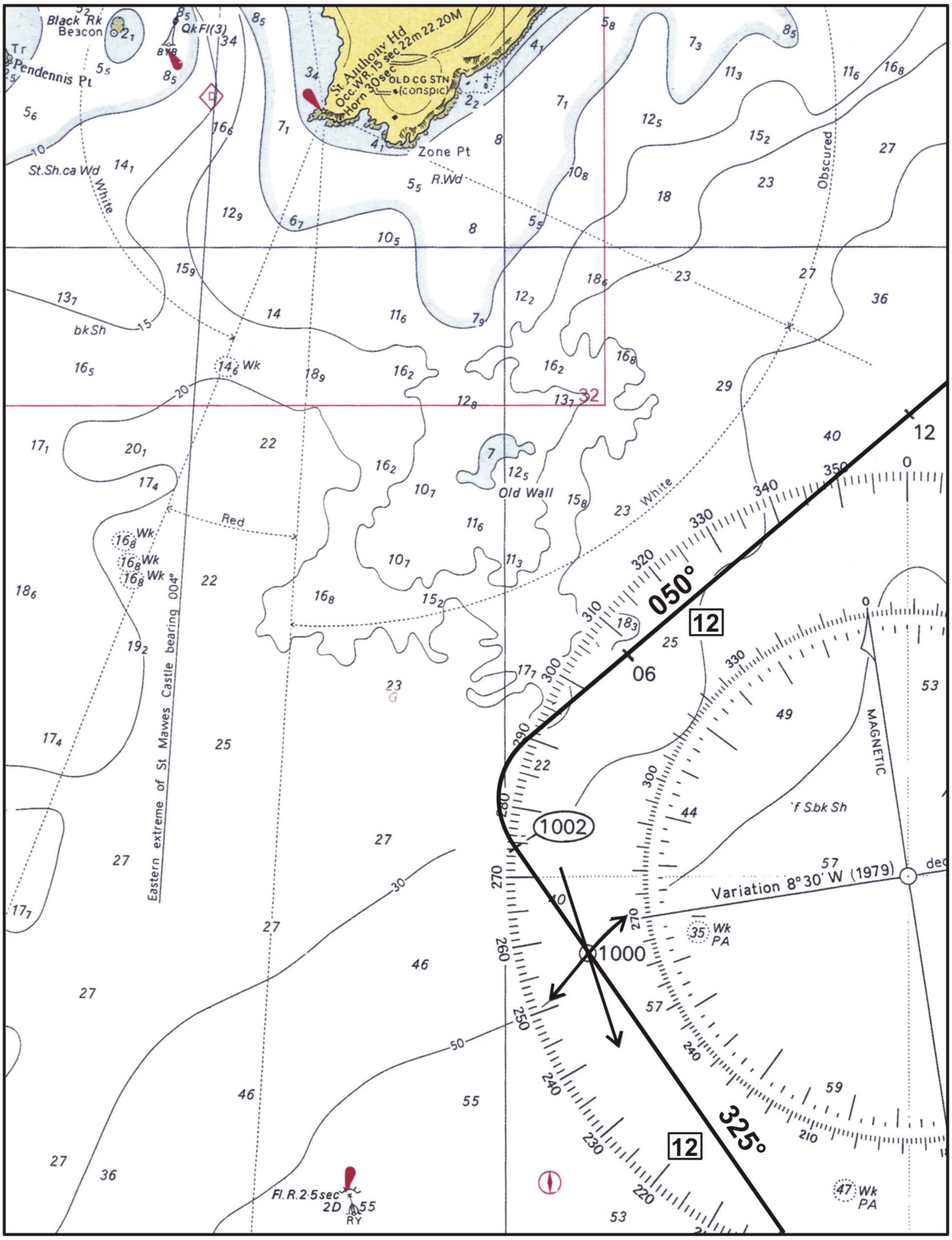

Fig 8-14. Fixing by Bearing and Sounding of Depth Contour

(0805) n. **Bottom Contour Soundings.** *Fixing* by the use of multiple bottom contour depth soundings alone is a specialized technique, normally only used by dived submarines. Details are at BR 45 Volume 9.

0806. Running Fixes

The concept of a *Transferred Position Line* was established at Para 0804. **The accuracy of the *Running Fix* is fundamentally dependent on the accurate assessment of the ship's course and speed (the *Water Track*) and the *Drift / Set* (effect of wind, *Tidal Stream / Current*) experienced over the period.** The biggest single source of error is likely to be an inaccurate assessment of the *Drift / Set*.

a. **'Standard' Running Fix.** A 'Standard' *Running Fix* is the point of intersection of a *Transferred Position Line* and a second *Position Line* on the same single object.

Example 8-9. 'Standard' Running Fix. A ship on course 090˚ speed 8 knots observes a lighthouse bearing 035˚ at 1600. At 1630 the same lighthouse bore 295˚. The *Tidal Stream* is estimated as 132˚ at 1.7 knots. Find the ship's position at 1630. See Fig 8-15 (below).

- Plot the *Transferred Position Line* for 1630 allowing for *Tidal Stream*, as in Examples 8-4 / 8-5 (Para 0804).
- Plot the 1630 *Position Line* (295˚). The 1630 *Running Fix* position is at the intersection of the *Transferred Position Line* and the 2nd *Position Line*.

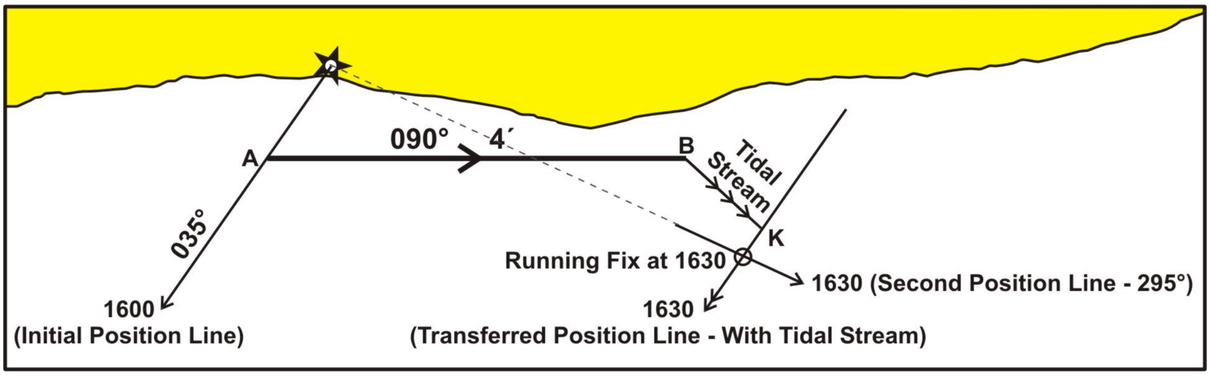

Fig 8-15. Example 8-9: A 'Standard' Running Fix

b. **'Special Case' Running Fix - Doubling the Angle on the Bow.** The range of an observed object is the distance run between observations when the 'angle on the bow' is doubled, provided there is no *Drift / Set*'(see Fig 8-16 below). Provided there is no *Drift / Set*, the distance run (AB) equals the range of the object (BL). If there is any *Drift / Set*, the observations should normally be plotted as a conventional *Running Fix* (see Example 8-9 at Para 0806a / Fig 8-15 above). The detailed theory of '*Doubling the Angle on the Bow*', both in still water and with a *Drift / Set*, is at Appendix 7.

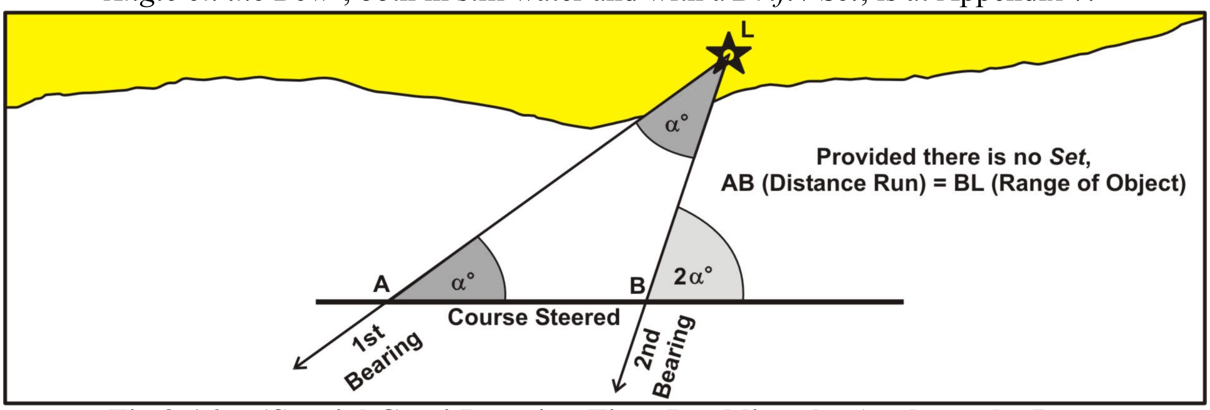

Fig 8-16. 'Special Case' Running Fix - Doubling the Angle on the Bow

(0806) c. **'Special Case' Running Fix - Establishing a Tidal Stream.** If starting from a known *Fix* position, the *Drift / Set* experienced may be estimated by *Running Fix*, provided it is certain that the strength and direction of the *Drift / Set* remain constant (see Example 8-10 / Fig 8-17 below).

Example 8-10. Running Fix from Fix Position with Unknown Tidal Stream. At 1700 a ship on course 180˚ was *Fixed* at *A*. At 1800 object *R* bore 090˚ and at 1836 it bore 053˚. Assuming it is constant, establish the *Tidal Stream* from 1700 to 1836. See Fig 8-17 (below).

- Plot the ship's *Water Track AE* (180˚) from the 1700 *Fix* at *A*.
- Plot the 1st (1800) *Position Line*, intersecting *AE* at *B*.
- Plot the *Transferred Position Line* at the *EP* position at 1836 using speed *AB*.
- Plot the 2nd *Position Line* (1836). The 1836 *Running Fix* position is at the intersection of the *Transferred Position Line* and the 2nd *Position Line*.
- The *Tidal Stream* from 1700-1836 is represented by *DP*, where *D* is the *DR* position at 1836, plotted from *A*.

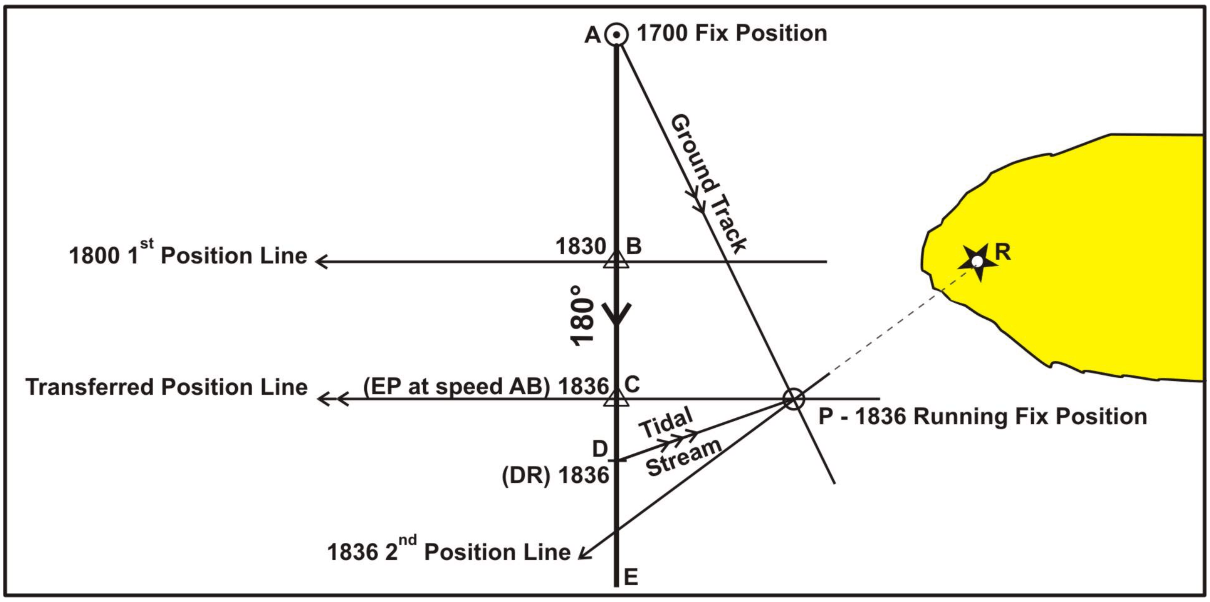

Fig 8-17. Example 8-10: 'Special Case' Running Fix - Establishing a Tidal Stream

(0806) d. **'Special Case' Running Fix - Estimating Distance Abeam.** When '*Doubling the Angle on the Bow*', if the initial angle on the bow is 45˚ the second angle will be 90˚, thus giving the distance abeam provided there is no *Drift / Set*; this is known as the '*Four-Point Bearing*' (ie one *Point* = 11¼˚ and thus four *Points* = 45˚ [see Para 0123c]). The *Four-Point Bearing* suffers the disadvantage that the distance abeam is not known until the object is abeam. However, provided the difference between the cotangents of the two measured angles is 1 [see formula (8.5) below], the distance run between the two angles equals the distance at which the object will pass abeam (see Fig 8-18 below); thus the distance abeam can be estimated before the object is abeam. A number of pairs of angles satisfy this criteria (see Table 8-1 below).

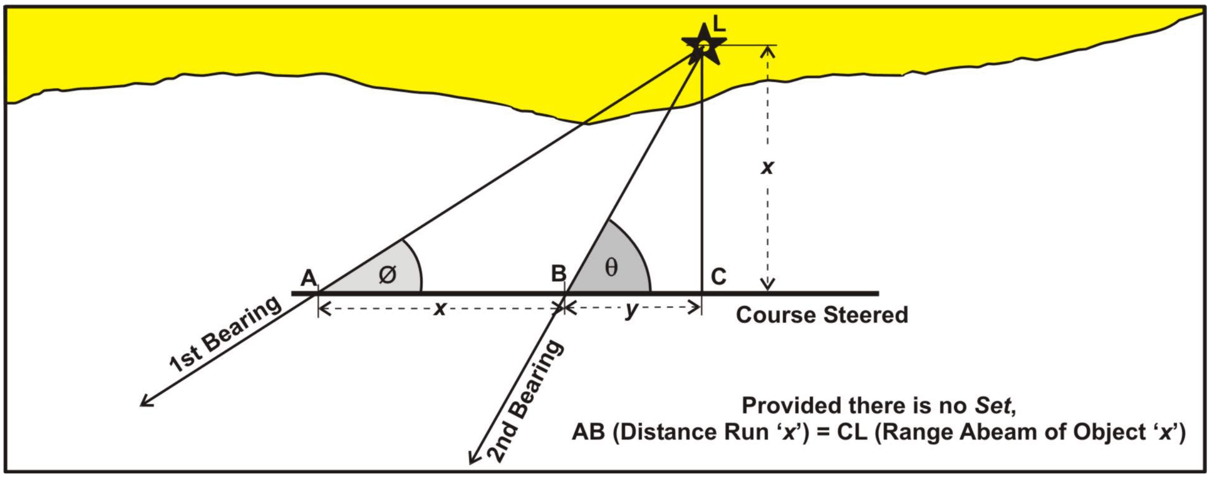

Fig 8-18. 'Special Case' Running Fix - Estimating Distance Abeam

In Fig 8-18: $cot\theta = \frac{y}{x}$ and $cot\phi = \frac{x+y}{x}$

$$cot\phi - cot\theta = \frac{x+y}{x} - \frac{y}{x}$$

$$cot\phi - cot\theta = 1$$ **. . . 8.5 (1987 Ed. . . 9.5)**

Table 8-1. Pairs of Angles for use in Estimating Passing Distance Abeam

ϕ	θ	**Remarks**
26½˚	45˚	Distance Run (AB) = Distance Abeam (CL) Distance Run (AB) = Distance to Abeam Position (BC)
30˚	53¾˚	
35˚	67˚	
40˚	79˚	

e. **Estimating Distance Abeam - Subtended Angle.** From the *Radian Rule*, 1˚ subtends approximately 1 *n.mile* at a distance of 60 *n.miles*. (or 2 cables at 12 *n.miles*). Thus the angle on the bow and range of an object may be used to calculate the distance it will pass abeam (*eg An object is 10˚ on the bow at 12 n.miles. Assuming no Tidal Stream or Leeway, it will pass at 10* x *2 cables = 2 n.miles*). Course may be altered to adjust the distance off, but allowance must be made for any *Tidal Stream* or *Leeway*.

0807. Selection and Use of Visual Fixing Marks

a. **Choosing Marks.** Marks selected for *Fixing* should be charted and identifiable. If possible, marks should be ahead of the ship and visible from the same repeater.

- **Bearing Separation.** Chosen marks should be well separated in bearing, normally with a minimum cut of 30˚. Fig 8-19 (below) illustrates the difference in position caused by an error of 5˚ with two cuts of 90˚ (*AB*) and 20˚ (*CD*). Ideally, when three objects are observed they should be 60˚ apart in bearing, and with two objects 90˚ apart.
- **Effect of Range.** The closer the object, the less will be the difference in position resulting from any error in the bearing.
- **Same Circle.** The marks and ship should NOT be on the circumference of the same circle, because any unknown compass error will NOT be revealed when the bearings are plotted (see details at Paras 0808f-h / Fig 8-27).
- **Channels.** When navigating in channels on older charts, marks should be selected from one side only to avoid any possible discrepancy arising from any different geographical *Datums* in use on opposite sides of the channel.

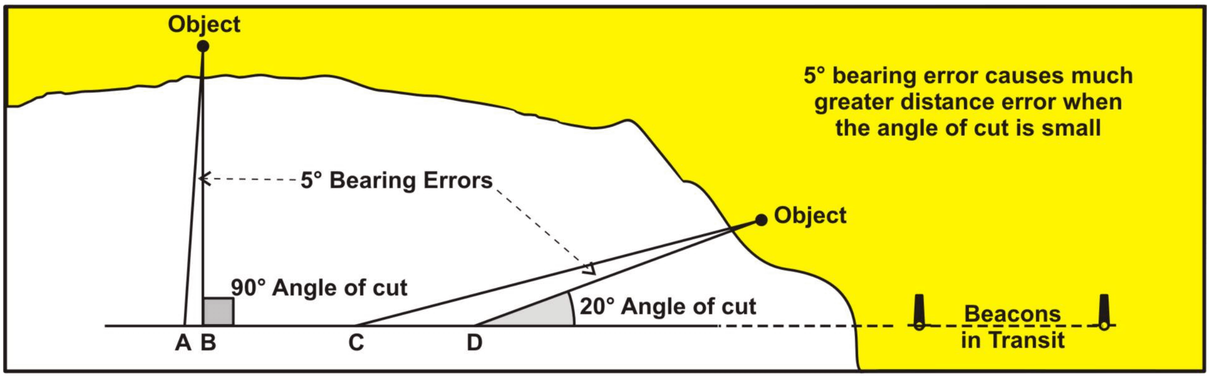

Fig 8-19. Effect of a 5˚ Error at 90˚ and 20˚ Angles of Cut

b. **Fixing Procedures.** The normal (single operator) procedure for *Fixing* is as follows, **and should NOT take more than 1 minute.** An assistant may be used to record and plot *Fixes*. *Fixing* procedure when anchoring are at Para 1415b.

- Select 3 or 4 marks from the chart and any additional marks to be 'shot-up'.
- Check the bearings of selected marks from the present *DR / EP* position.
- Positively identify the marks with binoculars. If in any doubt, identify them, either by transit bearing or from a previous *Fix* (see Para 0807d opposite).
- Write the names of selected marks in the *Navigational Record Book*.
- Observe the bearings as quickly as possible. Except when anchoring, the time of the *Fix* will be the last bearing, so take marks with slow bearing movement first (ie ahead / astern) and marks with rapid bearing last (ie beam). The time of *Fix* should ideally coincide with the *DR / EP* time on the chart.
- Note the bearings and time in the *Navigational Record Book* (see Fig 7-11). If using a magnetic compass, apply *Magnetic Deviation* and *Variation*.
- Plot the *Fix* and compare to the *DR / EP*. Lay off a further *DR / EP*.
- Verify time of *Wheel-Over* (if applicable). Plan marks for next *Fix*.
- Return to lookout.

(0807) c. **Shortcuts to Fixing.** Experienced OOWs may use shortcuts to *Fixing* procedures. Examples of these are as follows, but it should be emphasised that these are not recommended for beginners.

- The fastest changing bearing will be observed at the exact intended time for the *Fix*; the other bearings will be observed just before or just after this time.
- Only the last two digits of each bearing are memorised while taking the *Fix*; these are recorded in the *Navigational Record Book* <u>after</u> all bearings have been taken and the third (hundreds) figure of each added by inspection. This procedure allows 3 (or 4) bearings to be taken very rapidly, but does require mental agility and a practised memory for figures to achieve reliable results.

d. **Identifying Marks - Procedures.** The OOW / NO must plan ahead and positively identify any marks which are to be used for *Fixing* or as marks in *Pilotage*. **Chimneys, flagstaffs and radio masts are notorious for having been demolished and/or sometimes re-built in slightly different positions without the charting authority having been informed; such marks should be treated with particular caution.** Several methods are available to identify marks, as follows:

- **From the DR / EP.** Well-known, 'Conspicuous' marks (often marked 'Conspic' on the chart) may often be identified from the *DR / EP*, by looking down the expected bearing with binoculars. Such marks may be identified by this method alone, but less obvious marks will need further proving before being used.
- **By Transits.** A quick and most useful method of positively identifying an unknown mark is to select a bearing when it will be in transit with a known object and to observe the bearing of both at that moment. This can often be the 'opening bearing' when the object comes into transit with an edge of land. Compass error must be allowed for, as reliance is made of a single bearing. If doubt remains, another transit bearing will provide a 'cut', or it may be necessary to prove the mark by *Fix* (see Bullet below).
- **By Fix.** When taking a *Fix,* an additional bearing of an 'unknown' mark may be taken. Once the *Fix* is plotted, the extra bearing is plotted <u>from</u> the *Fix* <u>to</u> the mark to check whether it coincides with the mark's charted position. If doubt remains, this process may need to be repeated to provide a 'cut' with another check bearing, either using another *Fix* or a or a suitable transit (see Bullet above).

e. **Poor Fixes.** If the *Fix* does not fit, **it must NOT be 'fudged'**; <u>this is both dishonest and dangerous</u>. The *Fix* either needs to be re-worked to eliminate known errors (see Para 0805d / Fig 8-12), or to be retaken. **If there is any doubt about the ship's position in relation to immediate dangers, the ship should be turned into safe water or stopped using astern power (as appropriate) and the navigation situation resolved before proceeding further.** This may dent the OOW / NO's pride, but it may well prevent the ship itself from being severely 'dented' in a grounding.

(0807) f. **Identification of Uncharted Objects.** It sometimes happens that a shore object or buoy is visible from the ship but is not shown on the chart. Its position may be plotted on the chart by a series of bearings, using the *Fix* and / or transit methods at Para 0807d (previous page). Figs 8-20 and 8-21(below) illustrate these techniques in establishing the position of an uncharted buoy, by bearings and transits respectively. **In practice, these two methods may be mixed.** Once a fixed object has been positively identified and plotted on the chart, it may be used for *Fixing* the ship, subject to any accuracy limitations arising out of the processes used to chart the object.

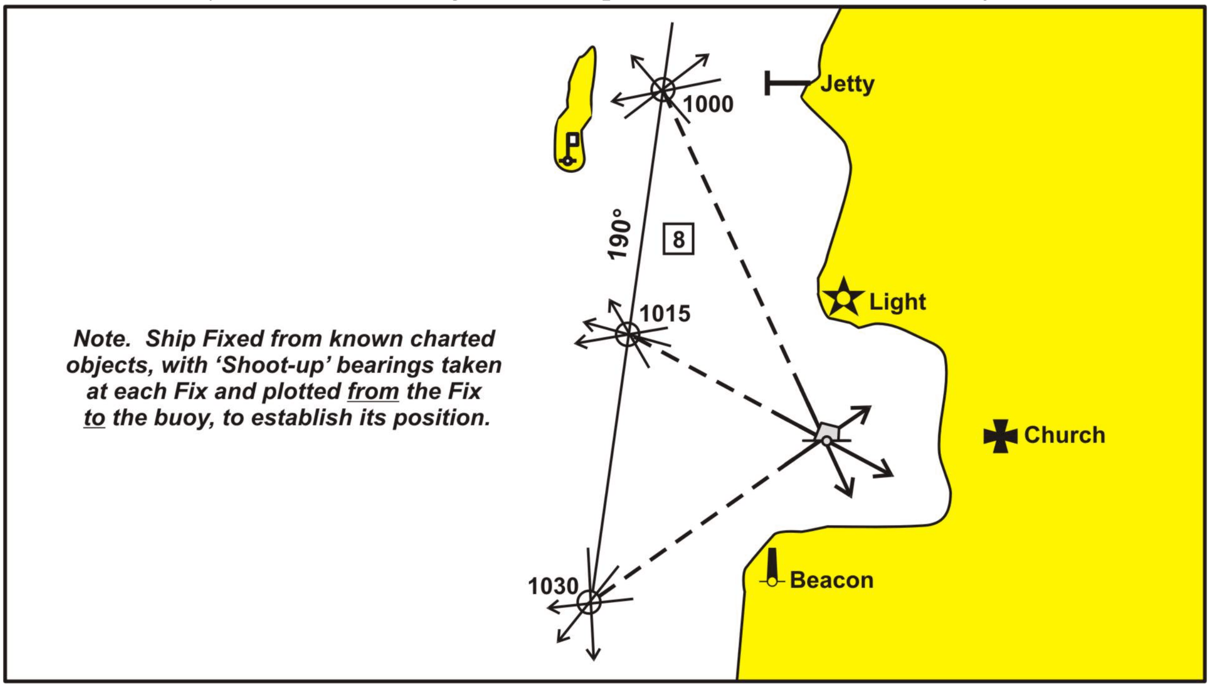

Fig 8-20. Establishing the Position of an Uncharted Buoy by Fixing Method

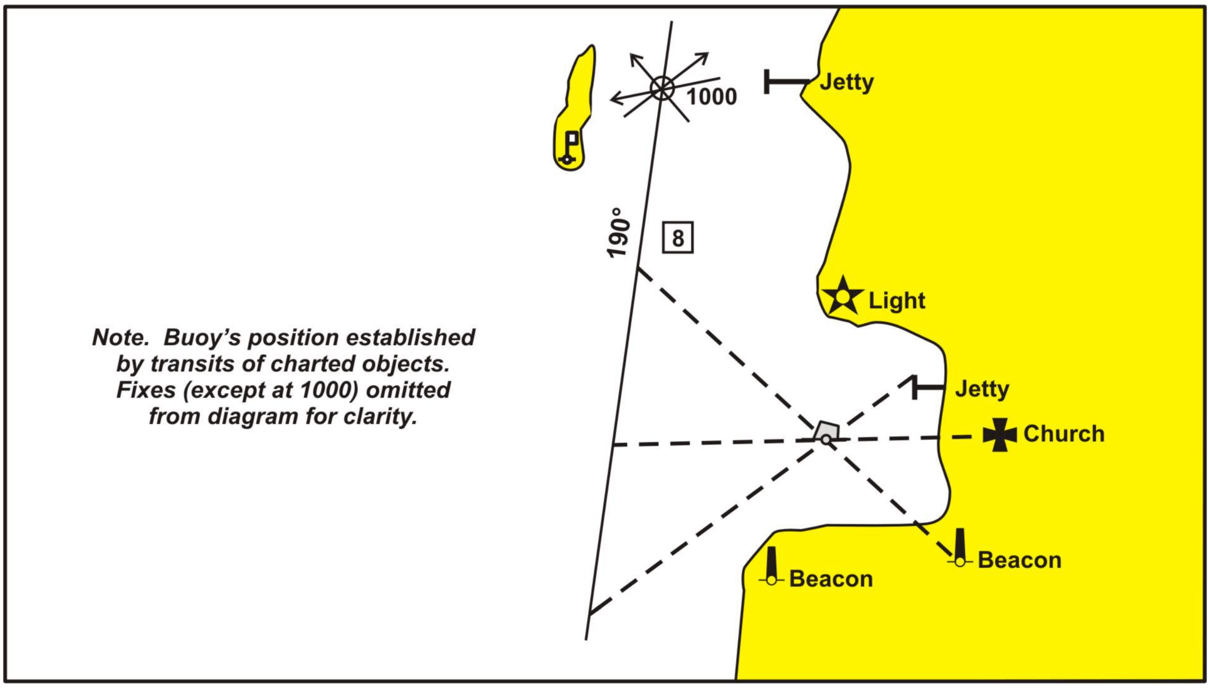

Fig 8-21. Establishing the Position of an Uncharted Buoy by Transit Method

0808. Horizontal Sextant Angle (HSA) Fixes

Despite the advent of *DGPS / GPS* and *WECDIS / ECDIS* equipments, in the event of *'GPS Denial'* or other circumstances (eg defects, hostilities, action damage etc), it may still be necessary to navigate to a higher degree of accuracy than that obtainable from normal *Fixing*, while at the same time plotting the *Fix* quickly and maintaining an accurate record of the ship's movements. Examples of the occasions when this may be necessary are mine countermeasure operations, *Pilotage* and the anchoring of ships in company. Two visual methods are available for this type of *Fixing*: *Horizontal Sextant Angles* (*HSAs*) and *Bearing Lattices*.

a. **Fixing by HSAs.** Observing and plotting *HSAs* subtended by three or more objects *Fixes* the ship's position by the intersection of two or more *Position Lines*. It is extremely useful for *Fixing* the ship accurately when moored or at anchor, and for *Fixing* the ship accurately at sea when two trained observers are available. The theory of *HSA Fixes* is at Appendix 6 and errors in *HSA Fixes* at Chapter 16 / Appendix 10.

- **Advantages.** The advantages of the *HSA Fix* are:
  - *HSA Fixes* are more accurate than a compass *Fix*, because a sextant can be read more accurately than a compass.
  - *HSA Fixes* are independent of compass errors.
  - *HSAs* can be taken from any part of the ship.
  - *HSA Fixes* are easy to take, particularly with trained observers.
- **Disadvantages.** The disadvantages of the *HSA Fix* are:
  - Plotting *HSA Fixes* can take longer than plotting compass bearings.
  - Three suitable objects are essential (see Para 0808e).
  - If the objects are incorrectly charted or incorrectly identified the *Fix* will be false and the error may not be apparent. For this reason *HSA Fixes* should not normally be used with a poorly surveyed chart.
  - The *HSA* marks and ship should NOT be on the circumference of the same circle, because the *Fix* can be plotted anywhere on the circle (see details at Para 0808f / Fig 8-27).

b. **HSA Fixes - Concept.** Fig 8-22 (below) shows a ship *'L'* and marks (*A, B, C, D*), all in roughly the same horizontal plane. The angles *ALB* and *BLC* are measured by *HSA*. *L* and *B* lie at the intersection of the circles *ALB* and *BLC*, which contain the observed angles *ALB* and *BLC* respectively. The ship must be at *L*, and not at *B*.

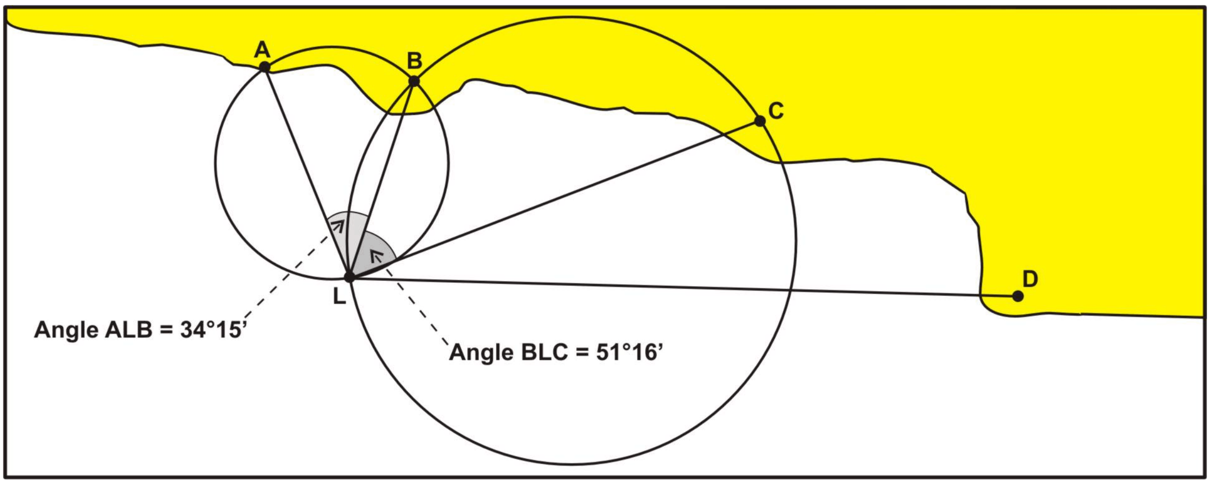

Fig 8-22. Fixing by Two HSAs

(0808) c. **HSA Fixes - Plotting.** To plot the *Fix* at Fig 8-22 (previous page), the angles *ALB* and *BLC* are set on a *Station Pointer* which is placed over the chart so that *LA*, *LB* and *LC* pass through the charted positions of *A*, *B*, and *C*. *L* is the ship's position.

- **Plotting Check Angles.** To guard against incorrect identification, a check angle may be taken between the centre and fourth objects (*B* & *D* in Fig 8-22). When a *Station Pointer* is used, the fourth angle may be plotted after the *Fix* has been obtained, by holding the *Station Pointer* steady and moving the appropriate leg to the check angle. This leg should then pass through *D*.

- **Recording Angles.** The *Fix* shown in Fig 8-22 (previous page) would be recorded in the *Navigation Record Book* as:
  HSA A 34˚15′ B 51˚46′ C
  HSA B 73˚49′ D

- **No Station Pointer.** If no *Station Pointer* is available, a *Douglas Protractor* (see Para 0811g) or tracing paper may be used instead. The observed angles are drawn from the centre of the *Douglas Protractor* or from any point on the sheet of tracing paper. The *Douglas Protractor* or tracing paper are placed on the chart and rotated until all the lines are in contact with the charted objects. The ship's position may then be pricked through onto the chart.

d. **Strength of an HSA Fix.** The mathematical strength or weakness of an *HSA Fix* is assessed by the angle of cut between the *Position Circles*; the closer the cut is to 90˚ the stronger the *Fix*. A major disadvantage of plotting by *Station Pointer*, *Douglas Protractor* or tracing paper (as at Para 0808c) is that none of these methods shows the *Position Circles*. The *Position Circles* may be drawn on the chart using a simple geometrical construction, allowing the angle of cut to be assessed (see Fig 8-23 below).

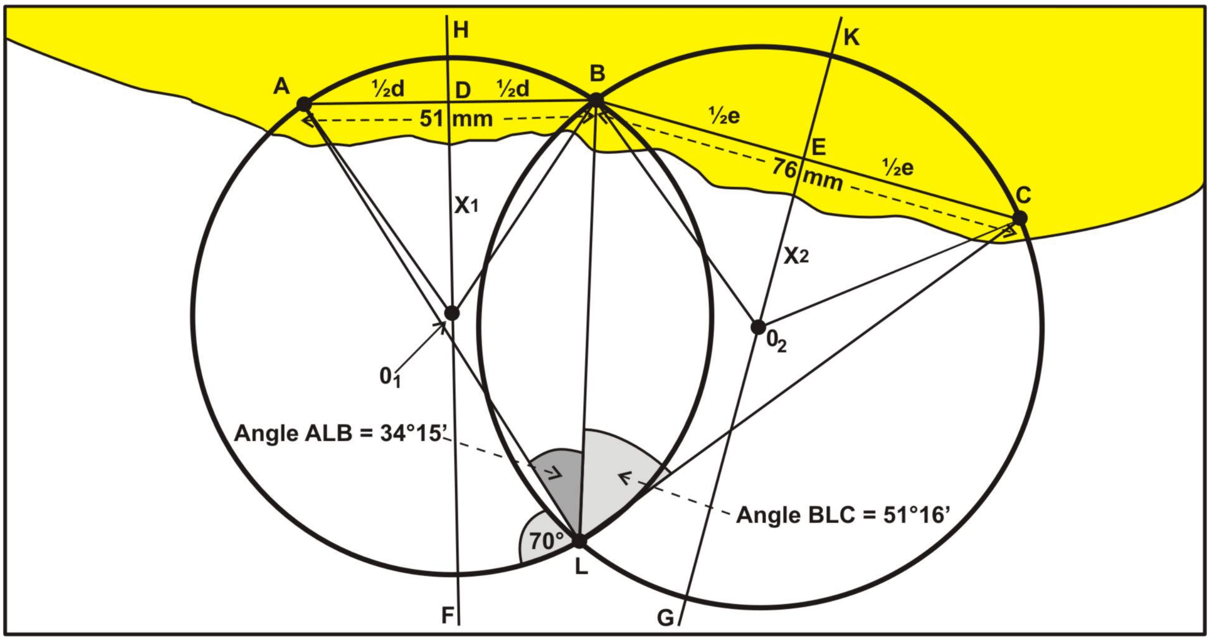

Fig 8-23. Plotting the Position Circles of an HSA Fix

- **Construction.** Perpendicular bisectors to *AB* & *BC* are *HDF* & *KEG*. The centres O_1 and O_2 of the two relevant *Position Circles* through *AB* and *BC* may be found from the calculations opposite, based on formula (A6.2):

(0808d) From example at Paras 0808c/d and Figs 8-22 / 8-23:

$$DO_1 = X_1 = \frac{\frac{1}{2}d}{\tan 34^\circ.25} = 37.45\ mm \qquad \text{. . . formula (A6.2)}$$

$$EO_2 = X_2 = \frac{\frac{1}{2}e}{\tan 51^\circ.2667} = 30.48\ mm \qquad \text{. . . formula (A6.2)}$$

- **Plotting the Position Circles.** The distance between *A* and *B* is represented by *d* the distance between *B* and *C* by *e*. The two *Position Circles*, radii AO_1 and BO_2, may now be plotted and the *Fix* at *L* established.
- **Angle of Cut.** The angle of cut between the two *Position Circles* is immediately apparent and the closer this is to 90˚, the stronger the *Fix*. Ideally, the angle of cut should never be less than 30˚. In Fig 8-23 (opposite), the angle of cut at *L* is about 70˚.
- **Third Position Circle.** If the two angles are small (eg 20˚ or 30˚) the weakness of the *Fix* may be overcome to some extent by plotting a third *Position Circle* through the two outer marks A and C (this would be circle *ALC* in Fig 8-23).

e. **Choosing Marks for an HSA Fix.** The sum of the two *HSAs* should be more than 50˚; better results will be obtained if neither *HSA* is less than 30˚. Marks for an *HSA Fix* should be chosen so that at least one of the following conditions applies:

(1) Marks are either all on or near the same straight line, and the centre mark is nearest the observer (see Fig 8-24 below).

(2) The centre mark is nearer the ship than the line joining the other two (see Fig 8-25 overleaf).

(3) The ship is inside the triangle formed by the marks or on the outer edge of the triangle formed by the marks (see Figs 8-26a/b overleaf).

(4) At least one of the *HSAs* should change rapidly as the ship alters position.

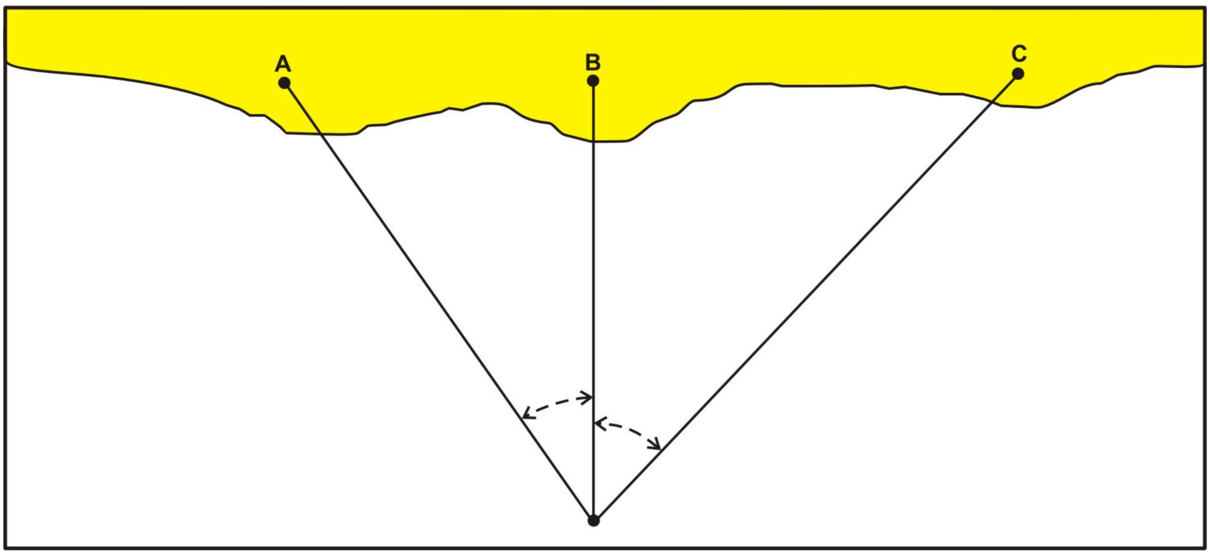

Fig 8-24. HSA Condition (1) - Marks are either all On or Near the Same Straight Line. Centre Mark is Nearest the Observer

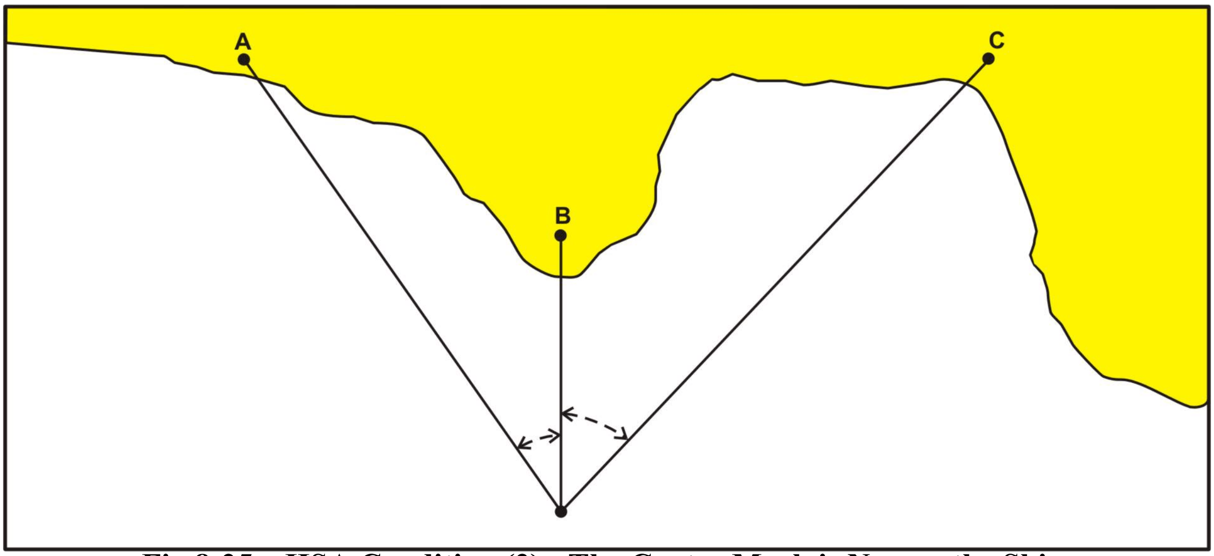

Fig 8-25. HSA Condition (2) - The Centre Mark is Nearer the Ship than the Line Joining the other Two

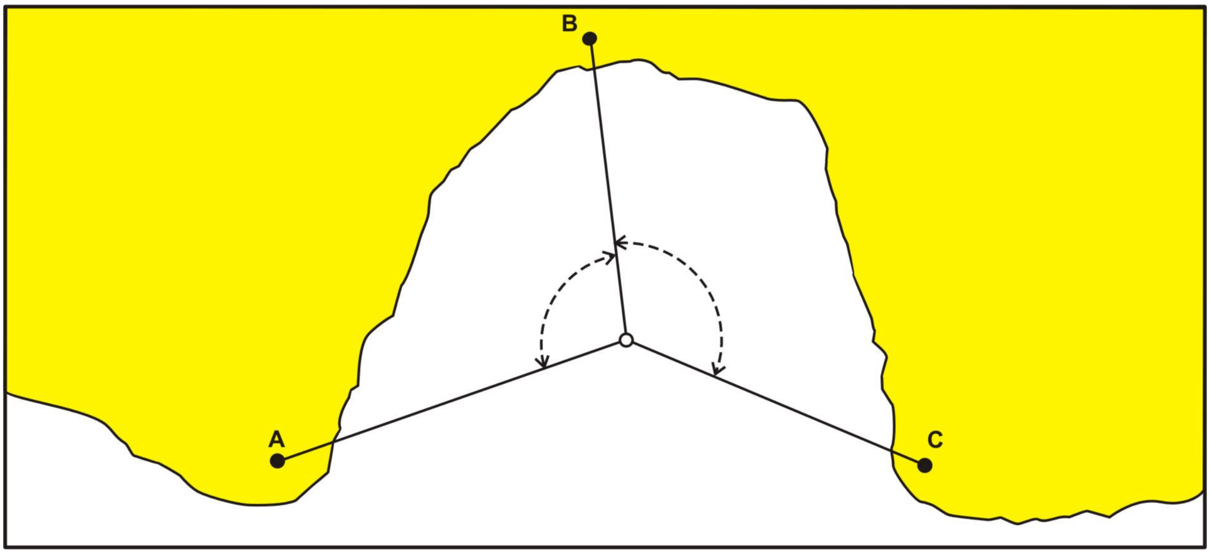

Fig 8-26a. HSA Condition (3a) - Ship is Inside the Triangle Formed by the Marks

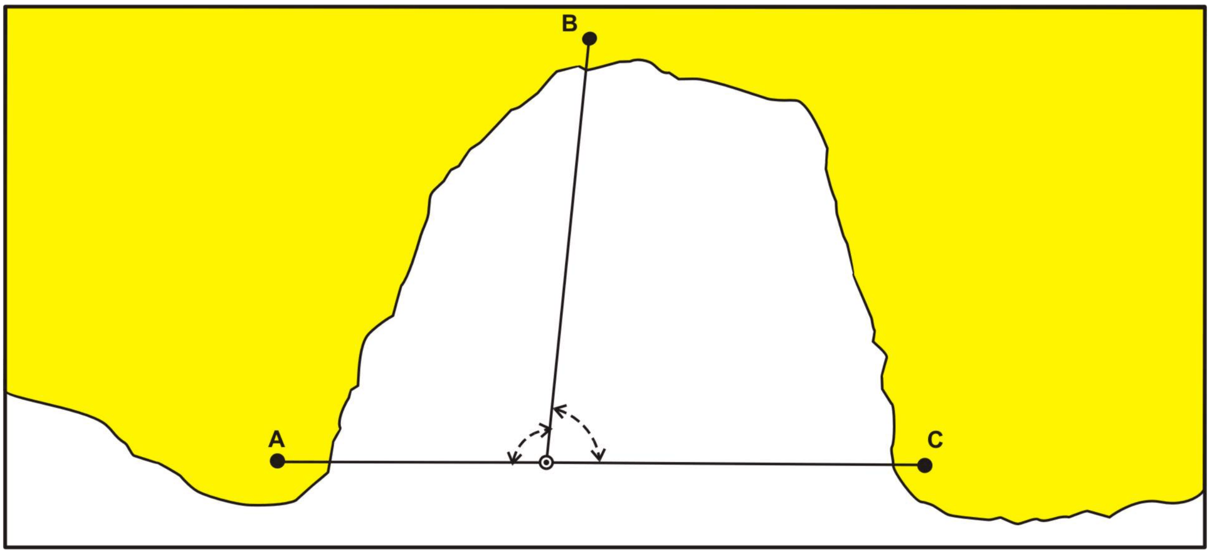

Fig 8-26b. HSA Condition (3b) - Ship is on Outer Edge of Triangle Formed by Marks

(0808) f. **When Not to Fix Using HSAs.** If the ship and the marks observed are all on the arc of the same circle (see Fig 8-27 below), the two *Position Circles* become one, and the two *HSAs* will cut at any point on the arc. **An *HSA Fix* is impossible in these circumstances.** In Fig 8-27, the beacon should have been chosen for the middle mark, and NOT the chimney.

g. **When Not to Fix Using Compass Bearings.** If the ship and the marks observed using compass bearings (instead of *HSAs*) are all on the arc of the same circle (as in Fig 8-27 below), and there is an unknown error in the compass**, this error will not be revealed by plotting.** The angles between the objects will be correct but the plotted bearings will always meet at some point on the arc of the circle. The plotted position will differ from the actual position dependent on the amount of the unknown error.

h. **Summary of When Not to Fix.** Never *Fix* the ship by *HSAs* or bearings when the ship and the objects observed are all on the arc of the same circle.

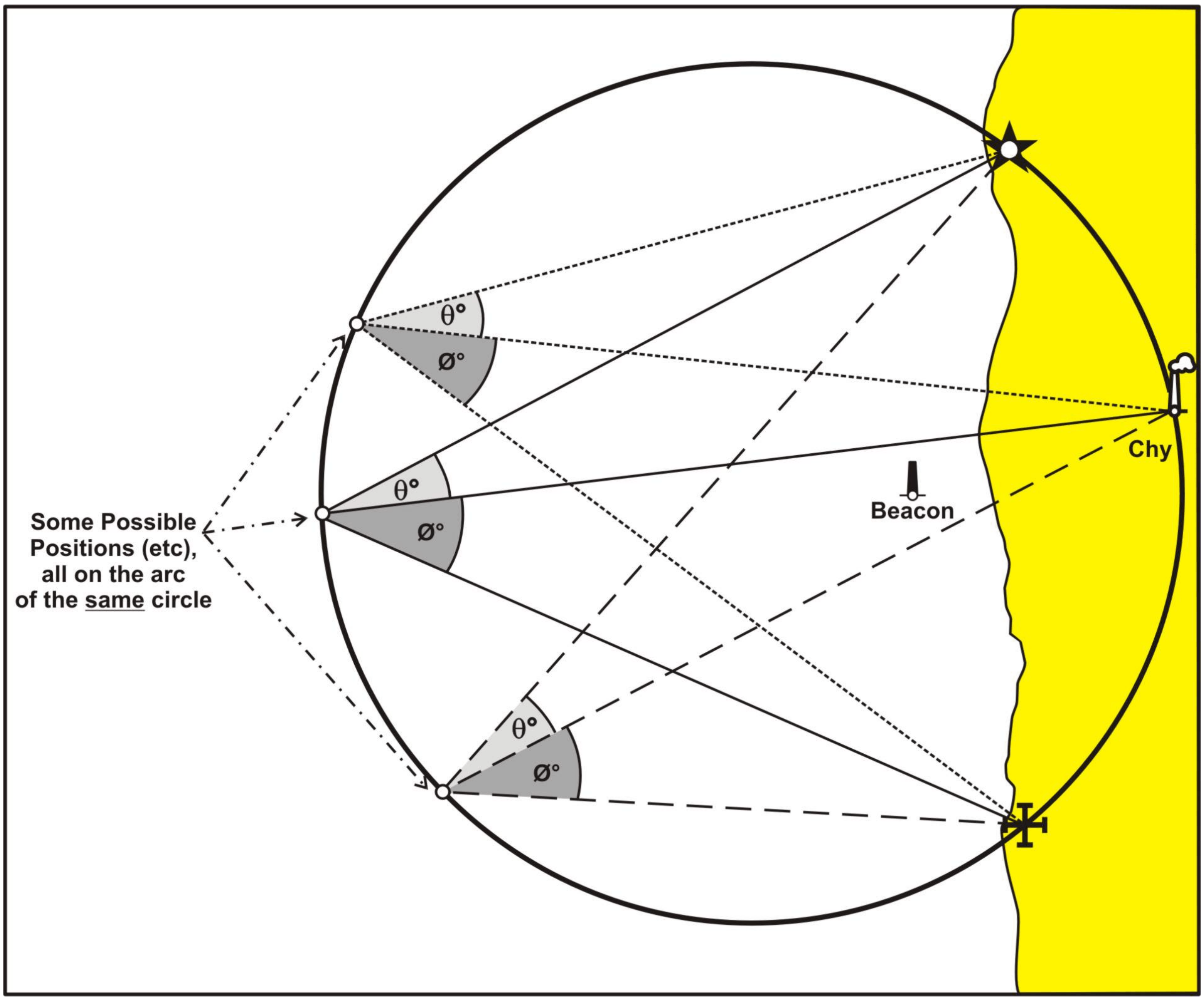

Fig 8-27. When Not to Fix Using HSAs or Bearings

i. **Rapid Plotting Without Instruments.** To enable *HSA Fixes* obtained to be plotted rapidly without instruments, an *HSA Lattice* of curves (see Fig 8-28 overleaf) may be constructed on the chart. Sets of curves are plotted from each of two pairs of marks and, if the angle between each pair is observed simultaneously, the *Fix* may be plotted immediately at the intersection of the two curves. The method for construction of the *HSA Lattice* is at Appendix 6.

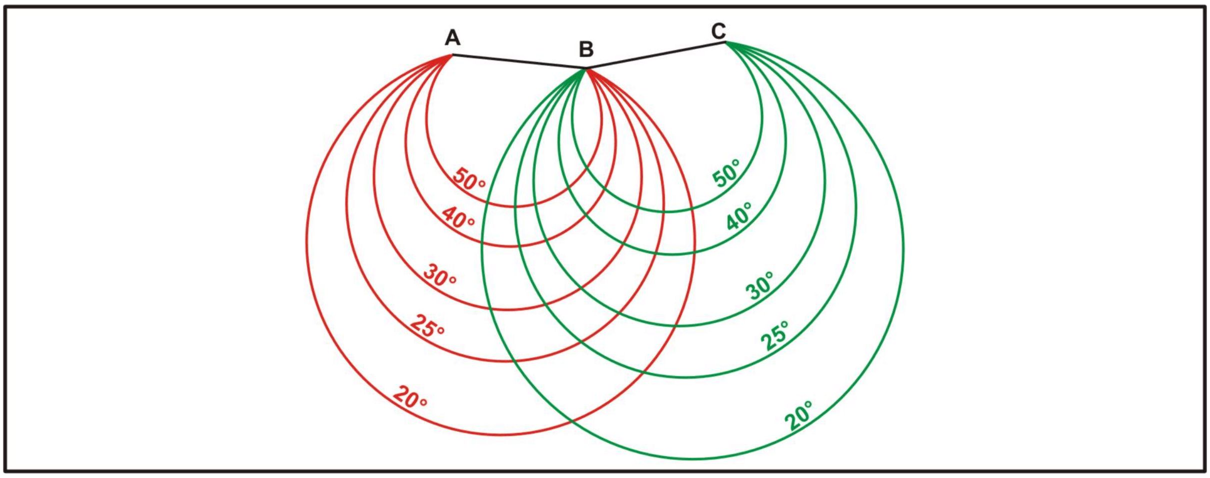

Fig 8-28. HSA Lattice of Curves

0809. Bearing Lattice Fixes

To produce a *Bearing Lattice* for use with rapid plotting of visual *Fixes*, a lattice of bearing lines from two visually conspicuous objects is drawn on the chart (see Fig 8-29 below).

a. **Marks.** Marks should be suitably placed to give an acute angle of cut, ideally between 60˚ to 90˚ (a minimum angle of cut of 30˚ is acceptable). In Fig 8-29 (below), the acute angle of cut varies between 55˚ and 90˚. Depending on the distance of the objects and the *Scale* of the chart, lines may be drawn 1˚ to 5˚ apart. In Fig 8-29, the lines are drawn at 5˚ intervals, while two 'boxes' are illustrated at 1˚ intervals.

b. **Procedure.** A team of 3 is required: a 'plotter' to coordinate, record and plot the bearings, and two 'bearing takers'. The 'plotter' controls the rate of *Fixing* which should be about one *Fix* every 30 to 60 seconds. Noise level is reduced if headphone communication is available.

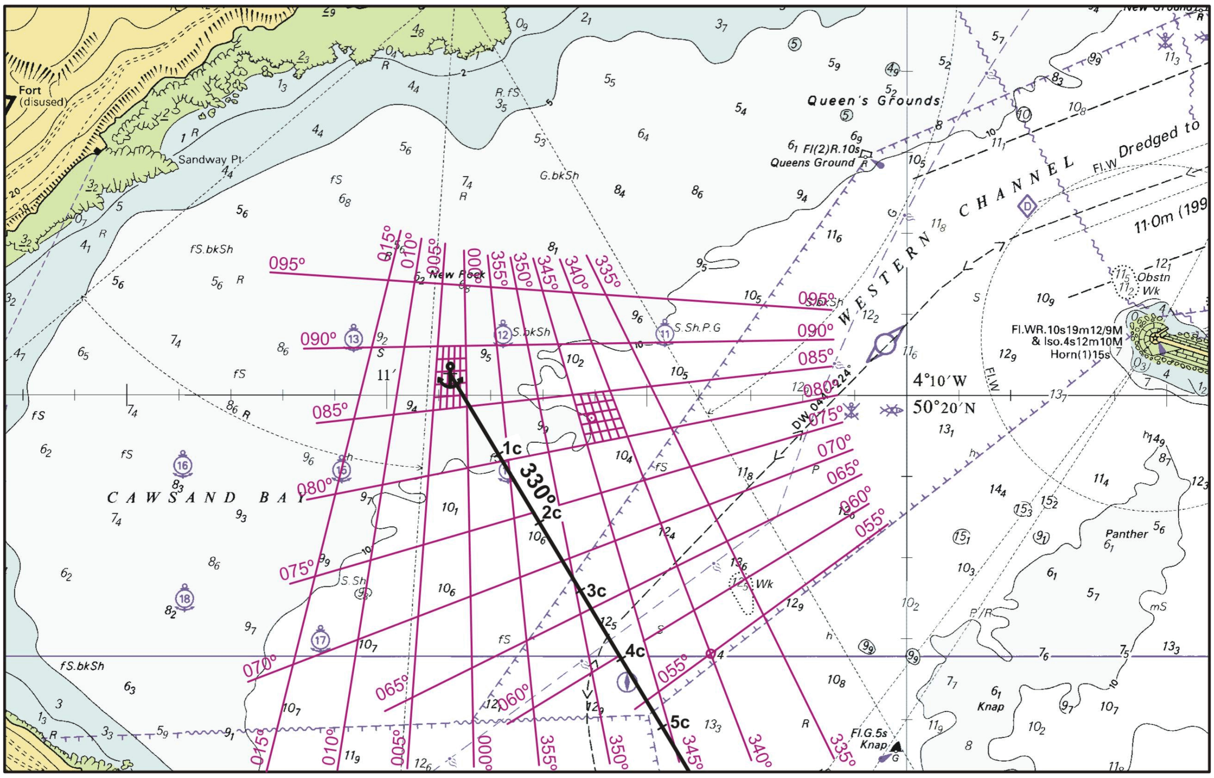

Fig 8-29. Bearing Lattice

0810. HSA Fixing Procedures

Successful rapid *Fixing* using *HSAs* and an *HSA Lattice* can only be achieved if there is a high standard of training and techniques. Special teams should be formed, as follows:

- Plotting/Course Control Officer
- Recorder
- Red Angle Taker
- Green Angle Taker

a. **Organisation.** When continuous *Fixing* is required, the angle takers should do a 20 minute trick. They should be positioned close together where they can best see the marks with good communications to the plotter; each *Fix* is recorded and plotted as the angle takers report their readings. *Fixes* should be plotted every 30-60 seconds. The *Index Error* of sextants must always be checked before use. The *Fixing* marks should be 'shot-up' beforehand to ensure they agree with the charted objects selected. Unless the *HSA Lattice* is drawn to a very large *Scale*, it will not usually be possible to plot to an accuracy of more than ± ½˚, so angles need only be reported to the nearest ½˚. But, if a *Station Pointer* is being used to plot the *Fixes* rather than an *HSA Lattice*, angles should be reported to the nearest 5' of arc. Either of the following *HSA Fixing* methods may be used, but it is easier to generate *DRs / EPs* with the 'Time Interval' method:

b. **Fixing by Time Intervals.** This method is <u>more difficult</u> to plot than Para 0810c.

- 5 - 10 seconds before the *Fix* is due, the recorder, using a stopwatch, gives the warning 'Standby' to the angle-takers.
- The angle-takers raise their sextants and align the marks. The lead angle-taker (whose angles are changing more quickly) and second are nominated.
- The second angle-taker reports 'On' when coincidence is obtained and then keeps the marks lined up.
- At the appropriate moment the recorder calls 'Time', and the lead angle-taker obtains coincidence and reports '*Fix*'. Both angle-takers record angles.
- The recorder notes the time of the *Fix* and the reported angles eg:
  'On the right, 60˚ 30′ (or Green 60˚ 30′)'.
  'On the left, 40˚ (or Red 40˚)'.
- The *Fix* is plotted.

c. **By Angle Intervals.** This method is <u>easier</u> to plot than Para 0810b.

- The lead angle is usually chosen to coincide with one of the *HSA* curves drawn on the chart.
- The lead angle-taker pre-sets the sextant.
- As the marks near coincidence, the lead angle-taker calls 'Standby' and the second angle-taker aligns the marks.
- The lead angle-taker calls '*Fix*' as the marks come into coincidence
- The time of the *Fix* is recorded.
- The angle-takers report their angles and the *Fix* is plotted.

0811. Adjustment of Fixes for Compass Errors

a. **Compass Error Checks.** If it is suspected that a *Cocked Hat* has been caused by a compass error, first check that any known corrections have been applied correctly.

- **Gyro Errors.** *Gyro* error corrections should be applied as in Para 0121 (ie if gyro is HIGH, <u>subtract</u> error from <u>observed</u> bearing; if LOW, <u>add</u> error).

- **Magnetic Variation and Deviation.** *Magnetic Variation* and *Deviation* should be applied as in Para 0124 (ie <u>subtract</u> westerly *Variation* and *Deviation* from the *Magnetic Compass* bearing and <u>add</u> it when easterly; the mnemonics *'CADET* 'and *'CDMVT*' are useful for the conversion rules).

b. **Methods of Checking Gyro Error and Magnetic Deviation.** The *Gyro* error and / or *Magnetic Deviation* may be checked by any of the following methods:

- **Transit.** The compass bearing of two charted objects may be observed when they are in line (see Para 0803c) and the true or magnetic bearing taken from the chart. The difference between the observed and charted bearings will be the *Gyro* error (see Paras 0121e/f) or the *Magnetic Deviation* (see Para 0125 and Example 1-7).

- **Bearing of a Distant Object.** The ship (ideally at anchor) may be *Fixed* by *HSA*s, or may be alongside a jetty in a known geographic location. An observed bearing of a distant object may then be compared with the bearing taken from the chart and the error calculated. From the *Radian Rule* (see Para 0127), ½˚ subtends approximately 100 yards at 6 *n.miles*; so, if the ship's position is known to an accuracy of 100 yards, the compass error can be established to an accuracy of ½˚ using an object 6 *n.miles* away.

- **True Bearing of a Heavenly Body.** The true bearing of a heavenly body may be most easily and accurately calculated with *NAVPAC* software (DP 330; see Para 0210a), using the procedures at BR 45 Volume 2 Chapter 3. With data from the Nautical Almanac (NP 314), other methods include the *Cosine* formula, Weir's Azimuth Diagrams (Charts 5000 and 5001), Marine Sight Reduction Tables (NP 401), Air Navigation Sight Reduction Tables (NP 303), Concise Nautical Almanac Reduction Tables (NP 314), and the ABC Azimuth Tables / Amplitudes and Corrections Table in Norie's Nautical Tables (NP 320). Calculation details are at BR 45 Volume 2 Chapter 5.

- **Reduction of the Cocked Hat.** If it seems certain that the *Cocked Hat* is due to compass error alone, and none of the above three methods is available to resolve it, then the *Cocked Hat* may be reduced and the error found, as at Paras 0811d-e (opposite).

- **Magnetic Deviation - Gyro 'Comparison Swing'.** See Paras 0122g / 0125c.

c. **Reciprocal Compass Checks (RAS Operations).** Reciprocal compass checks may be carried out with other ships prior to conducting *Replenishment at Sea* (*RAS*) or similar close stationing operations. The procedure for carrying out reciprocal compass checks is at BR 45 Volume 6, Chapter 5. This method is NOT relevant for terrestrial *Fixing*.

(0811) d. **Reduction of the Cocked Hat - Station Pointer.** Assume that on taking a *Fix*, a *Cocked Hat* results (see Fig 8-30 below) and that the same compass error exists (see Note 8-5) on all three bearings (*A, B* and *C*). To reduce the *Cocked Hat*, subtract the three observed bearings from each other to establish the two angles between them (in Fig 8-30, 345½˚-051˚= 65½˚ and 097˚-051˚= 46˚). These angles are then set on a *Station Pointer* and the ship's position found by rotating the instrument until each arm of the *Station Pointer* goes through the charted position of the relevant mark. In the absence of a *Station Pointer*, the angles can be drawn on a *Douglas Protractor* (see Para 0811g overleaf) or piece of tracing paper and the same procedure adopted. The charted bearing of the furthest object may then be compared with the observed bearing; the difference is the compass error (*Gyro* error 3˚ Low in Fig 8-30).

e. **Reduction of the Cocked Hat - Iteration.** An alternative (iterative) method for establishing a (fixed) compass error is to add (or subtract) 1˚ to each bearing and re-plot the *Fix*. Provided that the same compass error exists (ie not *Random Errors* - see Note 8-5) on all three bearings, the compass error may be established by an iterative process of gradually refining its size and direction. In Fig 8-30, an *Gyro* error of 3˚ Low is found. A similar process may be adopted with *WECDIS / ECDIS.*

Note 8-5. *Random Errors are explained at Paras 1611-1612; Cocked Hats resulting from different Random Errors on each bearing are covered at Appendix 10 Para 3e.*

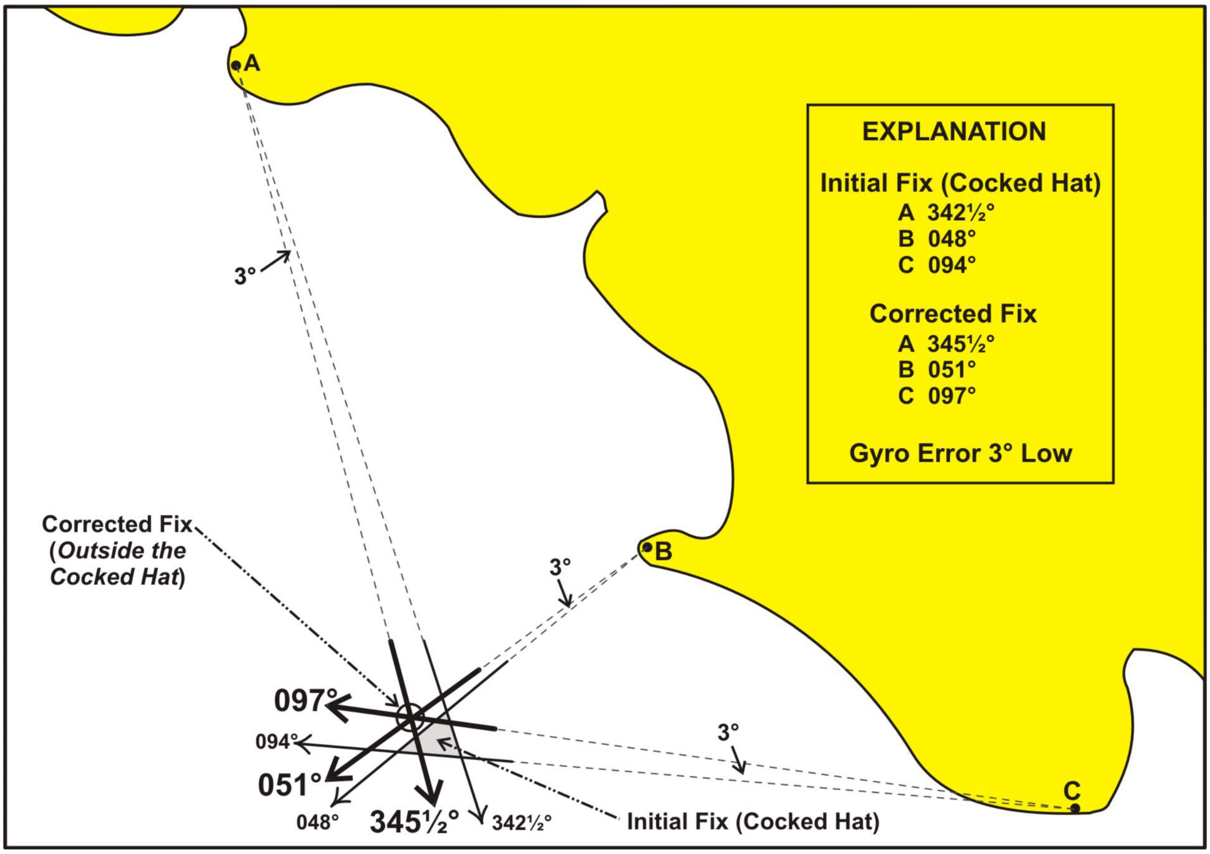

Fig 8-30. The Cocked Hat Resulting from Gyro Error (*Same Error on all Bearings*)

f. **Fix Position - Inside or Outside the Cocked Hat.** The ship's position may well be OUTSIDE the *Cocked Hat* (as demonstrated at Fig 8-30 above). There may be real danger in assuming that the ship's position will always be inside a *Cocked Hat*.

(0811) g. **The Douglas Protractor.** The *Douglas Protractor* is at Fig 8-31 (below), with bearings from Fig 8-30 (previous page) drawn on it.

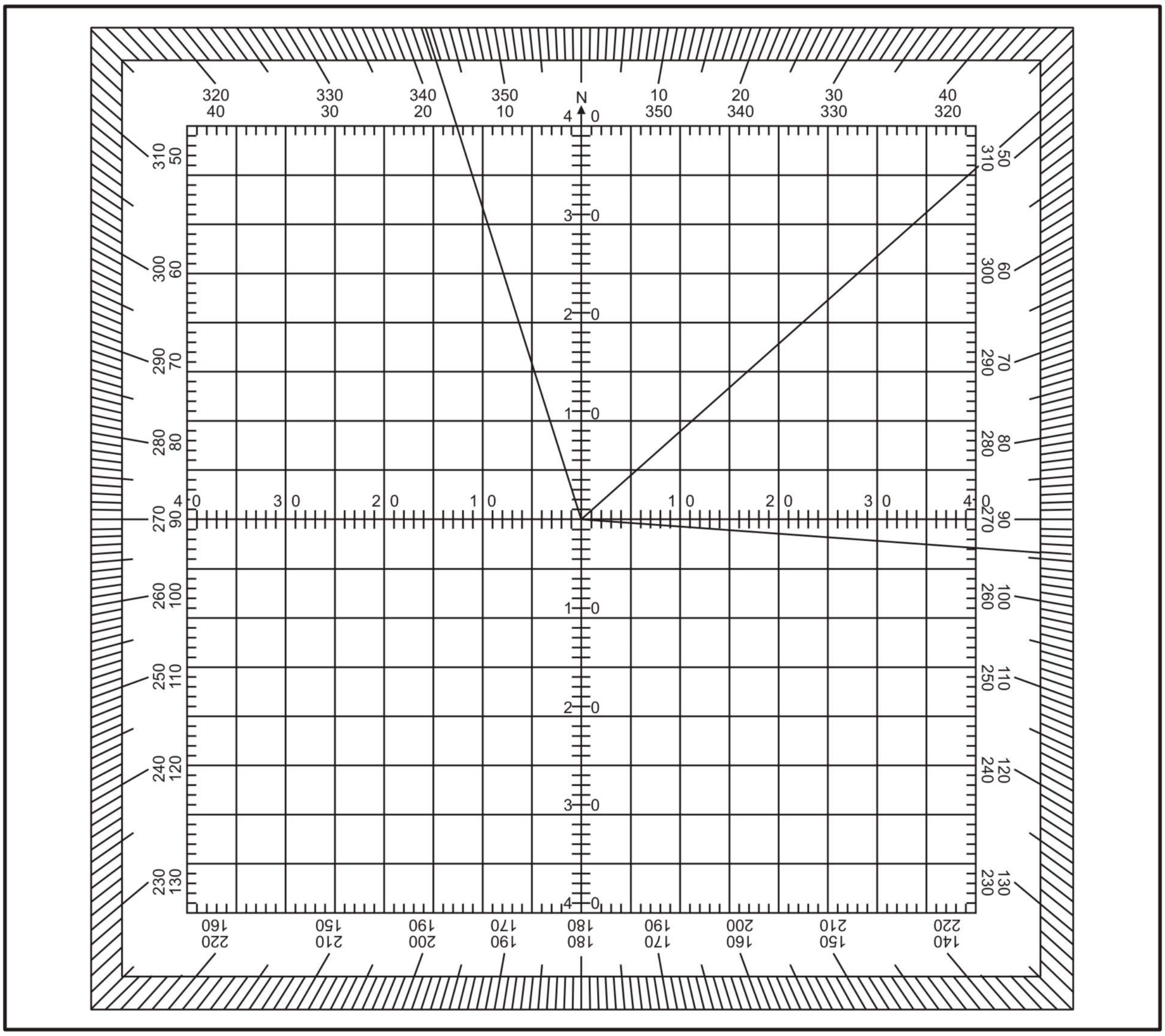

Fig 8-31. The Douglas Protractor (*with bearings from Fig 8-30 drawn on*)

0812. Action on Obtaining a Cocked Hat

Assessment of the size of a *Cocked Hat* is a subjective judgement. Noting the danger from automatically assuming (potentially in error) that the ship's position is inside a *Cocked Hat* (see Para 0811f / Fig 8-30 previous page), the following actions should be taken:

a. **Small Cocked Hat.** If the *Cocked Hat* is small and the ship is not endangered, the centre may usually be taken as the ship's position, without undue risk.

b. **Large Cocked Hat.** If the *Cocked Hat* is large and it seems clear that the same error is NOT applicable to each bearing, then either the *Fix* position should be taken as the corner of the *Cocked Hat* nearest to danger (taking the ship's subsequent movements into account) or, the *Fix* should be disregarded and reliance placed on the *DR / EP* until another *Fix* is obtained. In any case, if a large *Cocked Hat* has been obtained, a new *Fix* should be taken as soon as possible. **If the ship is in the vicinity of immediate danger, it should be turned into safe water or stopped using astern power (as appropriate) until the uncertainty about its position is resolved.**

CHAPTER 9

AIDS TO NAVIGATION

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CHAPTER 9

AIDS TO NAVIGATION

0901. Scope of Chapter

Chapter 9 provides a summary of 'Aids to Navigation', including definitions of *IMO* '*e-Nav*' and RN / RFA '*Digital Navigation*'. Further details are at BR 45 Volumes 3 and 8, and in *United Kingdom Hydrographic Office (UKHO*) publications, as specified in the appropriate paragraphs of this chapter. This chapter replaces Chapter 10 of the 1987 Edition of this book.

0902. Coverage of Digital Navigation, Electronic Charts, Radar and its Applications

Digital Navigation and '*Electronic Chart*' concepts are at Paras 0630-0633 and procedures for their use are at Paras 0720-0721. Use of radar is at Para 1232 *(Coastal Navigation*) and Paras 1316 / 1325 (*Blind Pilotage*). Radar and its applications is at Chapter 15.

0903-0909. Spare

SECTION 1 - SATELLITE NAVIGATION, LORAN AND E-NAV / DIGITAL NAV

0910. GPS (NAVSTAR Global Positioning System)

The following information on *Global Positioning System* (*GPS*) is a brief summary; for details see BR 45 Volume 3 and *Admiralty List of Radio Signals* (*ALRS*) Vol 2 (NP 282).

a. **GPS Configuration.** *NAVSTAR* in an acronym for 'NAVigation Satellite Timing And Ranging'. *NAVSTAR GPS* is a US Dept of Defence world-wide satellite navigation system providing very accurate continuous position, velocity and time. 24 operational satellites are uniformly distributed in 6 orbital planes, each inclined to the plane of the *Equator* at 55˚, at a height of 20,200 km (10,900 *n.miles*). This configuration ensures that at least 4 satellites with suitable elevations are 'visible' to a receiver anywhere on the Earth's surface at any time (except in *Polar* regions where coverage is reduced).

b. **Levels of Accuracy.** There are 3 levels of *GPS* accuracy (*PPS, SPS*, *DGPS*). For details of *Dilution Of Precision (DOP)* / *Estimated Position Error (EPE)* see Para 1806g.

- **PPS.** Encrypted *Precise Positioning Service* (*PPS*) for military users.
- **SPS.** *Standard Positioning Service* (*SPS*) for commercial users. Accuracies in the order of 8-13 metres (95%) may be expected with modern receivers.
- **DGPS.** *Differential GPS* (*DGPS*) for all users(see Para 0911). Accuracies in the order of 3-4 metres (95%) or better may be expected.

c. **Spheroid and Datum.** *GPS* is referenced to *WGS 84.* Most modern charts and all *WECDIS / ECDIS / ECS* equipments are referenced to *WGS 84*, but if *GPS* positions are used with older charts referenced to other horizontal *Geodetic Datum*s, a '*Datum Shift*' must be applied to *GPS* (*WGS 84*) positions before they are plotted. Details of the appropriate *Datum Shift* are normally found on the chart.

d. **Pseudo Ranging.** '*Pseudo Ranging*' is used to calculate the geographical position of the *GPS* receiver, using atomic clocks in the satellites and the propagation time of each satellite transmission. To obtain a two-dimensional *Fix*, the receiver must obtain a minimum of 3 *Pseudo Ranges* so that the processor can remove the effects of receiver clock offset error, satellite clock / *GPS* system time errors and atmospheric propagation delays. A minimum of 4 *Pseudo Ranges* will give a three-dimensional *Fix*.

0911. DGPS (Differential Global Positioning System)

The following information on *Differential GPS* (*DGPS*) is a brief summary; for details see BR 45 Volume 3 and *ALRS* Volume 2 (NP 282).

a. **DGPS Method.** Fixed *DGPS* base stations determine real-time errors in received *GPS Pseudo Ranges* for each 'visible' satellite. The corrections are then transmitted directly to *DGPS* receivers in vehicles in the vicinity (typically within about 1000 km / 540 *n.miles*), normally using selected maritime radiobeacon frequencies to do so. Corrections may also be transmitted by satellite links (eg INMARSAT), or in certain areas by *Eurofix* (see Para 0912) using *LORAN-C* stations. *DGPS* receivers in receipt of corrections apply them automatically to *GPS* data before establishing a *DGPS Fix*. *DGPS* will only work when the vehicle is within range of the *DGPS* station and when using the satellites being monitored by that station.

b. **DGPS Accuracy.** As stated at Para 0910b, *DGPS* accuracies in the order of 3-4 metres (95%) or better may be expected.

0912. Eurofix - Transmitting DGPS Corrections on LORAN-C

The following information on *Eurofix* is a brief summary; for details see BR 45 Volume 3, and *ALRS* Volume 2 (NP 282).

a. **Eurofix Method.** *Eurofix* uses selected *Northwest European LORAN-C System (NELS)* stations to transmit *DGPS* corrections (see Para 0911 above), without corruption of the *LORAN-C* signal, to satellite receivers in vehicles within about 1000 km / 540 *n.miles*. *Eurofix* signal channel allocations include future provision for *DGLONASS* , *eLORAN* and *DChayka* (Russian *LORAN* -C equivalent) corrections.

b. **Eurofix Availabilty.** *Eurofix* is currently (2008) under development and may NOT yet be available from all *NELS* stations. *ALRS* Volume 2 (NP 282) should be consulted for details of available transmitting stations.

0913. GLONASS

The following information on *GLONASS* (an acronym for 'GLObal NAvigation Satellite System') is a brief summary; for details see *ALRS* Volume 2 (NP 282).

a. **GLONASS Configuration..** *GLONASS* is operated by the Russian Federation Space Forces and is similar in nature to *GPS* (see Para 0910), except that *WGS 84* is not used. When fully operational (planned for 2009), it will provide a world-wide satellite navigation system giving very accurate continuous position, velocity and time. 24 operational satellites will be uniformly distributed in 3 orbital planes, each inclined to the plane of the *Equator* at 64.8°, at a height of 19,100 km (10,313 *n.miles*). This configuration improves *Polar* region coverage, as compared to *GPS*. In March 2008, 16 operational satellites were in orbit. *GLONASS* provides encrypted and non-encrypted services.

b. **Spheroid and Datum.** The *Spheroid* and *Datum* used by *GLONASS* is *PZ 90*, referenced to the *Soviet Geocentric Co-ordinate System 1990* (*SGS 90*). Differences between *PZ 90* and *WGS 84* are less than 15m with a mean average of about 5m.

0914. Galileo

The following information on *Galileo* is a brief summary; for details see *ALRS* Volume 2 (NP 282).

a. **Galileo Origin.** The European Union is developing an independent satellite navigation constellation named '*Galileo*'. The system is not yet (2008) fully operational. *ALRS* Volume 2 (NP 282) should be consulted for details of available satellites.

b. **Galileo Configuration.** *Galileo* is a world-wide satellite navigation system, similar to *GPS* (see Paras 0910), providing extremely accurate continuous position, velocity and time. 27 satellites (plus 3 active spares) will be distributed in 3 orbital planes, each inclined to the plane of the *Equator* at 56˚, at an altitude of 23,222 km (12,540 *n.miles*). This configuration will provide good cover up to 75˚ *Latitude*.

c. **Levels of Service / Accuracy.** Five levels of service / accuracy will be available:

- **Open Service (OS).** The *Open Service* (*OS*) will be a basic level dedicated to consumer applications and general interest navigation.
- **Commercial Service (CS).** The restricted access *Commercial Service* (*CS*) will be used for commercial and professional applications that require superior performance. Accuracies in the order of 1 metre (95%) may be expected.
- **Public Regulated Service (PRS).** The restricted access *Public Regulated Service* (*PRS*) will be used for governmental applications that require high continuity characteristics. Accuracies in the order of 1 metre (95%) may be expected.
- **Safety of Life Service (SoL).** The highly stringent *Safety of Life Service* (*SoL*) will be used where passenger safety is critical.
- **Search and Rescue Service (SAR).** The *Search and Rescue Service* (*SAR*) will be used for pinpointing the location of world-wide distress messages.

0915. GNSS (GPS, GLONASS, SBAS and Galileo)

The following information on *Global Navigation Satellite Systems* (*GNSS*) is a brief summary; for details see *ALRS* Volume 2 (NP 282). *GNSS* is the generic term for satellite navigation systems that provide autonomous global geo-spatial coverage.

a. **GPS and GLONASS Integrated Use.** *GLONASS* provides advantages for high-*Latitude* cover (see Para 0913) while *GPS* favours mid-*Latitudes* (see Para 0910), thus a receiver able to operate with both systems would offer faster acquisition times, optimum results at all *Latitudes* and an increased number of 'visible' satellites. Although there are technical complications, dual receiver technology continues to make significant advances and a wide range of products are commercially available.

b. **Satellite Based Augmentation System (SBAS).** The *Satellite Based Augmentation Systems* (*SBAS*), primarily developed for aeronautical purposes, are overlay systems offering users greater reliability, accuracy, availability, integrity and continuity. *SBAS* signals are broadcast from several geo-stationary communications satellites and provide corrections for *GPS* measurements which enhance the accuracy of the *GPS* receiver.

(0915) c. **SBAS Components.** The 3 major components of *SBAS* are:

- *EGNOS* (*European Geostationary Navigation Overlay Service*)
- *WAAS* (*Wide Area Augmentation System*) [American]
- *MSAS* (*MTSAT Satellite Augmentation System)* [Japanese]
  (*MTSAT = Multi-functional Transport Satellite*]

These systems are currently (2008) under development. See also Paras 1806g/h.

d. **GNSS: SBAS + GPS + GLONASS (+ Galileo).** The current and future proposed *GNSS-1* structure is illustrated at Fig 9-1 (below). In summary, *SBAS* is composed of *EGNOS, WAAS* and *MSAS*; *GNSS-1* is composed of *GPS, SBAS* and *GLONASS*. The integration of *Galileo* (see Para 0914) into a proposed *GNSS-2* structure is currently (2008) still under discussion.

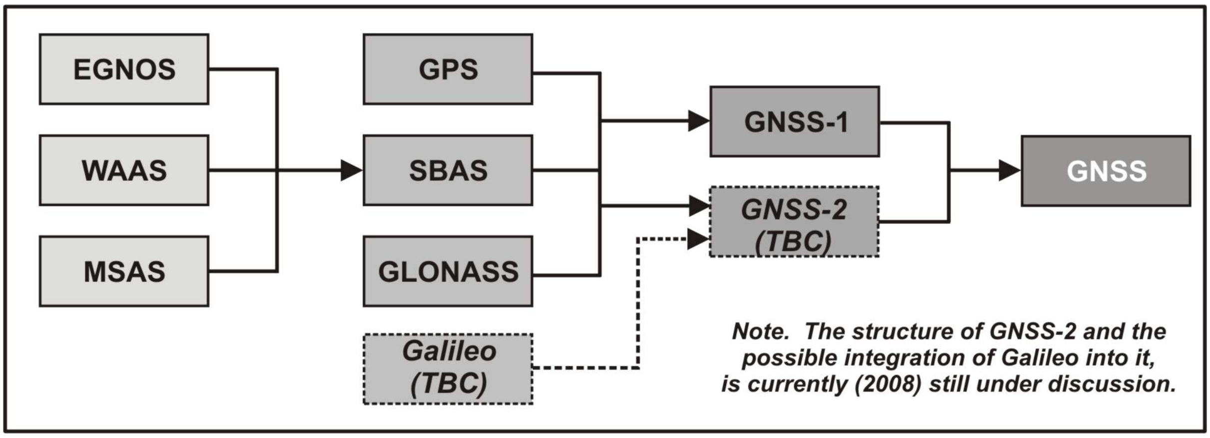


Fig 9-1. GNSS: SBAS + GPS + GLONASS (+ Galileo)

0916. Vulnerability of Satellite Navigation Systems to Jamming / Interference / Spoofing

a. **Threat.** Unless an anti-jamming antennae is fitted, *GPS / DGPS* and other satellite navigation systems are extremely vulnerable to degradation or denial ('*GPS Denial*'), due to deliberate 'Jamming', accidental interference (eg TV stations) or 'Spoofing' (*deliberate introduction of a signal to seduce and mislead the GPS / DGPS receiver with graceful positional degradation*). Even simple defects (eg a break in the aerial leads or connection) can result in the receiver switching to *DR / EP* automatically; if this change is not noticed and allowance is not made for any *Currents* or *Tidal Streams*, serious positional errors can result (see incidents at Para 0805a [Note 8-4]).

b. **Remedy.** To guard against such events, in all circumstances *GPS / DGPS* (and other satellite navigation systems) <u>should be checked frequently against other available *Fixing* sources and against the correct *DR / EP*</u>, in accordance with Para 0805a.

0917. GMDSS - Integral GPS Capability

Global Maritime Distress and Safety System (*GMDSS*) equipments usually have an integral *GPS* (*SPS*) receiver and *GPS* position information can be accessed from the *GMDSS* equipment control screens. However, on some *GMDSS* equipments this position may only be displayed to the nearest whole minute; if so, it should normally only be used to check the veracity of other installed *GPS* equipments.

0918. LORAN-C and 'Enhanced LORAN' (eLORAN)

The following information on *LORAN* is a brief summary; for details, see BR 45 Volume 3 and *ALRS* Volume 2 (NP 282).

a. **LORAN Acronym.** *LORAN* in an acronym for 'LOng RAnge Navigation'; the earliest version (in 1957) was *LORAN-A*, but this was later superseded by *LORAN-C*.

b. **LORAN-C Configuration.** *LORAN-C* is a 100 kHz electronic position *Fixing* system using pulse transmission, covering the North West Atlantic including the Gulf of Mexico, the North Pacific including the South China Sea, North West Europe and the Arabian Sea. A *LORAN-C* chain consists of a master station and two, three or four slave stations sited around it at distances of about 600-1000 *n.miles*. Ground wave cover extends to a range of 800-1200 *n.miles*. Sky wave cover extends to 1800 - 2400 *n.miles* at night; there is usually some sky wave cover by day.

c. **LORAN-C Levels of Accuracy.** *Fixing* accuracy is better than 0.25 n miles (95%) within the ground wave and may be as good as 0.1 n mile close to the baseline between the stations of a pattern. *Fixing* accuracy is reduced to about 10-20 n miles (95%) when using the sky wave.

d. **LORAN-C Spheroid / Datum.** *LORAN-C* uses *WGS 84* (see Para 0910c).

e. **'Enhanced LORAN' (eLORAN).** *Enhanced LORAN (eLORAN)* is currently (2008) under development; it includes differential *eLORAN* corrections in the signal, with positional accuracy <10 metres achieved in trials. Operational systems are unlikely immediately but *eLORAN* may soon be a possible alternative to *GPS* in some areas.

0919. IMO 'e-Navigation' and RN / RFA 'Digital Navigation' - Definitions

The concept of '*e-Navigation*' is an *IMO* -led initiative based on the harmonisation of marine navigation systems and supporting shore services driven by user needs. It is currently (2008) under development at *IMO* and does not have any legal force at present.

a. **Possible Confusion between '*e-Navigation*' and *'Digital Navigation'*.** The term '*e-Navigation*' should NOT be confused with the term '*Digital Navigation*' used in this book. Their respective definitions (and source data) are at Paras 0919b/c (below)

b. **Provisional Definition of 'e-Navigation'.** The term '*e-Navigation*' has been defined provisionally (2007) by *IMO* as:

> "*'E-Navigation'* is the harmonised collection, integration, exchange, presentation and analysis of maritime information onboard and ashore by electronic means to enhance berth to berth navigation and related services, for safety and security at sea and protection of the marine environment." *[Source: IMO NAV 53 WP.8 e-NAV3 Output-11]*.

c. **Digital Navigation.** The term '*Digital Navigation*' used in the RN / RFA does NOT change existing navigation planning / execution principles and is defined as:

> "*'Digital Navigation'* is navigation on *ENCs / RNCs* (or other electronic charting products) using *WECDIS / ECDIS* equipments, rather than on paper charts other than for back-up purposes." [*Repeated at Para 1911. Source: BR 45 Volume 8 Para 0102*].

SECTION 2 - COMPASSES, INERTIAL NAV SYSTEMS, ECHO SOUNDERS & LOGS

0920. Gyro Compass Principles

Details of *Gyro Compass* use and error correction are at Para 0121. The following information on *Gyro Compass* principles is a summary; for details see BR 45 Volumes 3 and 9.

a. **Gyroscope Properties.** A rotating body, such as a gyro wheel (*Gyroscope*), has two inherent properties, *Gyroscopic Inertia* and *Precession*, both of which are crucial to *Gyro Compasses*.

- **Gyroscopic Inertia.** A rotating body has angular momentum, and in accordance with Newton's First Law of Motion, its *Spin Axis* will point at a fixed position in space until an external force is applied. The larger its mass and higher its rotational speed, the larger will be the angular momentum and thus the more stable will be the *'Rigidity-in-Space*' of the body's *Spin Axis*.

- **Precession.** Any attempt to **Tilt** or **Turn** a rotating body (*Gyroscope*) by an external force results in a combination of the 'force vector' and the 'angular momentum vector' acting on the body. The effect is that the *Spin Axis* will always move in a plane that is 90˚ ahead of the applied force and in the direction of wheel rotation. This can be summarised as:
    - Attempt to **Tilt** a *Gyroscope* and it will **Turn.**
    - Attempt to **Turn** a *Gyroscope* and it will **Tilt**.

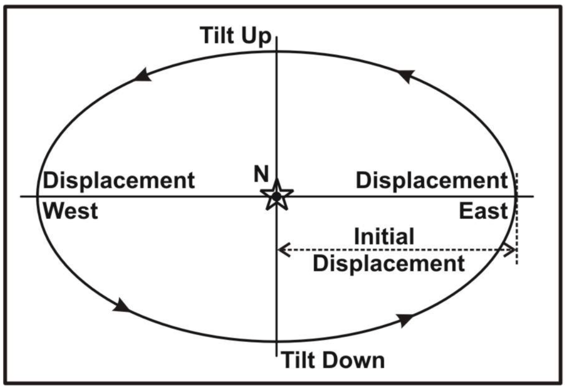
  This is easily demonstrated on a moving bicycle (the wheels of which are two simple *Gyroscope*s); to **Turn** left the rider leans (**Tilts**) inwards, and the front wheel **Turns** left automatically. The rear wheel would do the same but is constrained by the bicycle frame.

b. **North-Finding Gyro Compasses.** Unless started pointing directly at the *Celestial Pole*, the axis of a 'free' rotating *Gyroscope* on the Earth's surface will appear to rotate about the *Celestial Pole* with East-West 'Drift' and North-South 'Tilt', due to the rotation of the Earth beneath it. To make a free *Gyroscope* into a North-finding *Gyro Compass*, this 'Drift' rotation must be compensated, a 'Gravity Control' introduced for 'Tilt', and then 'Tilt' must 'damped' into a spiral which will settle on True North.

- **Earth's Rotation Drift Compensation.** The compensation necessary for the East-West'Drift' is proportional to both *Latitude* and Hemisphere. It is achieved by applying a suitable force to the side of the casing, originally as a weight called the '*Latitude Rider*', but now achieved electronically.

- **North-Seeking.** To make the *Gyro* 'North-seeking', a 'Gravity Control' was introduced for 'Tilt'; originally the casing was made top-heavy by a theoretical 'bail-weight', but is now achieved electronically. This resulted in the *Gyro* making an elliptical rotation about North and the horizontal (ie 'North-seeking' or 'undamped', but NOT 'North-finding'). See Fig 9-2a opposite.

- **North-Finding.** To make the *Gyro* North-finding, an unequal *Precession* must be introduced, resulting in the elliptical 'undamped' motion being changed into a spiral 'damped' motion, settling horizontally on True North. This was originally achieved by slightly offsetting the *Gyro*'s bottom cone bearing from the vertical plane, but is now achieved electronically. See Fig 9-2b opposite.



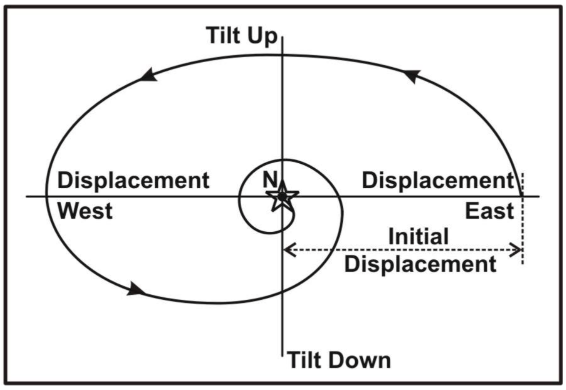


Fig 9-2a. North-Seeking Gyro Movement **Fig 9-2b. North-Finding Gyro Movement**

(The Pole Star is used above as an approximation for the North Celestial Pole)

(0920) c. **Gyro Compasses - Other Controls.** The Earth rotates to the East at about 900 kn at the *Equator* and 0 kn at the *Poles*; at intermediate *Latitudes* it is proportional to the Cosine of the *Latitude.* Provided the Earth's rotational speed is not too small (see Para 0920f below), a *Gyro Compass* will settle in the *Meridian*, normal to the Earth's rotation, but ship's movement (speed / *Heading*) and acceleration errors as a result of turns (*Ballistic Deflection* and *Ballistic Tilt*) will affect this. Additional controls are thus necessary, in the form of *Speed Log* and accelerometer inputs respectively.

d. **Gyro Compasses - User Inputs / Checks.** The correct *Latitude* setting (see Para 0920b 1st bullet) and *Speed Log* inputs are essential for accurate *Gyro Compass* results. Both can be applied manually by the user, but speed is normally input automatically; the *Latitude* on start-up normally has to be initialised by the user, thereafter it is automatic on most modern systems. Automatic *Gyro Compass* settings should be checked periodically. *Gyro Compass* errors should be checked frequently (see Para 0121).

e. **Modern Gyro Compasses.** *Gyro Compasses* have evolved over the last century from simple rotating wheels, to rotating spheres on almost frictionless gas bearings to the current generation of *Ring Laser Gyros* (*RLGs*) and *Fibre Optic Gyros* (*FOGs*) which have virtually no moving parts and rely on two coherent light beams passing around a closed path in opposite directions. Due to changes in interference patterns, *RLG / FOGs* can behave as a rotational sensor and thus may be used as a very accurate *Gyro Compass.* The most sophisticated *Gyro Compasses* have more than one gyro with *Spin Axes* 90˚ apart, and can provide not only a very accurate heading output but also the local vertical; they may thus be used for weapon and other stabilisation purposes.

f. **Very High (Polar) Latitudes - Directional Gyro Mode.** At very high (*Polar*) *Latitudes*, typically above about 80˚-84˚, a North-finding *Gyro Compass* becomes ineffective, partly due the greatly reduced rotational speed of the Earth at such *Latitudes* (see Para 0920c) and partly due to the convergence of *Meridians* near the *Poles*. The requirement for a North-finding *Gyro Compass* is thus replaced by one which will indicate and maintain a *Great Circle* course. This is achieved by making the *Gyro Compass* behave as a simple *Gyroscope*, pointing in a fixed direction in space (see Para 0920a 1st bullet); this is known as '*Directional Gyro*' (*DG*) mode and most modern *Gyro Compasses* have this facility. Once in *DG* mode, '*Grid Navigation*' is conducted on a *Polar Stereographic* chart (see details at BR 45 Volume 9).

0921. Inertial Navigation Systems (INS) Principles

The following information on *Inertial Navigation Systems* (*INS*) principles is a brief summary; for details see BR 45 Volume 3.

a. **Principles of Inertial Navigation Systems (INS).** An *INS* comprises the equivalent of 3 *Gyro Compasses* with their *Spin Axes* at 90˚ from each other; in practice, one of these *Gyro Compasses* may be replaced or augmented by accelerometers. It works by measuring accelerations produced by changes in course and speed; its computer integrates these accelerations into the distance and direction that the ship has moved from the starting position and converts these to *Latitude* and *Longitude* to give the present position. As an *INS* cannot measure absolute position, it is necessary to initialise it with a start position, and usual to monitor its performance against other navaids.

b. **Modern INS - Ring Laser Gyros (RLGs).** The limitations of gyros and accelerometers in early systems necessitated mounting them on stabilised gimbal platforms. *RLGs* do NOT require to be fitted on a stabilised gimbal platform, and so can be 'strapped down' (bolted) to the ship or aircraft structure.

c. **Use of INS.** *INS* are widely fitted in commercial and military aircraft, submarines, other warships and in many survey vessels. As well as position, *INS* also provides a very accurate heading output and the local vertical, and thus may be used for weapon and other stabilisation purposes.

0922. Magnetic Compasses

a. **Application.** A summary of the principles of magnetism and its application for the uses, errors and error corrections of *Magnetic Compasses* is at Paras 0122-0125.

b. **Magnetism and Magnetic Compass Principles.** A full explanation of the principles of magnetism, *Degaussing, Magnetic Compass* adjustment and correction, and the theory / limitations of *Fluxgate (Magnetic) Compasses* is at BR 45 Volume 3.

0923. Echo Sounder Principles

The following information on *Echo Sounder*s is a brief summary; for details see BR 45 Volume 3. **Standard depth / sounding reporting procedures are at Para 0924.** Guidance on specific uses of *Echo Sounder*s in *Coastal Navigation, Pilotage, Blind Pilotage* and *Minor Surveys* are at Chapters 12, 13 and 18 respectively.

a. **Importance of the Echo Sounder.** At sea, the nearest point of land is almost always the sea-bed. For example, consider a position in the busy English Channel, some 20 *n.miles* south of Plymouth Sound; the charted depths there are about 70m, which is generally regarded as relatively 'deep' water. However, when seen in a horizontal context, 70m is about one-third of a cable, and few ships would willingly venture within that (horizontal) distance of the *Limiting Danger Line* (*LDL*) unless berthing. Even in deepest ocean, depths rarely exceed 3 *n.miles* and are frequently very much less (minimum depths across the Mid-Atlantic Ridge are only a few hundred metres). **Seen in that context, the nearest point of land is almost always the sea-bed and thus the *Echo Sounder* is a vitally important piece of equipment for navigational safety.**

(0923) b. **Outline Concept.** In a vertical *Echo Sounder*, a pulse of sound is transmitted towards the sea-bed from a transducer on the underwater hull of a ship. On reaching the sea-bed, part of the sound is reflected back. Given the speed of sound in water (usually 1500 m/s) and the time interval between transmission and reception, the depth may be easily calculated. Depth may be displayed graphically or numerically.

c. **Waterline Depth v. 'Keel' Depth.** The pulse from the transmitting transducer is picked up almost instantaneously by the receiving transducer and is displayed on the recorder as a continuous 'transmission mark' line.

- **Waterline Depth.** If the transmission mark is set to the distance of the transducers below the sea surface, the recorder will show the depth of water below the waterline (this is the recommended default setting for most RN surface ships - see Para 0924e). Ships operating in shallow water areas with little clearance under the keel may experience *Squat* (see BR 45 Volume 6 Chapter 2) and this may affect the waterline depth setting in some cases.
- **'Keel' Depth.** If the transmission mark is set to zero of the scale, depth is recorded below the transducers (this is the recommended default setting for submarines and surface ships with substantially variable draughts - see Para 0924e). If the transducers are level with the keel, this depth may be considered as the 'keel depth'. If they are not level, a further adjustment to the transmission mark will be necessary.

d. **Separation Correction.** It may be necessary to apply a correction to recorded soundings in shallow water if the transducers are some distance apart laterally. See details at BR 45 Volume 3.

e. **Calibration of Echo Sounders.** *Echo Sounders* should be checked against the hand leadline periodically (see below). Several readings at various depths should be obtained for constructing a graph of *Echo Sounder* readings against leadline readings. Adjustments can then be made to the *Echo Sounder*, or the soundings themselves corrected. An *Echo Sounder* should be calibrated on each of the following occasions:

- On completion of a refit.
- When any part of the equipment is changed.
- Before any survey is carried out or a line of soundings obtained.
- If there is doubt about its accuracy.
- Annually.

f. **Interpretation of Soundings - False Echoes.** In general, hard sand, coral, chalk and rock give a good echo; thick mud gives a bad echo. *Echo Sounders* which can sound to great depths are subject to '2nd phase' errors, by reading on the 2nd (or even 3rd) pulse. In this case, the returning echo is not received until after the *Echo Sounder* has transmitted the next pulse(s); the prudent mariner should assume the vessel is standing into danger until the correct depth is established. Other false echoes may be caused by shoals of fish, water layers of different temperatures and density, salt and fresh water submarine springs, kelp or weed, side echoes, turbulence in the water, interference from incorrect 'Gain' settings or other sonars, or multiple reflection echoes between the ship's hull and the sea-bed in shallow water. See details at BR 45 Volume 3.

0924. Echo Sounder Reporting Procedures

Unless otherwise ordered by the NO or OOW, reports by the *Echo Sounder* operator should be made as follows. A copy of this instruction should be displayed at the *Echo Sounder*.

a. **Reporting Terms ("Depths" or "Soundings").**

- **Ships.** In surface ships, readings should be reported as 'Depth' (eg "Depth 20 metres shoaling").
- **Submarines.** In submarines 'Depth' means the depth of the submarine below the surface, and so in submarines all *Echo Sounder* readings should be reported as 'Sounding' (eg "Sounding 20 metres shoaling").

b. **Standard Reports.** When *Special Sea Dutymen* (*SSD*) or their equivalents are closed up, or at any other times when ordered, an additional person should close up to monitor and report depths / soundings. Standard reports should be at intervals of not greater than 1 minute, or as follows:

Depths / Soundings 0-20 metres:	**Report every 1 metre**
Depths / Soundings 20-40 metres:	**Report every 5 metres**
Depths / Soundings greater than 40 metres:	**Report every 10 metres**

c. **Use of Suffixes with Standard Reports.** The operator should suffix the depth / sounding with 'Steady', 'Deepening','Shoaling' or 'Below Minimum Depth / Sounding'.

d. **Briefing and Acknowledgement.** In *Pilotage*, the *Echo Sounder* operator should be briefed by the NO on the minimum expected depth / sounding on each leg. Each depth / sounding report is to be acknowledged by the Command Team (ideally in the order Chart Assistant, NO or OOW and CO in ships). If a sounding below the briefed minimum expected depth is reported but not acknowledged, the *Echo Sounder* operator is to repeat the report more loudly and urgently, until an acknowledgement is given.

e. **Soundings From Waterline / Transducer Depths.** Except where exempted below or where equipment limitations dictate otherwise, *Echo Sounder*s are to be adjusted to read depths from the waterline, unless specifically ordered otherwise by the NO. Exceptionally, as an exemption, in vessels which have substantially variable draughts (ie submarines and ships with flooding docks or equivalent) the CO / NO should normally order depths / soundings to be reported as transducer depths / soundings, because waterline depths / soundings may lead to substantial errors. In all cases each *Echo Sounder* is to be clearly labelled with the position from which it is reading.

f. **Reporting Units.** *Echo Sounder* readings should be reported in the units shown on the navigational chart in use. In the rare event of using a chart with soundings in feet or fathoms, the *Echo Sounder* operator should transpose the units before reporting the depth / sounding, by using a conversion table (eg NP 720).

g. **Annotation of Echo Sounder Trace.** In submarines, when an *Echo Sounder* operator is closed up it is mandatory for the paper trace to be annotated on the occasions listed below. This procedure is NOT mandatory for use in ships but is recommended.

- Every 6 minutes with a four figure time.
- At every *Fix* with a four figure time.
- All alterations of course and speed.
- Any change in a surfaced submarine's draught (eg trimming down).
- Any incident or other useful information (eg buoys abeam, boat transfers etc).

0925. Speed & Distance Measuring Equipment (Speed Logs)

The following information on 'Speed & Distance Measuring Equipment' (*Speed Logs*) is a brief summary; for details see BR 45 Volume 3.

a. **Concepts and Capabilities.** *Speed Logs* are used to record the ship's speed and distance run. This is usually through the water (ie not allowing for *Leeway*, *Tidal Stream*, *Current* or *Surface Drift*), although *Sonar Speed Logs* of various types may be used to determine the *Ground Track*. Provided they are properly calibrated and well sited on the hull, the accuracy of *Speed Logs* should be better than 2% (95% probability) for speed and distance through the water.

b. **Single-Axis Electromagnetic Speed Logs.** Single-axis electromagnetic (EM) *Speed Logs* are widely fitted in warships and other vessels. They have an electromagnet mounted in a sensing head which is fitted on the hull of a ship; the head may be on a fixed probe, a retractable rodmeter, or in a sensor flush with the hull. Probes are more accurate than a flush sensor because they usually protrude beyond the ship's *Boundary Layer* water flow. A small alternating current energises the electromagnet to set up a magnetic field. As the ship moves through the water, the sensor's magnetic field induces a voltage in the water. This voltage, which is proportional to the relative speed of the sensor to the water, is picked up by electrodes in the sensor and applied to electronic circuits where it is converted into speed and distance. The faster the water moves past the sensor the greater the voltage generated and the higher the speed recorded.

c. **Two-Axis EM Speed Logs.** A two-axis EM *Speed Log* has similarities to the single-axis version but has a discus shaped sensor, aligned so that each axis measures the ship's forward and athwartships speed through the water. Two pairs of electrodes pick up the induced voltages, which are proportional to the fore-and-aft and athwartships components of the water flowing past the sensor; these are applied to electronic circuits where they are converted into the resulting *Ground Track*. A two-axis EM *Speed Log* will therefore produce a good *DR* track through the water, but suffers from inaccuracies at very low speeds.

d. **Two-Axis Sonar Speed Logs.** Two-axis *Sonar Speed Logs* are typically fitted in vessels where high accuracy *Ground Tracks* are required, particularly at low speeds. They measure speed by processing the echoes derived from sonar pulses projected towards the sea bed. The signal processor can be based on either *Doppler* or *Correlation* principles. There are two types of *Correlation Speed Logs*, one being based on time (*Temporal Correlation Log*s), the other based on separation distance (*Spatial Correlation Logs*). *Sonar Speed Logs* are superior to conventional EM *Speed Logs* in several respects:

- *Sonar Speed Logs* measure the ship's ground speed over the sea bed in suitable depths of water.
- The accuracy of *Sonar Speed Logs* is typically of an order better than conventional EM *Speed Logs*, particularly at low speed.
- *Sonar Speed Logs* may be flush mounted without loss of accuracy and are thus less likely to be damaged.

(0925) e. **Pitometer Logs.** *Pitometer Speed Logs* use the difference between the static water pressure when stationary and the impact water pressure from a moving ship acting on a rodmeter probe protruding below the hull. They have superseded by EM *Speed Logs* in most large vessels.

f. **Impeller (Chernikeef) Logs.** *Impeller Speed Logs* (often referred to for historical reasons as *'Chernikeef Logs'*) use the rotation of an impeller caused by the water flow around a moving ship. They have mostly been replaced by EM or *Sonar Speed Logs*, although *Impeller Speed Logs* may still be found in some merchant vessels and in some small craft.

g. **Calibration of Speed Logs.** It is essential to calibrate *Speed Logs* over the whole speed range of the ship, on first installation, after a refit if the underwater shape of the hull has changed, or if a different type of probe or sensor is fitted. Percentage errors should always be calculated as a percentage of <u>log speed</u> and NOT true speed. Log speed is usually calculated in knots (ie *n.miles* [1852m] per hour); *Sea Miles* (1' of *Latitude*) vary in length (see Para 0113) and should not normally be used for *Speed Log* calibration. The detailed procedure for *Speed Log* calibration is at BR 45 Volume 3.

h. **The Dutchman's Log**. The *Dutchman's Log* is the oldest and simplest method of measuring the ship's speed through the water, but it may still be useful on occasion. A piece of wood is thrown overboard from a forward position and the time is taken when it passes two other points stationed along the fore and aft line of the ship at a known distance apart. The speed of the ship is then determined from the interval of time. When manoeuvring very slowly, a *Dutchman's Log* is useful to indicate whether the ship has headway or sternway.

0926. Night Vision Aids and Electro Optic Surveillance Systems (EOSS)

'Night Vision Aids' are now widely available and can be used to advantage in *Pilotage*, particularly if there are no shore lights (see Para 1312c); however, there is a risk of losing night-adapted vision for a short period after using them. In many RN warships, powerful *Electro Optic Surveillance Systems* (*EOSS*) are fitted; although controlled from the Ops Room, they can be used to provide a continuous bearing of a chosen mark. An *EOSS* bearing accuracy check should be carried out just before use or as soon as possible thereafter.

0927-0929. Spare

SECTION 3 - LIGHTS AND FOG SIGNALS

0930. Details, Characteristics and Nomenclature of Lights

a. **Availability of Light Information.** Details of lights may be found as follows:

- **Charts.** Lights are shown on Admiralty paper charts and *ARCS* charts (*Raster Navigation Charts* [*RNCs*]) as a light star with a magenta flare. The greatest detail will usually be found on the largest *Scale* paper / *ARCS* charts; the amount of detail reduces as the *Scale* of the chart decreases. *Electronic Navigation Charts* (*ENCs*) may be interrogated to display full details.

- **Admiralty List of Lights & Fog Signals (ALLFS).** The *ALLFS* (NPs 74-84) provides additional information not shown on paper / *ARCS* charts.

- **Admiralty Sailing Directions.** The *Admiralty Sailing Directions* ('*Pilots*') usually only provide the height and description of important light structures.

b. **Characteristics of Lights.** The appearance of a light is called its '*Character*' or '*Characteristic*'. The principal *Characteristics* are usually the sequence of light and darkness exhibited and, in some cases, the colour(s) of the light. Lights may be *Fixed, Rhythmic* and *Alternating*. See examples at Tables 9-1 to 9-3 (overleaf) and Fig 9-3.

- **Fixed Lights.** *Fixed* lights are those exhibited without interruption.

- **Rythmic Lights.** *Rythmic* lights show a sequence of intervals of light and dark, the whole sequence being repeated at regular intervals. Terms used to describe elements of *Rythmic* lights are:
  - **Period.** The *Period* is time taken to exhibit one complete sequence.
  - **Phase.** The *Phase* is one element (eg *Flash, Eclipse*) of a sequence.
  - **Flashing.** A *Flashing* light has a *Phase* of illumination <u>shorter</u> than that of darkness. *Quick, Very Quick* and *Ultra Quick Flashing* lights have flash rates per minute of 50-80, 80-160 and 'over 160' respectively.
  - **Eclipse.** An *Eclipse* is a *Phase* where no light is visible.
  - **Group Flashing.** A *Group Flashing* light is a *Flashing* light in which a group of flashes, specified in number, is regularly repeated.
  - **Isophase.** An *Isophase* light has a *Phase* of illumination the <u>same length</u> as that of darkness (*Eclipse*).
  - **Occulting.** An *Occulting* light has a *Phase* of illumination <u>longer</u> than that of darkness (eclipse).
  - **Group Occulting.** A *Group Occulting* light is a *Occulting* light in which a group of <u>eclipses</u>, specified in number, is regularly repeated.
  - **Composite Group Occulting.** A *Composite Group Occulting* light is similar to a *Group Occulting* light except that successive groups in a *Period* may have different numbers of eclipses.

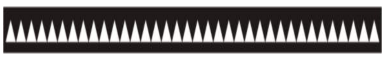
- **Alternating Lights.** *Alternating* lights are *Rhythmic* lights showing different colours during each sequence.
  - **Period.** The *Period* of an *Alternating* light is the time taken to exhibit the complete sequence, including the change of colour.

(0930b continued)

Table 9-1. Illustration of (White) Fixed Light Characteristics

Character	Abbreviation	Illustration
Fixed	F W	

Table 9-2. Illustration of (White) Rhythmic Light Characteristics

Character	Abbreviation	Illustration
Occulting	Oc W	
Group Occulting	Oc(2)W	
Composite Group Occulting	Oc(3+4) W	
Isophase	Iso W	
Flashing	Fl W	
Long Flashing	L Fl W	
Group Flashing	Fl(3) W	
Composite Group Flashing	Fl(3+2) W	
Quick Flashing	Q W	
Group Quick Flashing	Q(9) W	
‘Group Quick’ character, for IALA South Marks only.	Q(6)+L Fl W	
Interrupted Quick Flashing	IQ W	
Very Quick Flashing	VQ W	
Group Very Quick Flashing	VQ(3) W	
Interrupted Very Quick	IVQ W	

(0930b continued)

Table 9-2 (continued). Illustration of (White) Rhythmic Light Characteristics

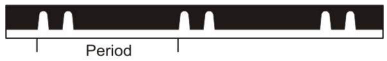
Character	Abbreviation	Illustration
Ultra Quick Flashing	UQ W	
Interrupted Ultra Quick	IUQ W	
Morse Code (example 'K') Morse Code (example 'AR') Flashing (example '4')	Mo(K) W Mo(AR) W Mo(4) W	  
Fixed and Flashing	FFl W	
Fixed and Group Flashing	FFl(2) W	

Table 9-3. Illustration of Alternating Light Characteristics

Character	Abbreviation	Illustration
Alternating	Al WGR	
Alternating and Flashing	AlFl WR	
Alternating Group Flashing Alternating Group Flashing	AlFl RW AlFl WWRR	 
Alternating Occulting	AlOc WR	
Alternating Group Occulting	AlOc WGR	
Alternating Fixed and Flashing Alternating Fixed and Flashing	AlF W Fl R AlF W Fl RG	 
Alternating Fixed and Group Flashing	AlF W Fl(3)G	
Alternating Fixed and Composite Group Flashing	AlF W Fl WRR	

(0930) c. **Nomenclature of Lights.** See *ALLFS* for full details of nomenclature of lights. All distances in *ALLFS* are in *Sea Miles*. See examples from charts at Fig 9-3 opposite.

- **Intensity.** The *Intensity* of a light is its 'brightness' and is either expressed in candelas or converted to *Nominal Range* (see below).
- **Elevation.** The *Elevation* of a light is the vertical distance between the focal plane of the light and the level of *Mean High Water Springs* (*MHWS*) or *Mean Higher High Water* (*MHHW*) or, on some charts, above *Mean Sea Level* (*MSL*).
- **Luminous Range.** The *Luminous Range* of a light is the maximum distance at which it can be seen, determined only by its intensity and the prevailing visibility. *Luminous Range* takes no account of *Elevation*, observer's height of eye or the curvature of the Earth.
- **Nominal Range.** The *Nominal Range* of a light is its *Luminous Range* for a meteorological visibility of 10 miles. The light's *Intensity* in candelas may be converted to *Nominal Range* with the *Luminous Range* diagram at Fig 9-4a.
- **Geographical Range.** The *Geographical Range* of a light is the maximum distance at which a light can reach an observer as determined by the height of the observer, the light's *Elevation* and the curvature of the Earth.
- **Loom.** The *Loom* of a light is the diffused glow observed from a light below the horizon or hidden behind an obstacle, due to atmospheric scattering.
- **Main Light.** The *Main Light* is the major of two lights on the same or adjacent supports.
- **Subsidiary (Auxiliary) Light.** A *Subsidiary (Auxiliary) Light* is one placed near a *Main Light* and having a special use in navigation.
- **Sector Light.** A *Sector Light* presents different colours or *Characteristics* in different directions. Sector limits are stated in true bearings (0˚-360˚) from seaward. If no sectors are stated it may be assumed to be visible all round.
- **Leading Lights.** *Leading Lights* are two or more lights forming a *Leading Line*; their alignment is stated in true bearings (0˚-360˚) from seaward.
- **Direction Light.** A *Direction Light* is a light showing over a very narrow sector, forming a single *Leading Light*. This sector may be flanked by sectors of greatly reduced intensity, or by sectors of different colours or *Character*.
- **Moiré Direction Light.** A *Moiré Direction Light* gives a yellow background to a screen on which a vertical black line is seen when on the centre-line; when off the centre-line, the vertical black line changes to black arrows indicating the direction to turn to regain the centre-line.
- **Vertical Lights.** *Vertical Lights* are two or more lights disposed vertically, horizontally or in a geometric shape, to distinguish them from single lights.
- **Occasional Lights.** *Occasional Lights* are only exhibited when needed.
- **Structure Descriptions - Heights, Bands, Stripes & Diagonal Stripes.** *Heights* are measured from the ground to the top of the structure. The terms '*Bands*', '*Stripes*' and '*Diagonal Stripes*' are used to describe horizontal, vertical or diagonal markings respectively.

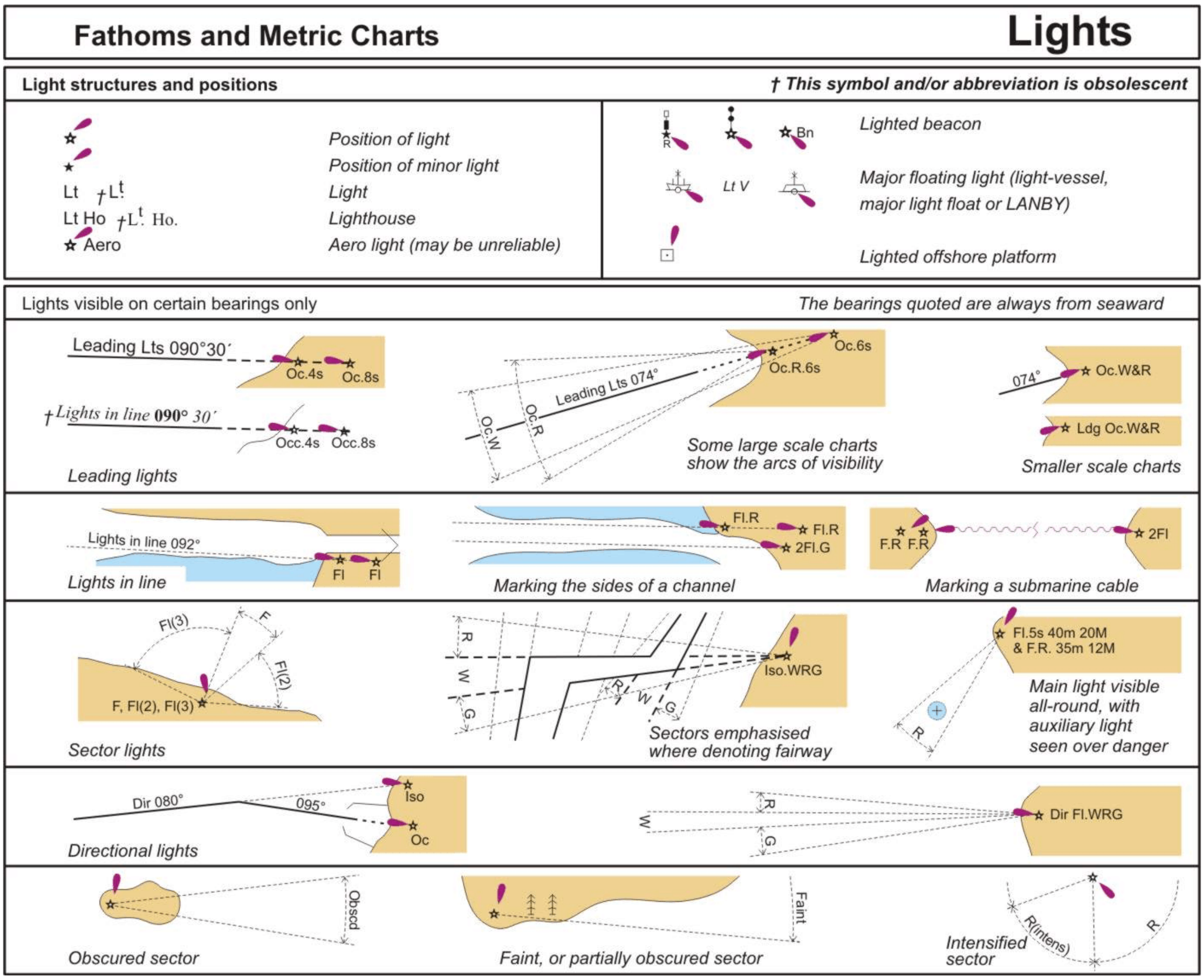


Fig 9-3. Examples of Lights shown on Admiralty (Paper / ARCS) Charts

0931. Admiralty List of Lights & Fog Signals - Paper and Digital Versions

ALLFS Volumes A-L (NPs 274-284) and its digital equivalent provide details of lights / *Fog* signals worldwide, with instructions for their use. Light buoys with a height of 8m or more may be listed, but mobile Oil / Gas / Drilling Platforms are not. *ALLFS* are published annually and are updated weekly by *Notices to Mariners*. *ALLFS* uses the following terms:

- **Positions.** Positions of lights are referenced to *WGS 84* unless otherwise stated but may be approximate. Charts should be consulted for more authoritative positions.
- **Aeromarine Lights.** *Aeromarine Lights* included are marine-type lights in which part of the beam is deflected 10˚-15˚ above the horizon for aircraft use.
- **Aero Lights.** *Aero Lights* are primarily for aircraft use; they are often of great intensity and *Elevation*. They are included where relevant for marine use, but details may change without notice and **they should be used with caution.**
- **Obstruction Lights.** *Obstruction Lights* mark radio towers, chimneys etc. Details may change without notice and **they should be used with caution.**
- **Daytime Lights.** *Daytime Lights* are operated throughout 24 hours.
- **Fog Lights.** Lights shown only in *Fog* are marked accordingly.
- **Fog Detector Lights.** *Fog Detector Lights* are often bluish in colour.

0932. Maximum Ranges of Lights

a. **Calculating the Maximum Range of Lights.** There are two criteria for determining the maximum range at which a light can be seen:

- **1st Criterion.** First, the light must be above the horizon. This depends on the *Geographical Range* (see definition at Para 0930c).
- **2nd Criterion.** Secondly, the light must be powerful enough to be seen. This depends on the *Luminous Range* (see definition at Para 0930c)

The range at which a light will be seen by the observer will be either the *Geographical* or the *Luminous Range*, **whichever is the less**. It is necessary to work out each range. Examples of the calculations are at Para 0932g overleaf.

b. **Geographical, Nominal and Luminous Ranges.** *Geographical, Nominal and Luminous Ranges* are defined at Para 0930c. The ranges quoted in *ALLFS* for nominated countries is the *Nominal Range,* but for other countries *Luminous Range* is used instead (in most cases with a meteorological visibility of 20 *Sea Miles*, equivalent to a transmission factor of 0.85); the list of nations for which *Nominal Range* is used is given in the preamble pages for each volume of *ALLFS*.

c. **Calculating Geographical Range.** *Geographical Range* may be calculated from a table in *ALLFS*, a copy which is at Fig 9-4b opposite.

d. **Calculating Luminous Range from Nominal Range and Visibility.** *Luminous Range* may be calculated from a diagram in *ALLFS* (see Fig 9-4a below). If *Nominal Range* is quoted in *ALLFS*, enter the diagram using *Nominal Range*, but if *Luminous Range* is quoted, enter using *Intensity* (candelas). **See also Note 9-2 (Para 0932g).**

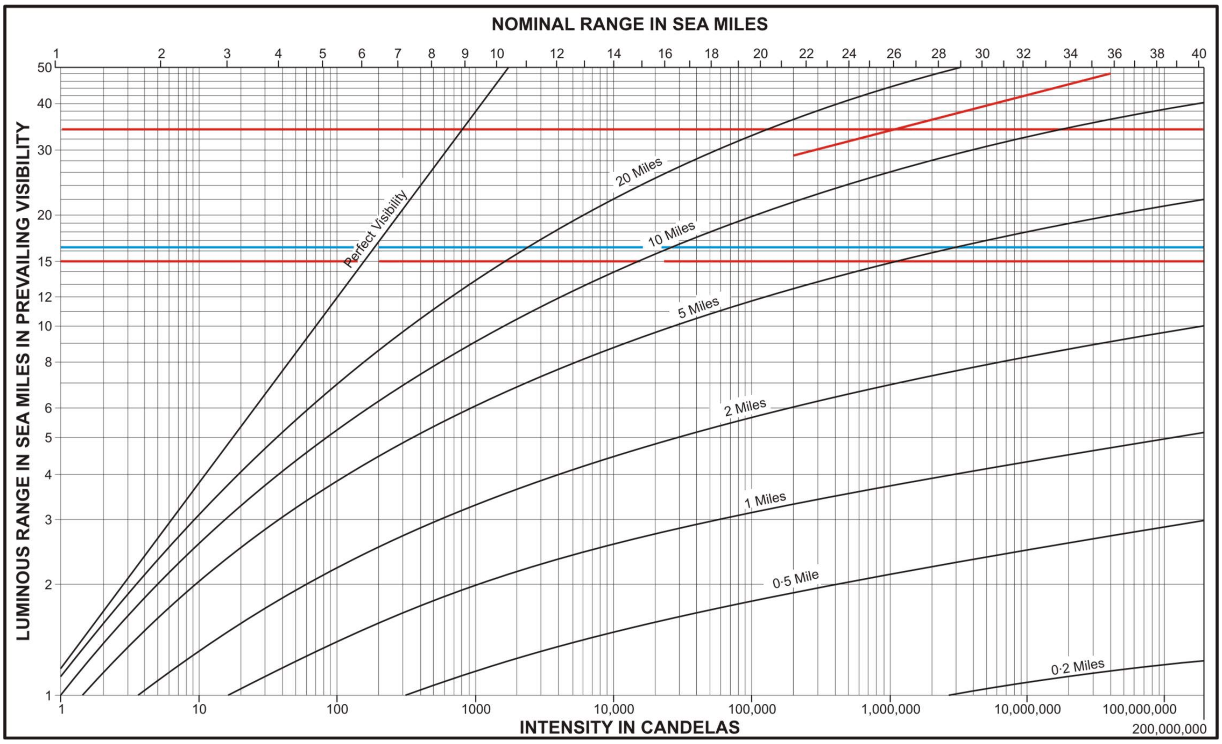


Fig 9-4a. Luminous Range Diagram (reproduced from ALLFS) *[Examples in colours]*

(0932 continued)

Elevation ft	Elevation m	Height of Eye of Observer in feet/metres: 3 ft / 1 m	7 ft / 2 m	10 ft / 3 m	13 ft / 4 m	16 ft / 5 m	20 ft / 6 m	23 ft / 7 m	26 ft / 8 m	30 ft / 9 m	33 ft / 10 m	39 ft / 12 m	46 ft / 14 m	52 ft / 16 m
		Range in Sea Miles												
0	**0**	2.0	2.9	3.5	4.1	4.5	5.0	5.4	5.7	6.1	6.4	7.0	7.6	8.1
3	**1**	4.1	4.9	5.5	6.1	6.6	7.0	7.4	7.8	8.1	8.5	9.1	9.6	10.2
7	**2**	4.9	5.7	6.4	6.9	7.4	7.8	8.2	8.6	9.0	9.3	9.9	10.5	11.0
10	**3**	5.5	6.4	7.0	7.6	8.1	8.5	8.9	9.3	9.6	9.9	10.6	11.1	11.6
13	**4**	6.1	6.9	7.6	8.1	8.6	9.0	9.4	9.8	10.2	10.5	11.1	11.7	12.2
16	**5**	6.6	7.4	8.1	8.6	9.1	9.5	9.9	10.3	10.6	11.0	11.6	12.1	12.7
20	**6**	7.0	7.8	8.5	9.0	9.5	9.9	10.3	10.7	11.1	11.4	12.0	12.6	13.1
23	**7**	7.4	8.2	8.9	9.4	9.9	10.3	10.7	11.1	11.5	11.8	12.4	13.0	13.5
26	**8**	7.8	8.6	9.3	9.8	10.3	10.7	11.1	11.5	11.8	12.2	12.8	13.3	13.9
30	**9**	8.1	9.0	9.6	10.2	10.6	11.1	11.5	11.8	12.2	12.5	13.1	13.7	14.2
33	**10**	8.5	9.3	9.9	10.5	11.0	11.4	11.8	12.2	12.5	12.8	13.5	14.0	14.5
36	**11**	8.8	9.6	10.3	10.8	11.3	11.7	12.1	12.5	12.8	13.2	13.8	14.3	14.9
39	**12**	9.1	9.9	10.6	11.1	11.6	12.0	12.4	12.8	13.1	13.5	14.1	14.6	15.2
43	**13**	9.4	10.2	10.8	11.4	11.9	12.3	12.7	13.1	13.4	13.7	14.4	14.9	15.4
46	**14**	9.6	10.5	11.1	11.7	12.1	12.6	13.0	13.3	13.7	14.0	14.6	15.2	15.7
49	**15**	9.9	10.7	11.4	11.9	12.4	12.8	13.2	13.6	14.0	14.3	14.9	15.5	16.0
52	**16**	10.2	11.0	11.6	12.2	12.7	13.1	13.5	13.9	14.2	14.5	15.2	15.7	16.2
56	**17**	10.4	11.2	11.9	12.4	12.9	13.3	13.7	14.1	14.5	14.8	15.4	16.0	16.5
59	**18**	10.6	11.5	12.1	12.7	13.2	13.6	14.0	14.4	14.7	15.0	15.7	16.2	16.7
62	**19**	10.9	11.7	12.4	12.9	13.4	13.8	14.2	14.6	14.9	15.3	15.9	16.5	17.0
66	**20**	11.1	12.0	12.6	13.1	13.6	14.1	14.5	14.8	15.2	15.5	16.1	16.7	17.2
72	**22**	11.6	12.4	13.0	13.6	14.1	14.5	14.9	15.3	15.6	15.9	16.6	17.1	17.7
79	**24**	12.0	12.8	13.5	14.0	14.5	14.9	15.3	15.7	16.0	16.4	17.0	17.6	18.1
85	**26**	12.4	13.2	13.9	14.4	14.9	15.3	15.7	16.1	16.4	16.8	17.4	18.0	18.5
92	**28**	12.8	13.6	14.3	14.8	15.3	15.7	16.1	16.5	16.8	17.2	17.8	18.3	18.9
98	**30**	13.2	14.0	14.6	15.2	15.7	16.1	16.5	16.9	17.2	17.5	18.2	18.7	19.2
115	**35**	14.0	14.9	15.5	16.1	16.6	17.0	17.4	17.8	18.1	18.4	19.1	19.6	20.1
131	**40**	14.9	15.7	16.4	16.9	17.4	17.8	18.2	18.6	18.9	19.3	19.9	20.4	21.0
148	**45**	15.7	16.5	17.1	17.7	18.2	18.6	19.0	19.4	19.7	20.0	20.7	21.2	21.7
164	**50**	16.4	17.2	17.9	18.4	18.9	19.3	19.7	20.1	20.5	20.8	21.4	22.0	22.5
180	**55**	17.1	17.9	18.6	19.1	19.6	20.0	20.4	20.8	21.2	21.5	22.1	22.7	23.2
197	**60**	17.8	18.6	19.3	19.8	20.3	20.7	21.1	21.5	21.8	22.2	22.8	23.3	23.9
213	**65**	18.4	19.2	19.9	20.4	20.9	21.4	21.7	22.1	22.5	22.8	23.4	24.0	24.5
230	**70**	19.0	19.9	20.5	21.1	21.5	22.0	22.4	22.7	23.1	23.4	24.0	24.6	25.1
246	**75**	19.6	20.5	21.1	21.7	22.1	22.6	23.0	23.3	23.7	24.0	24.6	25.2	25.7
262	**80**	20.2	21.0	21.7	22.2	22.7	23.1	23.5	23.9	24.3	24.6	25.2	25.8	26.3
279	**85**	20.8	21.6	22.2	22.8	23.3	23.7	24.1	24.5	24.8	25.1	25.8	26.3	26.9
295	**90**	21.3	22.1	22.8	23.3	23.8	24.2	24.6	25.0	25.4	25.7	26.3	26.9	27.4
312	**95**	21.8	22.7	23.3	23.9	24.3	24.8	25.2	25.5	25.9	26.2	26.8	27.4	27.9
328	**100**	22.3	23.2	23.8	24.4	24.9	25.3	25.7	26.1	26.4	26.7	27.3	27.9	28.4

Fig 9-4b. Geographical Range Table (reproduced from ALLFS) ***[Examples in RED]***

e. **Atmospheric Refraction.** The *Geographical Range* table in the *ALLFS* is based upon a particular allowance for *Atmospheric Refraction* (see Para 0803n, Note 8-3).

f. **Sighting of Lights at Ranges in Excess of Visibility.** From Examples 9-1 and 9-2 (overleaf), it can be seen that lights may be sighted at a range in excess of the estimated meteorological visibility, dependent on the light's *Intensity*.

(0932) g. **Calculating the Maximum Range of Lights - Examples.**

Example 9-1. Geographical Range less than Luminous Range

Height of eye 12 metres, estimated visibility 15 miles; disregarding *Height of Tide (HOT)*, at what range should the Lizard Light (A0060) be sighted? The *Elevation* of the light is 70 metres. *Nominal Range* is 26 *Sea Miles*.

Geographical Range. This can be established as 24.0 *Sea Miles* from the *Geographical Range* table (see Fig 9-4b) for an *Elevation* of 70 metres and height of eye 12 metres.

Luminous Range. Enter the *Luminous Range* diagram (see Fig 9-4a) from the top border for 26 *Sea Miles* (*Nominal Range*) and note where the vertical line from this point cuts the visibility range curve for 15 miles (which must be interpolated between the 20 mile and 10 mile visibility curves). From this second point move horizontally left and read off the *Luminous Range*; 34.0 *Sea Miles* in this case.

Maximum Range. The maximum range at which the light will be sighted is therefore the lesser of the *Geographical and Luminous Ranges*, and is thus 24.0 *Sea Miles*.

Example 9-2. Geographical Range greater than Luminous Range

Given the same situation as in Example 9-1 but with the visibility now reduced to 5 miles, at what range should the light be sighted?

Geographical Range. As before, *Geographical Range* is 24.0 *Sea Miles*.

Luminous Range. Follow the same procedure as in Example 9-1, but this time use the 5 mile visibility curve, move horizontally left and read off the *Luminous Range*; 15.0 *Sea Miles* in this case.

Maximum Range. The maximum range at which the light will be sighted is therefore the lesser of the *Geographical and Luminous Ranges*, and is thus 15.0 *Sea Miles*.

Example 9-3. Use of Luminous Range and Intensity (not Nominal Range)

Height of eye 10 metres, estimated visibility 5 miles. Disregarding *HOT*, at what range should Punta Gobernadora Light (J4836) be sighted? The *Elevation* of the light is 33 metres, *Luminous Range* 46 *Sea Miles*, *Intensity* 3 million candelas (see Note 9-2 below).

Geographical Range. This can be established as 18.0 *Sea Miles* from the *Geographical Range* table (see Fig 9-4b) for an *Elevation* of 33 metres and height of eye 10 metres.

Luminous Range. Enter the *Luminous Range* diagram (see Fig 9-4a) from the bottom border for 3 million candelas (*Intensity*) and note where the vertical line from this point cuts the visibility range curve for 5 miles, then move horizontally left and read off the *Luminous Range*; 16.4 *Sea Miles* in this case.

Maximum Range. The maximum range at which the light will be sighted is therefore the lesser of the *Geographical* and *Luminous Ranges*, and is thus 16.4 *Sea Miles*.

Note 9-2. Procedure for Obtaining Actual Luminous Range Without Intensity. *If Luminous Range is listed without Intensity, enter the Luminous Range diagram (see Fig 9-4a) at the left with the <u>tabulated</u> Luminous Range, move horizontally right until the 20 Sea Mile visibility curve is reached, then vertically up or down until the actual visibility curve is met, then read back across to the left, where <u>actual</u> Luminous Range for the prevailing visibility may be read.*

0933. Using Lights - Aide Memoire

The following aide memoire of practical tips is useful as a check-list when using lights for navigation.

- **Checking.** The *Characteristics* of a light should always be checked on initial sighting.
- **Actual Geographical Range.** *Atmospheric Refraction* and *HOT* may affect the *Geographical Range* expected.
- **Raising or Dipping Range.** The '*Rising or Dipping Range*' of a light can only be approximate, and must be used with **particular caution** if being used as a *Position Line*.
- **Obscured High Lights.** Lights placed at high *Elevations* are often obscured by cloud (eg lights on the Spanish coast).
- **Estimating Distances.** The distance of an observer from a light cannot be estimated from its apparent brightness.
- **Effect of Atmospheric Conditions.** The distance at which lights are sighted varies greatly with atmospheric conditions. It may be increased (or decreased) by *Abnormal Refraction*. It will be reduced by *Fog*, haze, dust, smoke or rain; a light of low *Intensity* may easily be obscured in any of these conditions and even the range of a light of great *Intensity* may be considerably reduced. **Thus ranges at which lights first appear can only be approximate.** There may also be *Fog* or rain in the vicinity of the light even though it is clear at the ship, and this may affect its initial sighting range.
- **Cold Weather.** In cold weather, and more particularly with rapid changes of weather, a shore-light's lantern glass and screens may be covered with moisture, frost or snow, which can greatly reduce the sighting range. Coloured sectors may appear more or less white, the effect being greatest with green lights of low intensity.
- **Sector Limits.** The limits of light sectors should not be relied upon and should always be checked by compass bearing. At the boundaries of sectors there may be a small arc in which the light may be obscured, indeterminate in colour or white; however, more modern lights usually define sector boundaries to a much greater degree of accuracy and precision than older lights.
- **Arcs of Visibility.** The limits of arcs of visibility of lights are rarely exact, especially at short ranges.
- **Reddish Hue.** In certain atmospheric conditions, white lights may have a reddish hue.
- **Glare from Background Lighting.** Glare from background lighting may considerably reduce the range at which lights are initially sighted. The approximate sighting range in such circumstances may be found by first dividing the *Intensity* of the light by 10 for minor background lighting, by 100 for major background lighting, and then using the *Luminous Range* diagram (Fig 9-4a) to establish a likely initial sighting range.

0934. Fog Signals - Types and Uses

ALLFS Volumes A-L (NPs 274-284) and its digital equivalent (DP 565) contain details of *Fog* signal and their operation. Brief details are also given on the chart.

a. **Fog Signal Types.** The following types of *Fog* signals are likely to be encountered.

- **Diaphone.** The *Diaphone* uses compressed air to issue a powerful low note, usually with a characteristic 'grunt' at the end of the note (a brief sound of suddenly reduced pitch). If the *Fog* signal does not end in this 'grunt', *ALLFS* will state 'No Grunt'.
- **Horn.** The *Horn* uses compressed air or electricity. *Horns* exist in many forms, differing greatly in sound and power. Some forms, particularly those at major *Fog* signal stations, simultaneously produce sounds of different pitch which are often very powerful. Others produce a single steady note, or vary continuously in pitch.
- **Siren.** The *Siren* uses compressed air and exists in many forms varying greatly in sound and power.
- **Reed.** The *Reed* uses compressed air and emits a weak (particularly if hand-operated) high-pitched sound.
- **Explosive.** This signal produces short reports by firing explosive charges.
- **Bell, Gong & Whistle.** *Bell, Gong & Whistle* may be operated by machinery, (sounding regularly), by hand (sounding irregularly) or by wave action (sounding erratically). *Bells, Gongs & Whistles* are frequently used as *Fog* signals on buoys.
- **Morse Code.** *Morse Code Fog* signals consist of one or more letters of the *Morse Code*. In a similar manner to lights, the abbreviation 'Mo' may be included in the abridged description of a *Fog* signal.

b. **Using Fog Signals - Cautions.** Sound travels through air in an unpredictable way and *Fog* signals <u>should never be relied upon implicitly</u>. *Fog* lookouts should be placed where noises in the ship are least likely to interfere with the hearing of a *Fog* signal.

- **Varying Distances.** *Fog* signals may be heard at greatly varying distances. *Fog* signal emitters vary greatly in power; reserve emitters are often weak.
- **Apparent Direction.** The apparent direction of a *Fog* signal is NOT always a correct indication of its true direction.
- **High and Low Notes.** If a *Fog* signal is a combination of high and low notes, one of the notes may be inaudible in certain atmospheric conditions.
- **Inaudible Areas.** There are occasionally areas around a station in which the *Fog* signal is quite inaudible.
- **Patchy Fog.** *Fog* may exist a short distance from a station and not be observable from it, so that the signal may not be operated.
- **Starting.** Some *Fog* signals cannot be started immediately *Fog* is detected.

0935-0939. Spare

SECTION 4 - BUOYS, OTHER FLOATING STRUCTURES AND BEACONS

0940. Buoy and Beacon Types

a. **Buoys.** Buoys are floating structures moored to the sea bed, used to mark channels and fairways, shoals, banks, rocks, wrecks and other dangers to navigation. Buoys have a distinctive colour and shape, they may carry topmarks and/or radar reflectors, exhibit lights and/or sound *Bells, Gongs, Whistles* or *Horns.*

b. **Beacons.** Beacons are navigational marks, erected as an aid to navigation. They may be placed on or in the vicinity of danger, or onshore. Beacons have a distinctive colour and shape, they may carry topmarks and/or radar reflectors and may exhibit lights; these features all have the same meaning as for buoys. Large unlit beacons are often referred to as *Daymarks* (*Daybeacons* in the USA and Canada). In its simplest form, a beacon is known as a 'Pile Beacon' and consists of a single wooden or concrete pile identified only by colour and possibly a number.

c. **Sources of Information.** The definitive guide to buoys and beacons for any area is the largest *Scale* paper chart or *ENC* / *RNC* of the place concerned. The *Admiralty Sailing Directions* ('*Pilots*') describe the buoyage system and *Direction of Buoyage* in use in the area covered by the particular volume and often refer in the text to individual light-buoys without giving a detailed description. Details of beacons may also be found in the *Admiralty Sailing Directions* ('*Pilots*'). *ALLFS* gives details of lighted beacons, and of most buoys with an of an *Elevation* of 8 metres or more.

0941. IALA Maritime System of Buoyage - Regions A and B

The *IALA Maritime Buoyage System* covers most (but not all) of the world and applies to buoys and beacons. It consists of *Lateral Marks, Cardinal Marks, Isolated Danger Marks, Safe Water Marks, Special Marks* and *Emergency Wreck Marking Buoys*. The following information is a brief summary; for full details, see '*IALA Maritime Buoyage System*' (NP 735).

a. **Regions A and B.** To reflect long-standing practices, the system is divided into *Region A* and *Region B* (see details in NP 735 and in the *Admiralty Sailing Directions* ['*Pilots*']). In broad terms, areas using *Region B* comprise N & S America, Japan, Korea and the Philippines; most of the rest of the world comprises *Region A*. The only differences are that the in *Region A* red is used for port hand *Lateral Marks* with green for starboard, and in *Region B* these colours are reversed; the shapes of *Lateral Marks* are the same in both *Regions* (can to port, conical to starboard).

b. **Direction of Buoyage.** *Lateral Marks* are generally used for well-defined channels with a '*Conventional Direction of Buoyage*', which indicates the port and starboard hand sides of the route to be followed. This is defined by either the *Local Direction of Buoyage* (used when approaching a harbour, estuary etc from seaward), or a *General Direction of Buoyage* (created on the principle of a clockwise direction around continents). The relevant *Direction of Buoyage* (if established) is normally shown on the chart and mentioned in *Admiralty Sailing Directions* ('*Pilots*') for a particular area.

c. **Retroflectors.** High-visibility *Retroflector* material is fitted to many unlit buoys.

(0941 continued)

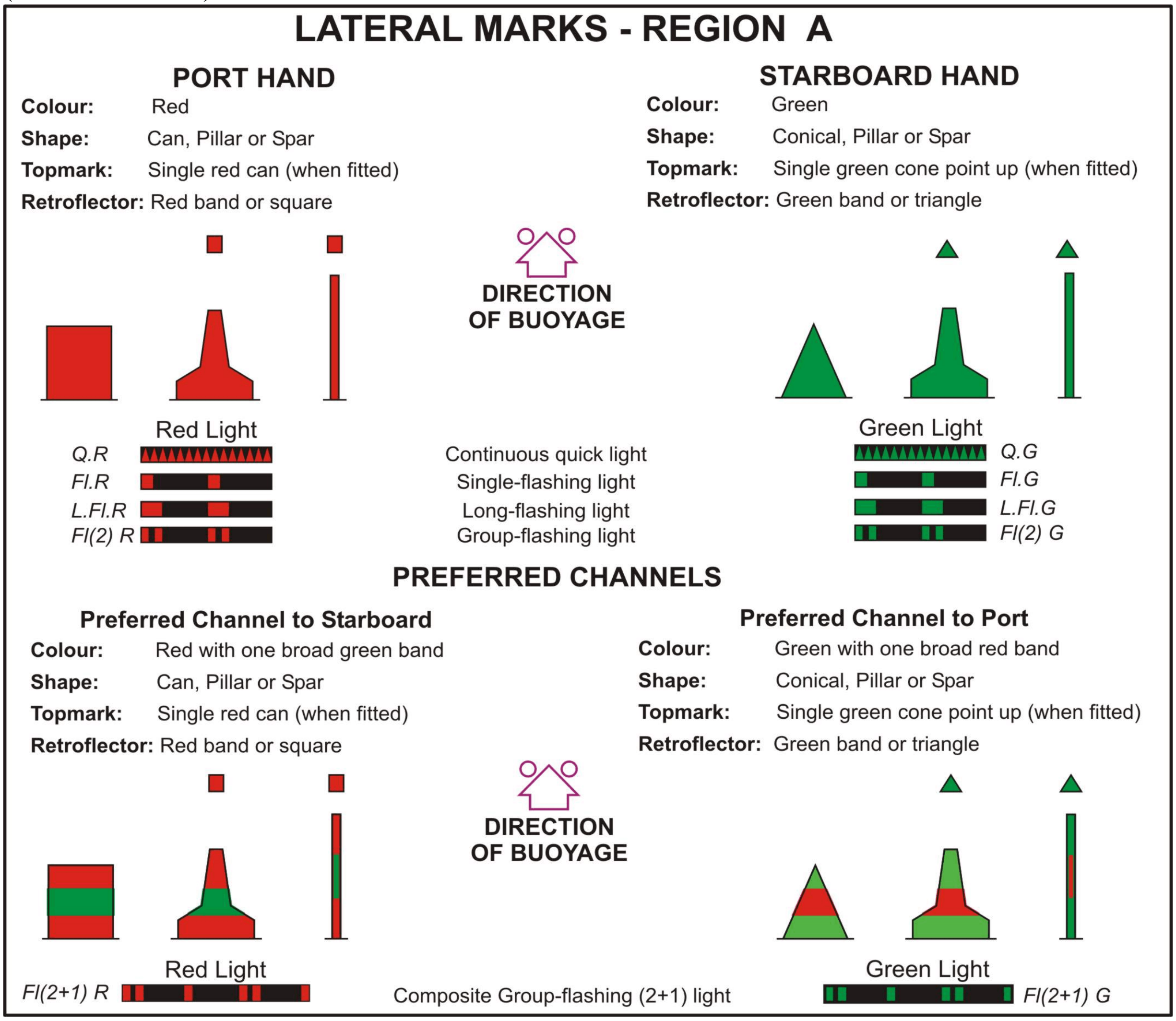


Fig 9-5. IALA Lateral Marks - Region A

(0941) d. **IALA Lateral Marks - Region A.** *Lateral Marks*, with colours, shapes and lights as in Fig 9-5 (above), are used in well defined channels in conjunction with *Direction of Buoyage* and indicate the port and starboard sides of the route to be followed. Where a channel divides to form two alternatives for the same destination, the 'Preferred Channel' is indicated by a modified *Lateral Mark*. In *Region A,* red is used for port hand *Lateral Marks* with green for starboard; the shapes of *Lateral Marks* are the same in both *Regions* (can to port, conical to starboard).

- **Features.** Fig 9-5 is schematic and buoy features may vary.
- **Shapes.** Where buoys do not rely on shape for identification, they carry the appropriate topmarks where practicable.
- **Numbers.** Numbering / lettering of buoys follows the *Direction of Buoyage*.

e. **IALA Lateral Marks - Region B.** As stated in Para 0941a, in *Region B Lateral Marks* are identical to those in *Region A* except that the colours are reversed with green used for port hand *Lateral Marks* and red for starboard; the shapes of *Lateral Marks* are the same in both *Regions* (can to port, conical to starboard).

(0941 continued)

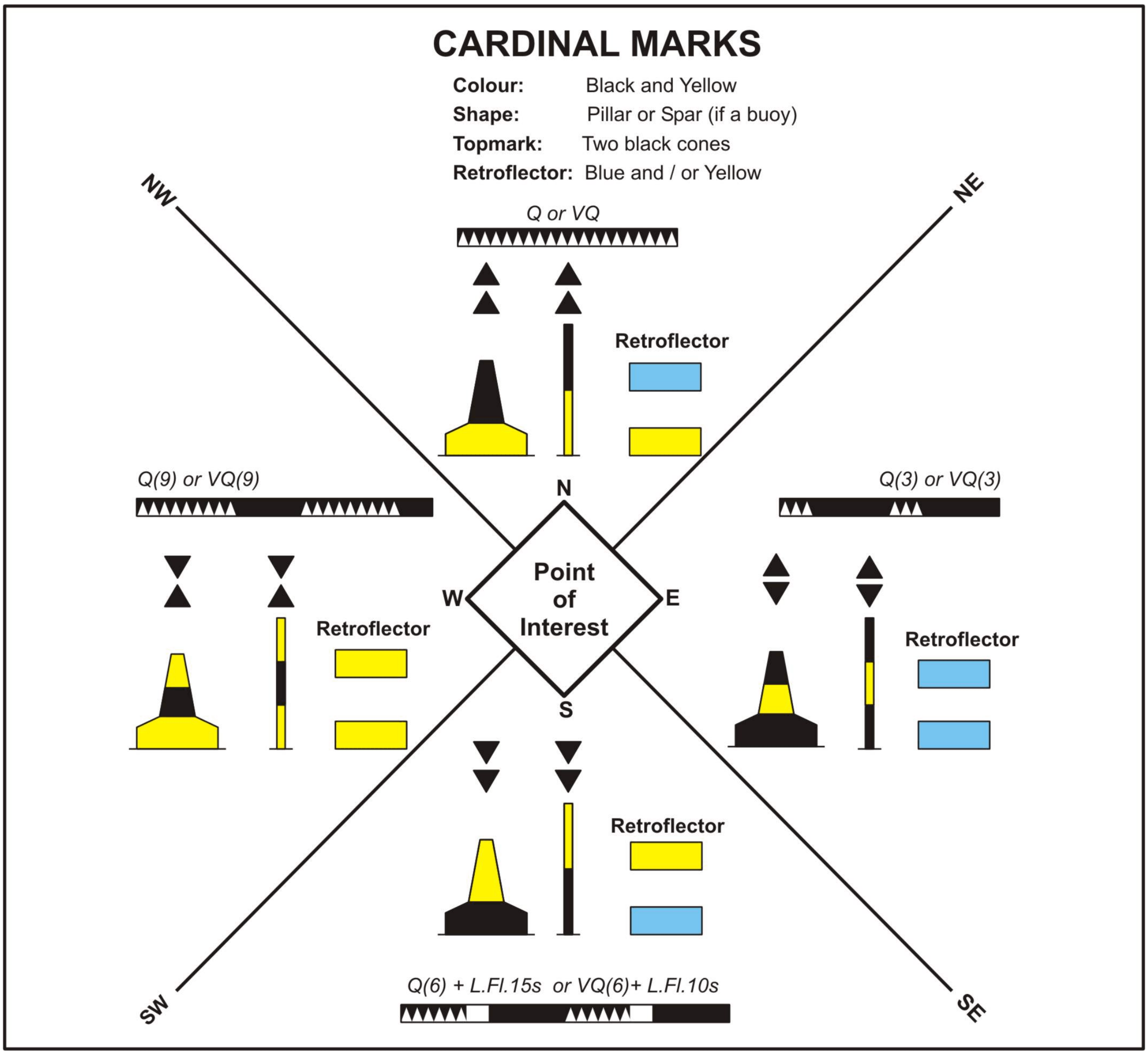


Fig 9-6. IALA Cardinal Marks

(0941) f. **IALA Cardinal Marks.** *Cardinal Marks*, with colours, shapes and lights as in Fig 9-6 (above), indicate that safe navigable water lies to the named side of the mark (ie safe water lies to the North of a North mark etc). *Cardinal Marks* are used to indicate that the deepest water in an area is on the named side of the mark, or to indicate the safe side on which to pass a danger (eg rocks, shoals or a wreck), or to draw attention to a feature in a channel such as a bend or junction, or the end of a shoal.

- **Topmarks.** *Cardinal Marks* are always painted in black and yellow bands, with black double-cone topmarks. The points of the cones indicate the position of the black-painted section relative to the yellow (see Fig 9-6).

- **Lights.** *Cardinal Marks* always show a white light based on a group of *Quick* (Q) or *Very Quick* (VQ) flashes using a clock face pattern which indicates the quadrant (ie North = continuous, East = groups of 3, South = groups of 6 plus a *Long Flash* (L Fl), and West = groups of 9) - see Fig 9-6 (above).

(0941) g. **IALA Isolated Danger Marks.** *Isolated Danger Marks*, with colours, shapes and lights as in Fig 9-7 (below), are placed at isolated dangers of limited extent surrounded by navigable water. On the chart, the position of the danger is the centre of the symbol or sounding indicating the danger; the symbol for the buoy will be slightly displaced.

- **Shape and Topmark.** *Isolated Danger Mark* buoys are pillar or spar in shape. A black double-sphere, topmark is fitted wherever practicable as an important feature.

- **Colours.** *Isolated Danger Marks* have horizontal stripes (red and black).

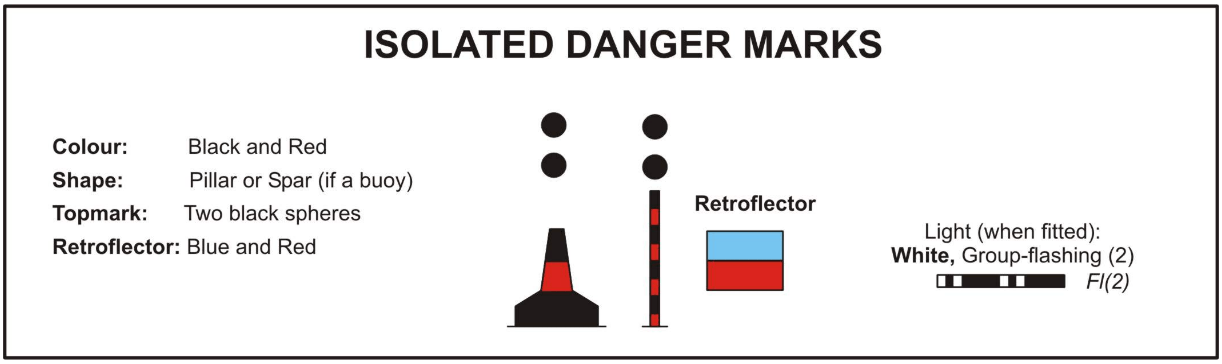


Fig 9-7. IALA Isolated Danger Marks

h. **IALA Safe Water Marks.** *Safe Water Marks*, with colours, shapes and lights as in Fig 9-8 (below), are used to indicate that there is navigable water all around the mark. They may be used as a centre-line, mid-channel or landfall mark or to indicate the best point of passage under a fixed Bridge.

- **Shape and Topmark.** If the *Safe Water Mark* is not spherical, a red spherical topmark is fitted wherever practicable as an important feature.

- **Colours.** *Safe Water Marks* have vertical stripes (red and white) and should not be confused with the *Emergency Wreck Marking Buoy* (see Para 0941j) which also has vertical stripes (blue and yellow).

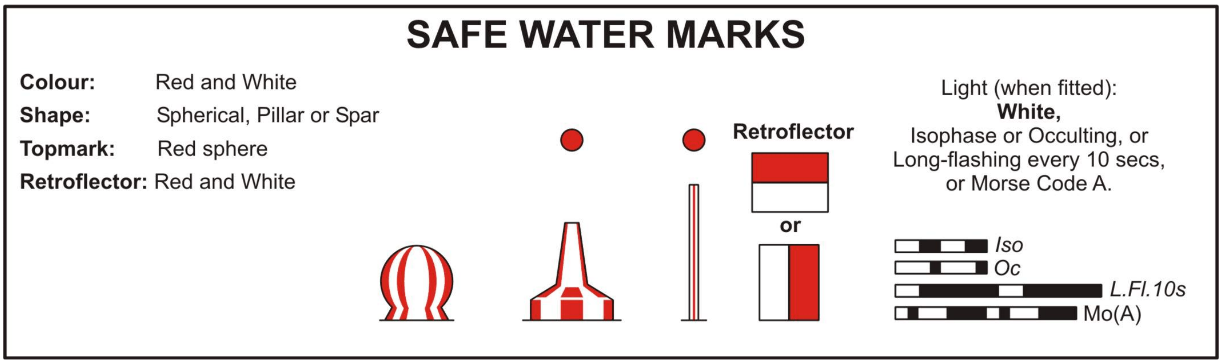


Fig 9-8. IALA Safe Water Marks

(0941) i. **IALA Special Marks.** Yellow *Special Marks*, with shapes and lights as in Fig 9-9 (below) are used to indicate a special area or feature, the nature of which is apparent from a chart, the *Admiralty Sailing Directions* (‘*Pilots*’), or *Notices to Mariners (NM)*. These include (but are not limited to): ‘Ocean Data’ marks, *Traffic Separation Schemes* (*TSS*), spoil grounds, recreation and military exercise zones, cables, pipelines, outfall pipes and ‘channels within a channel’.

- **Colour, Shape and Topmarks.** *Special Marks* are always yellow, can be a variety of shapes and may have lettered topmarks to indicate their purpose.

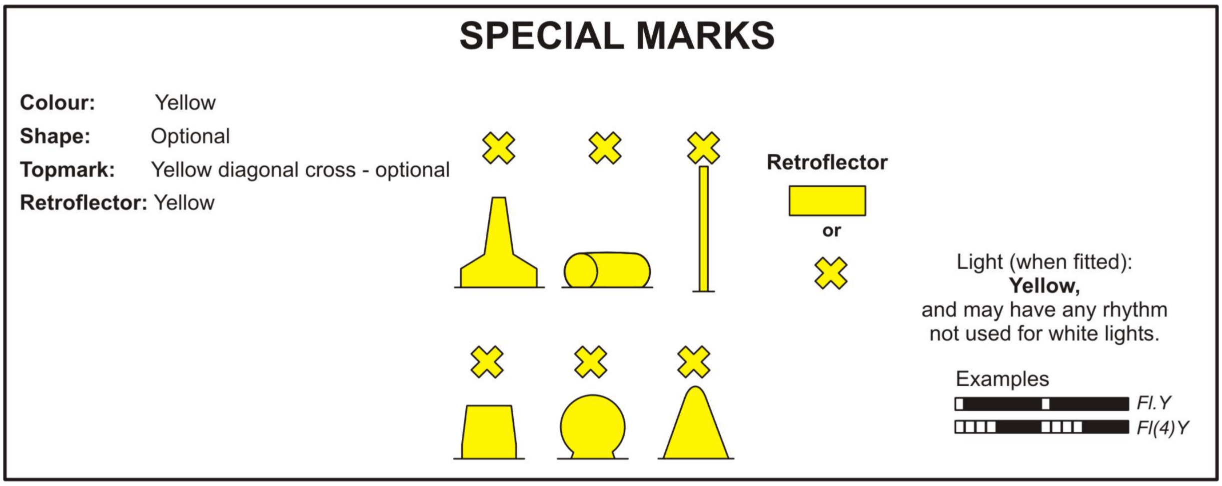


Fig 9-9. IALA Special Marks

j. **IALA Emergency Wreck Marking Buoys.** New *Emergency Wreck Marking Buoys* (see Fig 9-10 below) were introduced by *IALA* on a trial basis from 2006-2010 to provide clear and unambiguous marking of a new wreck, as a temporary response in the first 24-72 hours. Full incorporation of the buoy into the *IALA Maritime Buoyage System* will depend on the trial results.

- **Characteristics.** *Emergency Wreck Marking Buoy*s are fitted with alternating *Occulting* blue and yellow lights, a *Racon* (Morse Code ‘D’) and an *AIS* transponder. If multiple buoys are deployed, the lights will be synchronised
- **Retroflector.** As *Emergency Wreck Marking Buoy*s are always intended to be lit, they are NOT normally fitted with *Retroflectors*.
- **Colours.** *Emergency Wreck Marking Buoys* have vertical stripes (blue and yellow) and should not be confused with the *Safe Water Mark* (see Para 0941h) which also has vertical stripes (red and white).

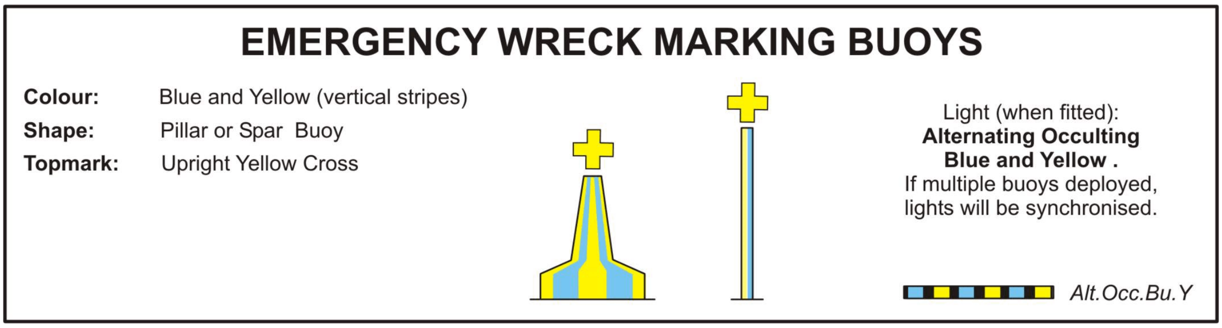


Fig 9-10. IALA Emergency Wreck Marking Buoys

0942. Using Buoys and Other Floating Structures for Navigation - Procedures

a. **Limitations of Buoys and Other Floating Structures.** The position of buoys and other floating structures (ie Light vessels, light floats and LANBYs [Large Automatic Navigation BuoYs]) **must always be treated with caution**, even in narrow channels where buoys positions may be monitored by harbour authorities (etc) ashore. Buoys (and to some extent, other floating structures as above) are liable to the following:

- **Dragging and Moving.** Buoys may quite easily drag or break adrift and they are frequently moved as a shoal extends.
- **Charted Position.** The charted position can only show the approximate position of the buoy mooring and takes no account of the length of mooring chain used; this may result in the buoy being substantially away from its charted position, particularly at LW or where a large (tidal) *Range* or strong *Tidal Stream* is experienced.
- **Light Characteristics.** Buoys may not always display the correct light *Characteristics*.

b. **Advantages of Buoys.** Despite the known limitations of buoys (see Para 0942a above), buoys are still a useful aid to navigation, particularly in the following circumstances:

- **Pattern.** If buoys are in a pattern, their relative position to each other may be assessed or *Fixed* using identification techniques (see Para 0807d), thus enhancing confidence (or otherwise) in their position.
- **Shore Radar Monitoring.** If buoys (particularly if they are in a pattern) are monitored by radar from shore by an appropriate harbour authority (etc), it may be possible to confirm (eg by a VHF call) if they are all in position.
- **Supplementary Visual Checks.** Buoys provide an instant visual confirmation (or otherwise) of other position source information. The *Echo Sounder* depth of water should also be used for correlation of position.

c. **Use of Buoys with Caution.** Buoys should NOT be treated as infallible aids to navigation, particularly when in an exposed position. Whenever possible:

- **Fixing & Pilotage.** Navigate by *Fixing* from charted shore objects or high accuracy electronic systems, and in *Pilotage* also run a *Headmark / Sternmark* with *Clearing Lines (Clearing Bearings* or *Clearing Ranges*)
- **Echo Sounder.** Always monitor the *Echo Sounder* (as this measures the proximity of the nearest point of land) and correlate it to the charted position.
- **DR / EP.** Always check the *DR / EP* against the other positions indicated.
- **Cross Checking.** As with all navigation, the concept of cross-checking all available sources of positional information applies to buoys; **use buoys, but do NOT rely implicitly on them.**

0943-0949. Spare.

SECTION 5 - AUTOMATIC IDENTIFICATION SYSTEM (AIS) AND VHF RADIO

0950. Concept of AIS and VHF Radio Use

The following information on *Automatic Identification System (AIS)* and the use of VHF radio is a summary; for details, see BR 45 Volumes 3 & 8, and UK *Maritime and Coastguard Agency* (*MCA*) documents. Current *MCA* documents are available on the *MCA* website.

a. **AIS Summary.** *AIS* was introduced in 2002 and is a shipboard broadcast transponder system, using common digital VHF channel(s) by which ships continually transmit identity, course, speed and other data to other ships and coastal authorities.

b. **AIS - Aim and Objectives.** The aim and objectives of *AIS* are to enhance the following 3 areas of maritime activity:

- **Aim.** The aim of *AIS* is to enhance the safety of life at sea, the safety and efficiency of navigation, and the protection of the marine environment.
- **Objectives.** In support of the aim, the objectives of *AIS* are:
  - To help identify vessels.
  - To assist in contact tracking.
  - To simplify information exchange by reducing VHF verbal reporting.
  - To provide additional information to assist situation awareness.

c. **Classes of AIS - Features and Vessel Carriage Requirements.** Four classes of *AIS* are in use, with the following features and vessel carriage requirements:

- **AIS Class A.** *AIS* Class A provides full maritime functionality (see details at Para 0951) and must be carried by all vessels over 300 grt, from 1 July 2007. The baseline display of *AIS* data is by *Minimum Keyboard Display* (*MKD*), but *AIS* data may instead be interfaced to *Automatic Radar Plotting Aids* (*ARPA*) and *WECDIS / ECDIS*, for graphical display. An *AIS* 'Pilot Port' is required for an external *AIS* display (eg laptop) to be connected to the ships' system.
- **Warship AIS (W-AIS).** *Warship AIS* (*W-AIS*) is an enhanced version of *AIS* Class A and has additional military functionality. See BR 45 Volume 8(1).
- **Inland AIS (Derivative of AIS Class A).** *Inland AIS* is a derivative of *AIS* Class A, specifically authorised for use in inland waters. *Inland AIS* covers the main features of *AIS* Class A while covering the specific requirement of inland navigation and enabling direct data exchange between seagoing and inland vessels in mixed traffic areas. *Inland AIS* specifications may vary regionally but, typically, may replace the *MKD* with a graphical display, remove VHF *Digital Selective Calling* (*DSC*), add some facilities of specific relevance to inland navigation (eg dedicated *Persons on Board* [*POB*] data field) and increase the message transmission rate. See Note 9-1 (below).
- **AIS Class B.** *AIS* Class B provides a reduced maritime functionality and may be carried optionally by vessels not covered by *AIS* Class A. In summary, *AIS* Class B has much reduced message fields and transmission intervals; tracks may also be automatically inhibited in areas of high traffic density.

Note 9-1. Thames AIS. *An Inland AIS ('Thames AIS') is operated by the Port of London Authority (UK) in the River Thames, with mandatory carriage requirements from 1 June 2007.*

0951. AIS - Operation

a. **Operation of AIS.** *AIS* Class A should always be in operation when ships are underway or at anchor, although it may be switched off if the Master / CO considers its continued use may compromise the security of the ship (eg piracy areas). Such action should be noted in the Ship's Log with reasons; in mandatory reporting systems, the competent authority should also be informed unless security would be compromised.

b. **Modes of Operation.** *AIS* operates in 3 modes -'Static', 'Dynamic' and 'Voyage Related', as follows:

- **Static Mode.** In 'Static Mode', *AIS* Class A transmits the vessel's identity, size and type, and the precise location of the navaid antenna (usually *GPS*) in use on board.
- **Dynamic Mode.** In 'Dynamic Mode', *AIS* Class A transmits the vessel's Position, Position 'time stamp' in UTC, *Course Over the Ground* (*COG) / Ground Track*, *Speed Over the Ground* (*SOG*) / *Ground Speed,* heading, rate of turn and 'Navigational Status' (which may include Underway [power / sail], alongside, at anchor, 'Not under Command' [NUC], 'Restricted in Ability to Manoeuvre' [RAM], 'Constrained by Draught [CBD]', aground or fishing).
- **Voyage Related Mode.** In 'Voyage Related Mode', *AIS* Class A transmits the vessel's draught, any hazardous cargo, destination, ETA and *Waypoints*.

c. **Data Input.** Most *AIS* data is input automatically, either at installation set-up or from navaids / gyro compass. <u>However, before sailing or when changes occur, the NO / OOW should manually input the following</u>: draught, cargo, destination, *Waypoints*, ETA, navigational status and text messages. **Failure to input this data accurately may lead to incorrect information being broadcast** (see Paras 0953 [opposite] and 1320a).

d. **Data Transmission Slots.** *AIS* Classes A and B use a *Time Division Multiple Access* (*TDMA*) system to transmit *AIS* messages in slots, each of 29.7 milliseconds.

- **AIS Class A / Inland AIS.** *AIS* Class A vessels use *'Self Organising TDMA'* (*SOTDMA*) to organise transmission slots automatically. *AIS* Class A 'Base Stations' and 'Aids to Navigation' use *'Fixed Access TDMA'* (*FATDMA*) with reserved transmission slots. *Inland AIS TDMA* is based on *AIS* Class A *SOTDMA*, but regional variations in the transmission rate may occur.
- **AIS Class B.** *AIS* Class B vessels use *'Carrier Sense TDMA'* (*CSTDMA*) to listen for a free slot before transmitting.

e. **Range of Operation and AIS Loading.** *AIS* coverage is usually 20-30 *n.miles*, unless substantial shielding by high land masses occurs. If high traffic densities occur and the available *AIS* broadcast time-slots become overloaded, *AIS* will automatically drop contacts furthest away.

f. **Short Text Messages.** *AIS* may be used to send short (maximum 158 characters), safety-related text messages, addressed either to a specific addressee or broadcast to all ships and shore stations (eg iceberg sighted, buoy off station etc).

0952. Virtual (or Pseudo) AIS Contacts

a. **Virtual AIS Contact Facility.** Certain equipments, used primarily by coastal authorities (eg *VTS* Centres etc) may be fitted with a 'Base-Station' *AIS* capability, allowing them to provide *Virtual AIS* (or *Pseudo AIS*) symbols / data in any position. This allows the positions to be broadcast for:

- Vessels whose transmissions to other ships are shielded by land masses.
- Vessels which are not transmitting on *AIS* but whose positions are known and need to be disseminated (eg disabled vessel in distress etc).

b. **False AIS Contacts.** The *Virtual AIS* contact facility could lead to the appearance of false *AIS* contacts. The *Virtual AIS* contact facility also has the potential for misuse (eg spoofing) if such equipment comes under the control of those wishing to confuse, either for illegal purposes (eg piracy) or warfare. Particular care is necessary with *AIS* contacts (**see CAUTION below).**

CAUTION

FALSE AIS CONTACTS. The *Virtual AIS* contact facility could lead to the appearance of false *AIS* contacts. Particular care is necessary when an *AIS* contact is not complemented by a radar contact.

0953. Incorrect AIS Data

Since the introduction of *AIS*, many instances have occurred of incorrect *AIS* data being broadcast, due <u>either</u> to incorrect setting up of navaids and other interfaced equipment, <u>or</u> to incorrect manual inputs. One example of erroneous *AIS* data is at Fig 9-11 (below), where a ship broadcast its *AIS* position some 4.8 cables from its real position, causing a potentially dangerous situation in the busy approaches to the ports of Portsmouth and Southampton (UK). Many other examples of incorrect *AIS* data being broadcast have been reported.

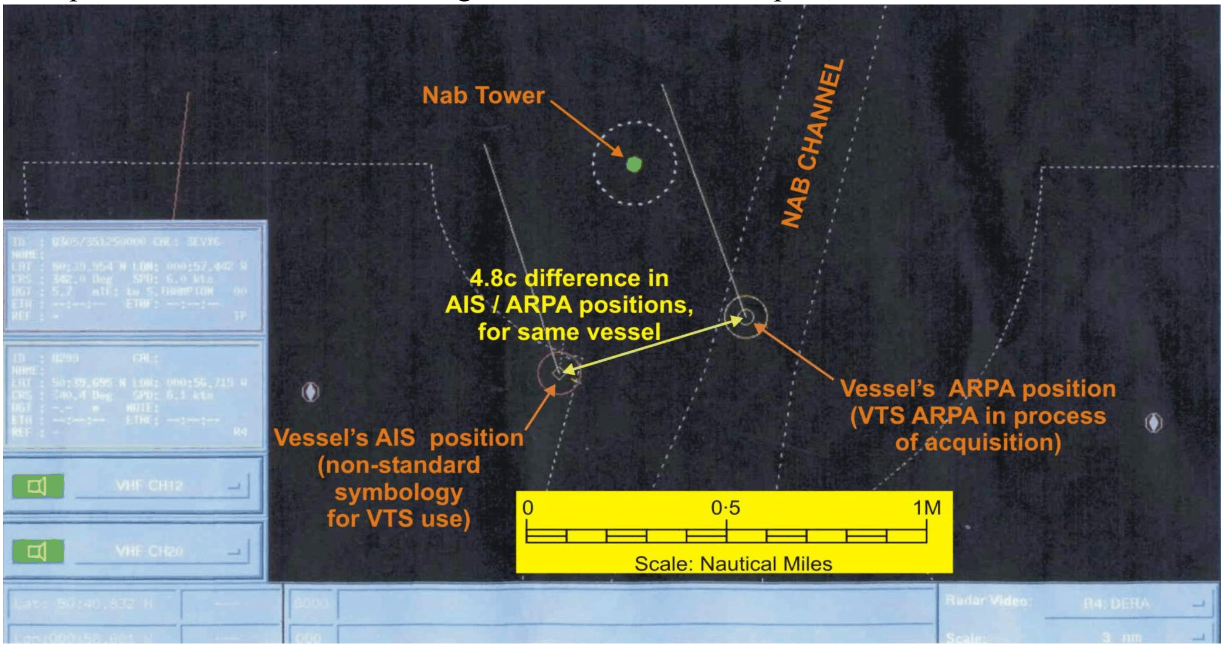


Fig 9-11. Example of Dangerously Incorrect AIS Information Transmitted by a Vessel
(Reproduced by kind permission of Associated British Ports, Southampton)

0954. Collision Avoidance - Use of AIS and VHF Radio

a. **Limitations of AIS.** *AIS* may not provide a complete picture of the situation around a vessel, for a variety of possible reasons. Not all ships are fitted with *AIS* (particularly small craft and fishing boats); in others, *AIS* may be switched off or be transmitting incorrect data. Other floating objects without *AIS* (eg navigational marks etc) may give a radar echo but will not give an *AIS* response. If a sensor is not installed (eg a gyro compass is not required in ships under 500grt) or fails, *AIS* cannot transmit its data. *AIS* positions are derived from the vessel's *GNSS* equipment (usually *GPS*) and may not exactly coincide with the radar contact.

b. **Misuse of AIS for Collision Avoidance.** Collision avoidance must be carried out strictly in compliance with the *International Regulations for Prevention of Collisions at Sea 1972* (the *'ColRegs'*). There is no direct provision in the *ColRegs* for the use of *AIS* data, except broadly under the 'all available means' provisions of *ColRegs* Rules 5 and 7 **(but see CAUTION below)**.

CAUTION

AIS DATA. Current (2008) UK (*MCA*) guidance is that *AIS* may be used to assist collision avoidance decision making, but only as an additional source of information; decisions should be based primarily on visual and/or radar data.

c. **Correlating Virtual AIS Contacts with Radar Contacts.** *AIS* 'Base-station' equipments are capable of providing *Virtual AIS* (or *Pseudo AIS*) contact symbols / data in any position on *AIS* displays (see Para 0952). Mariners should therefore take particular care if an *AIS* contact does not correlate with an appropriate radar contact.

d. **Misuse of VHF Radio for Collision Avoidance.** In a significant number of collisions, one or both parties had used VHF radio in an attempt to avoid collision, and this was later found to be a contributory cause. Time can be wasted making VHF contact and establishing the correct identity of each party, instead of complying with the *ColRegs*. Subsequent VHF conversations are open to misunderstanding. **See CAUTION below.**

e. **Misuse of AIS with VHF Radio for Collision Avoidance.** Notwithstanding availability of *AIS*, the uncertainties that can arise over the identification of vessels and the interpretation of VHF messages present a real danger (see Para 1954d above). **See CAUTION below.**

CAUTION

MISUSE OF VHF RADIO AND AIS. The use of VHF radio to discuss action to be taken between approaching ships is fraught with danger and is actively discouraged by the UK MCA; identification of a contact by *AIS* does NOT remove this danger.

CHAPTER 10

TIDES AND TIDAL STREAMS

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CHAPTER 10

TIDES AND TIDAL STREAMS

1001. Scope of Chapter

Chapter 10 summarises the theory and practice of *Tides* and *Tidal Streams*, including predictions from the Admiralty Tide Tables and Admiralty *TotalTide*® software. For further details see BR 45 Volume 2 (Astro Navigation) and 'The Admiralty Manual of Tides' (NP 120). This chapter replaces Chapter 11 of the 1987 Edition of this book.

1002. Primary Definitions

The terms '*Tides*' (vertical), *'Tidal Streams'* (horizontal), *Currents* and the American usage *'Tidal Currents'* are frequently mixed up; their definitions (repeated at Para 1040a) are as follows. Further information is at Paras 1040-1046 and Paras 1120-1125.

- **Tides.** '*Tides*' are **periodic vertical reversing** movements of the water on the Earth's surface, caused by the *Tide Raising Forces* of the Moon and Sun.
- **Tidal Streams.** *'Tidal Streams'* are the **periodic horizontal reversing** movements of the water accompanying the vertical rising and falling of *Tides*.
- **Currents.** Ocean *Currents* are **non-tidal** movements of water, which may flow **steadily** at all depths in the oceans and may have both horizontal and vertical components; a *Surface Current* can only have a horizontal component. In rivers and estuaries, there is often a permanent *Current* caused by the flow of river water.
- **Tidal Currents.** *'Tidal Currents'* is the American usage for *'Tidal Streams'*. The term *'Tidal Currents'* is NOT used in this book.

1003-1009. Spare

SECTION 1 - TIDAL THEORY

1010. Newton's Universal Law of Gravitation

Tides are caused by the gravitational attraction of other heavenly bodies on the Earth and on the water over the Earth. The two heavenly bodies which have the greatest *Tide Raising* effect are the Sun and the Moon, while the effect of other heavenly bodies is negligible.

a. **Magnitude.** The magnitude of the gravitational attraction between two bodies is defined in *Newton's Universal Law of Gravitation*, which states that:

> ***"For any two heavenly bodies, a force of attraction is exerted by each one on the other, the force being:***
> 1. ***Proportional to the product of the masses of the two bodies.***
> 2. ***Inversely proportional to the square of the distance between them.***
> 3. ***Directed from the centre of the one to the centre of the other."***

b. **Calculation.** *Newton's Universal Law of Gravitation* may be expressed as:

$$F \propto \frac{m_1 m_2}{d^2} \qquad \textbf{. . . 10.1 (1987 Ed . . . 11.1)}$$

where F is the force, m_1 and m_2 the masses of the two bodies, and d their distance apart.

1011. Earth-Moon System

a. **Barycentre.** The common *Centre of Gravity* of two bodies in a system is known as its *'Barycentre'*.

b. **Earth-Moon Barycentre.** The Earth and Moon form an independent system rotating about a common *Centre of Gravity* known as the Earth-Moon *Barycentre* (see Fig 10-1 below). The Earth-Moon *Barycentre* lies on a line joining the *Centres of Gravity* of the Earth and Moon (at a point about 1000 miles below the Earth's surface).

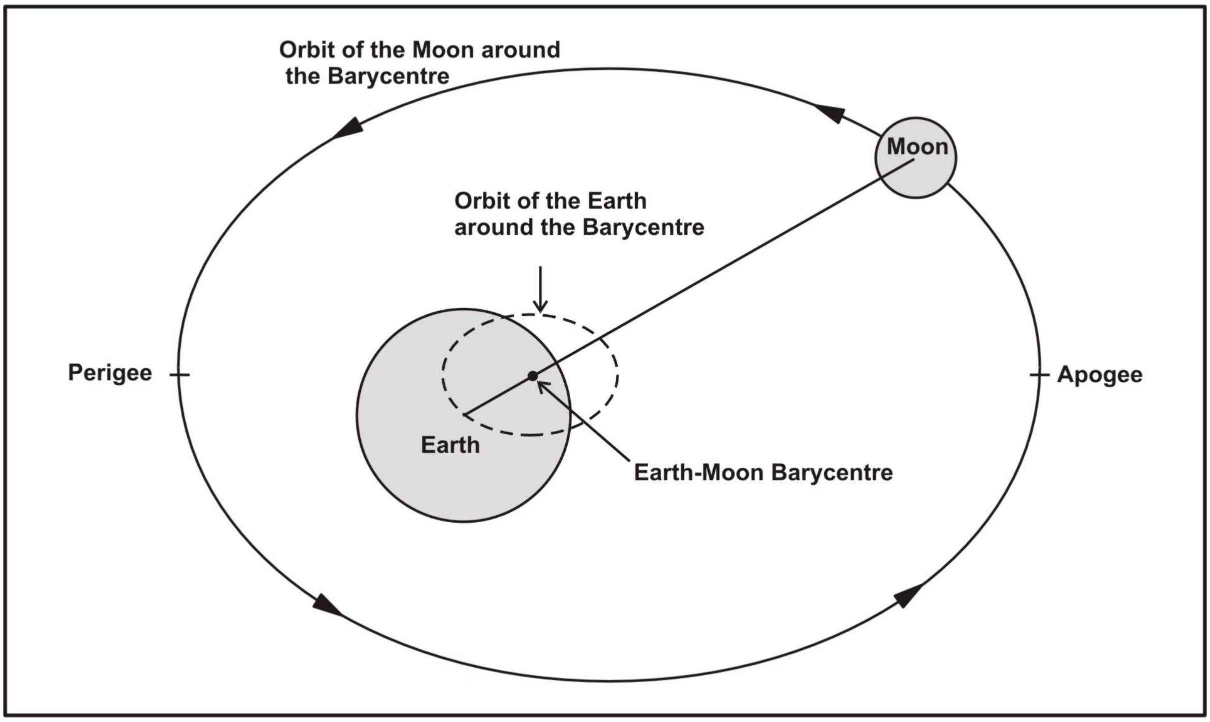


Fig 10-1. The Earth-Moon System (*not to scale*)

c. **Earth-Moon Orbits.** The Earth describes a very small ellipse about the Earth-Moon *Barycentre*, while the Moon describes a much larger ellipse about the same *Barycentre*, taking approximately 27½ days to complete one orbit. The Moon revolves around the Earth with respect to the Sun approximately once every 29½ days; this period is known as the *Lunar Month* (see details at BR 45 Volume 2).

d. **Earth-Sun Orbits.** The explanation of Earth-Sun orbits is at Para 1016.

1012. Gravitational Force

a. **Gravitational Force of the Moon Acting on Water.** Although the *Gravitational Force* of the Moon acts on the Earth as a whole (affecting the structure of the Earth itself) and also on the atmosphere, its effect on the water on the Earth's surface is the important factor for the causes of *Tides*.

b. **Gravitational Force of the Moon Acting on the Earth.** In Fig 10-2 (below):

- MM_1 is the diameter of the Earth on the line joining the centres of the Earth and Moon.
- *M* is the point on the Earth's surface directly under the Moon and known as the *Sublunar Point*.
- M_1 is on the opposite side of the Earth away from *M* and is known as the *Antipode* (or *Antipodal Point).*
- *A* and *B* are two points on the *Great Circle* whose plane is perpendicular to MM_1, and at all points on this *Great Circle* the distance from the Moon is effectively the same (see Note 10-1) as that from the centre of the Earth.

Thus, the *Gravitational Force* exerted by the Moon anywhere on *AB* is the same and is denoted by *G*. At *M* the distance to the Moon has decreased; thus, the *Gravitational Force* acting at *M* is increased by a small amount δG, while at M_1 the *Gravitational Force* has decreased by a similar amount. Thus, the total *Gravitational Force* acting at *M* is $(G + \delta G)$ and that at M_1 is $(G - \delta G)$.

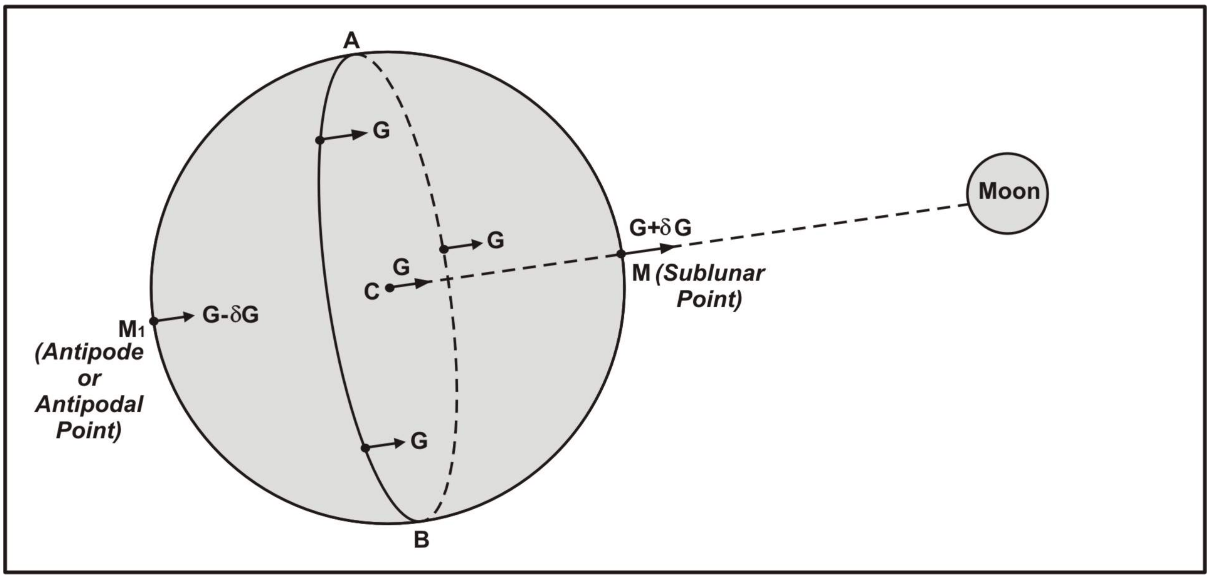


Fig 10-2. Gravitational Force of the Moon Acting on the Earth (*not to scale*)

Note 10-1. *The distance of A and B from the Moon is very slightly more than that at C but, as the radius of the Earth is small compared with the distance of the Moon (approximately 1:60), the differences are also small.*

(1012) c. **Differential Gravitational Force on the Earth's Surface.** If it is assumed that the Earth is a smooth *Sphere* completely covered by water, the *Gravitational Force* acting on the waters may be considered as the difference between the *Gravitational Force G* acting at the centre of the Earth and the actual *Gravitational Force* anywhere else on the Earth's surface (see Fig 10-3 below).

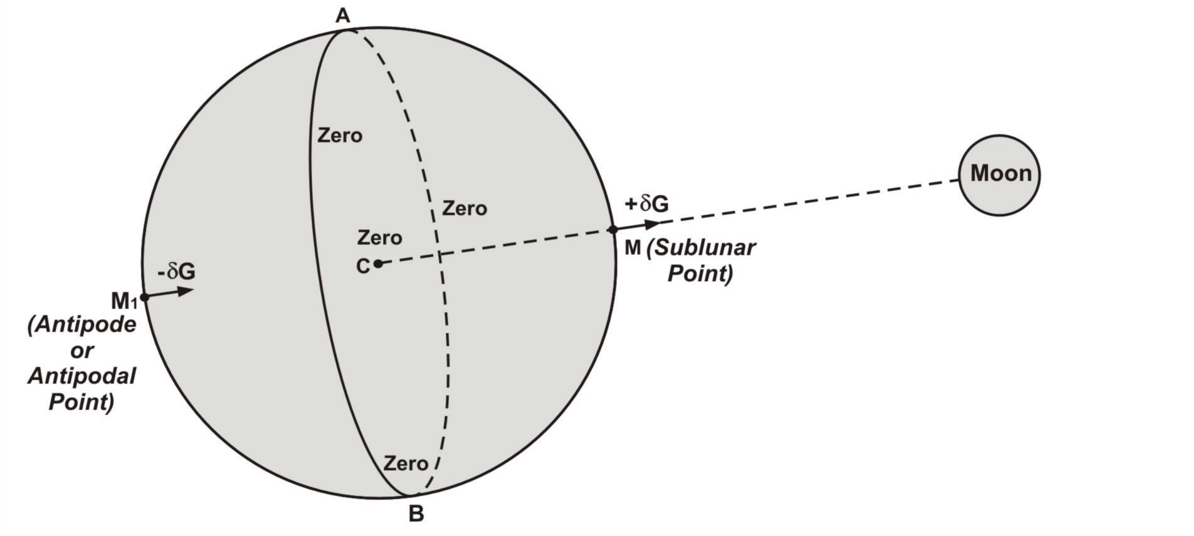


Fig 10-3. Differential Gravitational Force on the Earth's Surface at M / M_1 (*not to scale*)

d. **Direction of Gravitational Force at Antipodal Point.** At the *Antipodal Point M_1*, the differential *Gravitational Force* is negative (ie '-*δG'*). This is equivalent to the differential *Gravitational Force* at M_1 being positive (ie '+-δG') but acting in the opposite direction (see Fig 10-4 below).

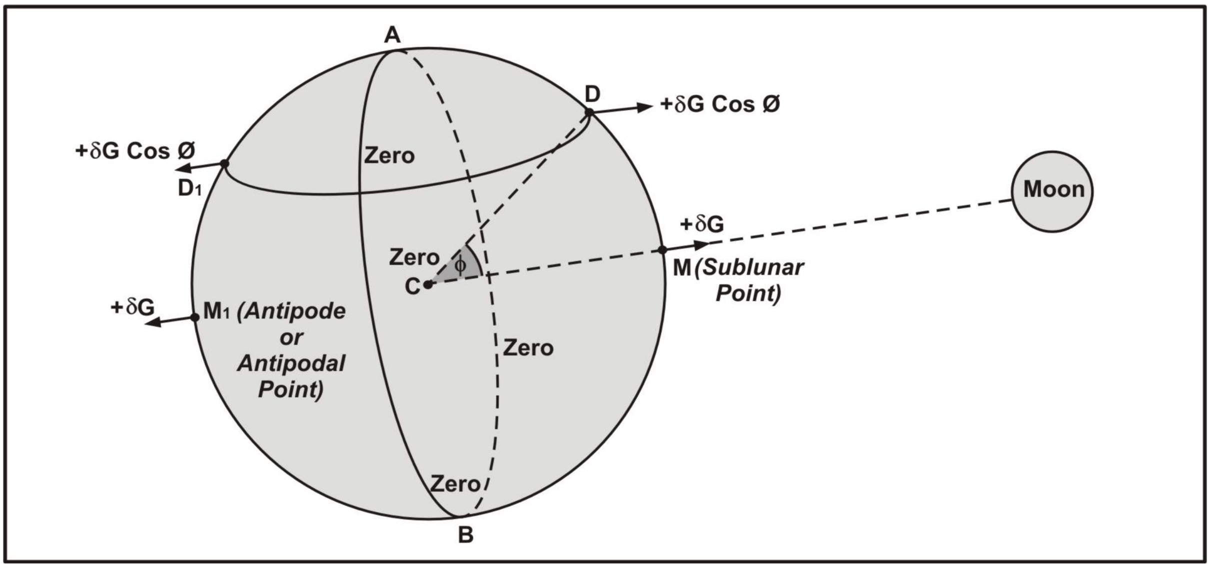


Fig 10-4. Differential Gravitational Force at Other Points (D etc) on Earth's Surface (*Not to scale*)

e. **Differential Gravitational Force at Other Points.** At some other point *D* on the Earth's surface (see Fig 10-4 above), the differential *Gravitational Force* acting on the waters must be somewhere between δG and zero. If *D* is ϕ° above the *Sublunar-Antipodal* plane, then the differential *Gravitational Force* at *D* is equal to $\delta G \cos \phi^\circ$. Similarly, at D_1 it is also equal to $\delta G \cos \phi^\circ$, but acting in the opposite direction.

(1012) f. **Centrifugal Force.** As the Earth and Moon orbit around a *Barycentre*, *Centrifugal Force* acts at M_1 and M_2. The effect is to reinforce the differential *Gravitational Forces.*

1013. Tide Raising Force

a. **Vertical and Horizontal Components of Differential Gravitational Forces.** If it is assumed that the entire surface of the Earth is covered with a uniform layer of water, the differential *Gravitational Forces* may be resolved into a vertical component (*FV*) at right angles to the Earth's surface and a horizontal component (*FH*) directed towards the *Sublunar* or *Antipodal Points* (see Fig 10-5 below).

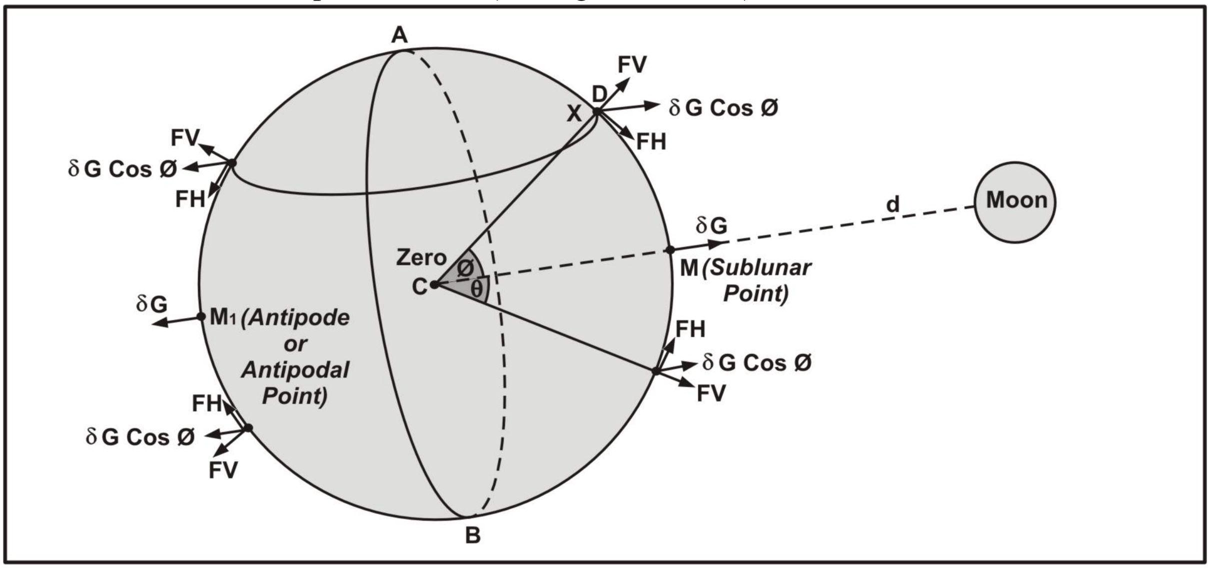


Fig 10-5. Vertical and Horizontal Components of Differential Gravitational Forces (*Not to scale*)

b. **Vertical and Horizontal Components.** The vertical component is only a very small portion of the Earth's gravity, so that the actual lifting of the water against gravity is infinitesimal. *Tides* are produced by the horizontal component which cause the water to move across the Earth and pile up at the *Sublunar* and *Antipodal Points* until an equilibrium position is found. The horizontal component of the differential *Gravitational Forces* is known as the *Tide Raising* (or *Tractive) Force*. Its magnitude at a given point (X in Fig 10-5) may be expressed as:

$$F_H \propto \frac{3}{2} \mathrm{x} \frac{m_2 r}{d^3} \sin 2\phi \qquad \textbf{... 10.2 (1987 Ed ... 11.2)}$$

where F_H is the magnitude of the horizontal *(Gravitational) Tide Raising Force;*
m_2 is the mass of the Moon;
r is the radius of the Earth;
d is the distance between the Earth's and Moon's centres;
ϕ is the angle at the centre of the Earth between the line joining the *Sublunar* and *Antipodal Points*, and the line joining the Earth's centre with point X.

From formula (10.2), it may be seen that the *(Gravitational) Tide Raising Force* caused by the Moon varies directly with the mass of the Moon and the radius of the Earth, but is inversely proportional to the cube of the distance between Earth and Moon.

(1013) c. **Effect of the Tide Raising (or Tractive) Force.** The effect of the *Tide Raising Force* is shown at Fig 10-6 (below).

- **Minimum (Zero).** The *Tide Raising Force* is zero at the *Sublunar* and *Antipodal Points M* and M_1 and along the *Great Circle AB* the plane of which is perpendicular to MM_1.
- **Maximum.** The maximum *Tide Raising Force* may be found along the *Small Circles EF* and *GH*, which are 45˚ from the *Sublunar* and *Antipodal Points* respectively.

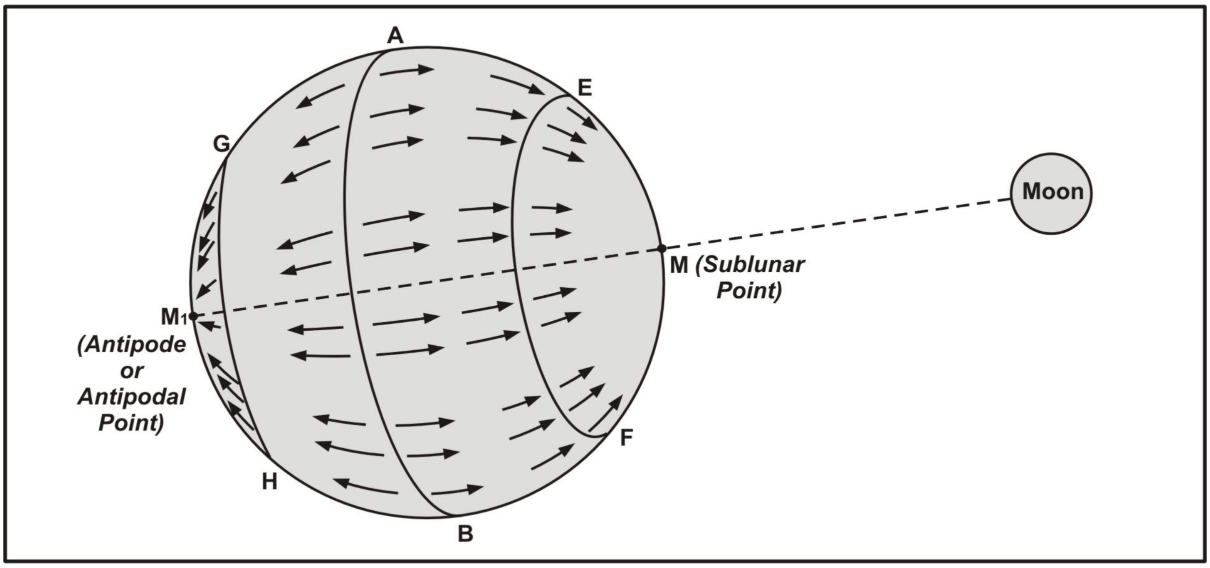


Fig 10-6. Effect of the Tide Raising (or Tractive) Force (*not to scale*)

d. **Lunar Equilibrium Tide.** Equilibrium is reached when the *Tides* formed at the *Sublunar* and *Antipodal Points* are at such a level that the tendency to flow away from them is balanced by the *Tide Raising Force*. The *Tide* caused in these circumstances is known as the *Lunar Equilibrium Tide* (see Fig 10-7), with a 'High Water'(HW) at *M* and M_1 and a 'Low Water'(LW) at *A* and *B*.

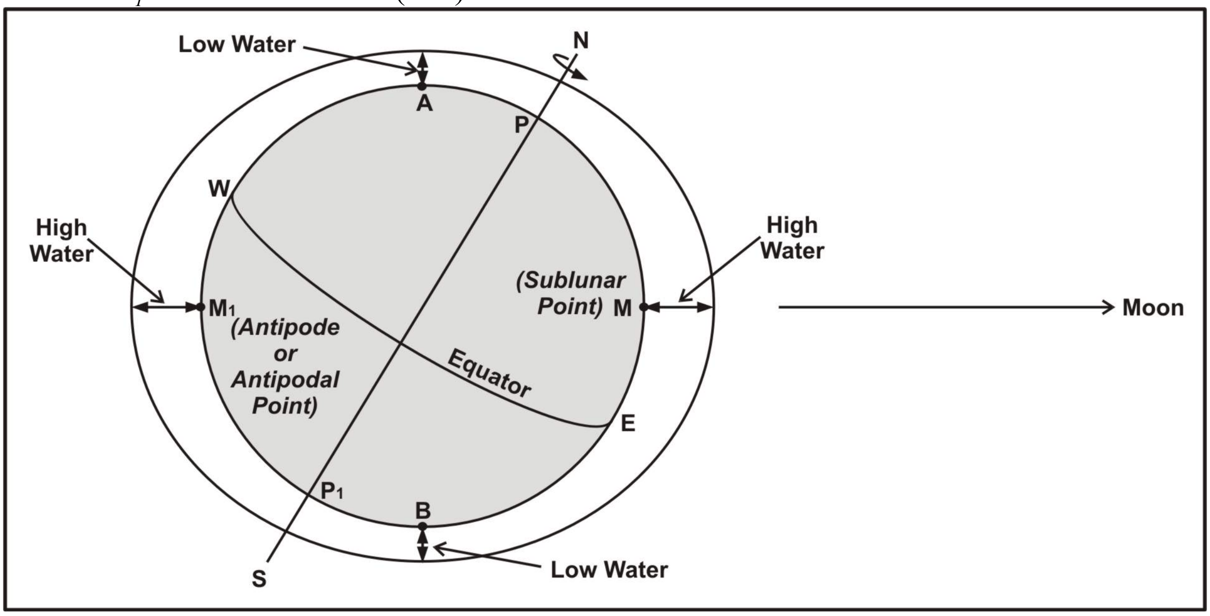


Fig 10-7. Lunar Equilibrium Tide (*not to scale*)

1014. Effect of the Earth's Rotation

a. **Moon's Declination Zero - Lunar Equilibrium Tide.** The effect of *Tide Raising Forces* on the Earth and at points *M* and *P* / P_1-P_4 when the Moon is above the Earth's *Equator* (ie the Moon's *Declination* is zero - see Note 10-2 below) gives a *Lunar Equilibrium Tide* (see Fig 10-8 below). However, the Earth rotates relative to the Moon once every *Lunar Day* (approximately 24 hours 50 minutes) and thus during this period observers at points *M* and P_1-P_4 will experience two equal HWs at intervals of 12 hrs 25 mins, interspersed with two equal LWs also 12 hrs 25 mins apart. Such *Tide*s, with 1 cycle per ½ day, are called *Semi-Diurnal Tide*s (see Para 1020 for further details).

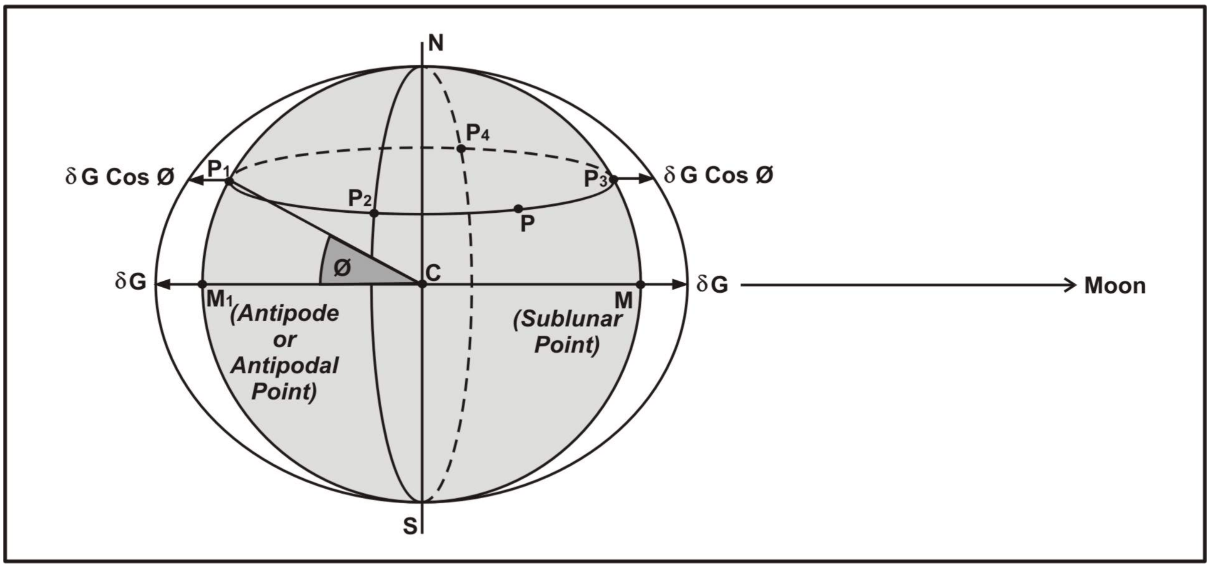


Fig 10-8. Moon's Declination Zero and Effect of the Earth's Rotation (1) (*not to scale*)

b. **Lunar Equilibrium Tide - Parameters.** *Tide Raising Force* magnitudes vary with the cosine of *Latitude* ϕ ° (see Fig 10-9 below). HW occurs shortly after the Moon's transit (upper and lower) of the observer's *Meridian*; the slight delay is a side effect of the Earth's rotation. The (tidal) *Range* (ie HW minus LW heights) of the *Lunar Equilibrium Tide* is less than 1 metre at the *Equator*.

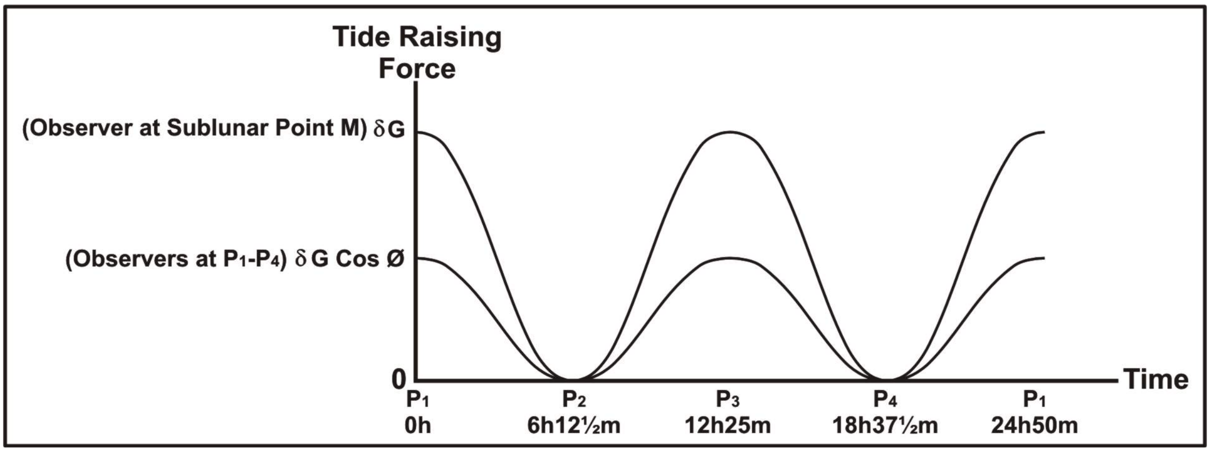


Fig 10-9. Moon's Declination Zero and Effect of the Earth's Rotation (2)

Note 10-2. *Declination is the angular distance of a heavenly body North or South of the Celestial Equator, and corresponds to Latitude on the Earth (see details at BR 45 Volume 2).*

1015. Change of Moon's Declination

a. **Diurnal Inequality.** When the Moon's *Declination* is <u>NOT</u> zero, maximum *Tides* still occur at the *Sublunar* and *Antipodal Points M* and M_1. But at any point *P* on the Earth's surface (see Fig 10-10 below), not only are the heights of successive HWs and LWs different, but the time intervals between them also change (see Fig 10-11 below). This effect is known as *Diurnal Inequality* and affects *Diurnal* and *Semi-Diurnal Tides*.

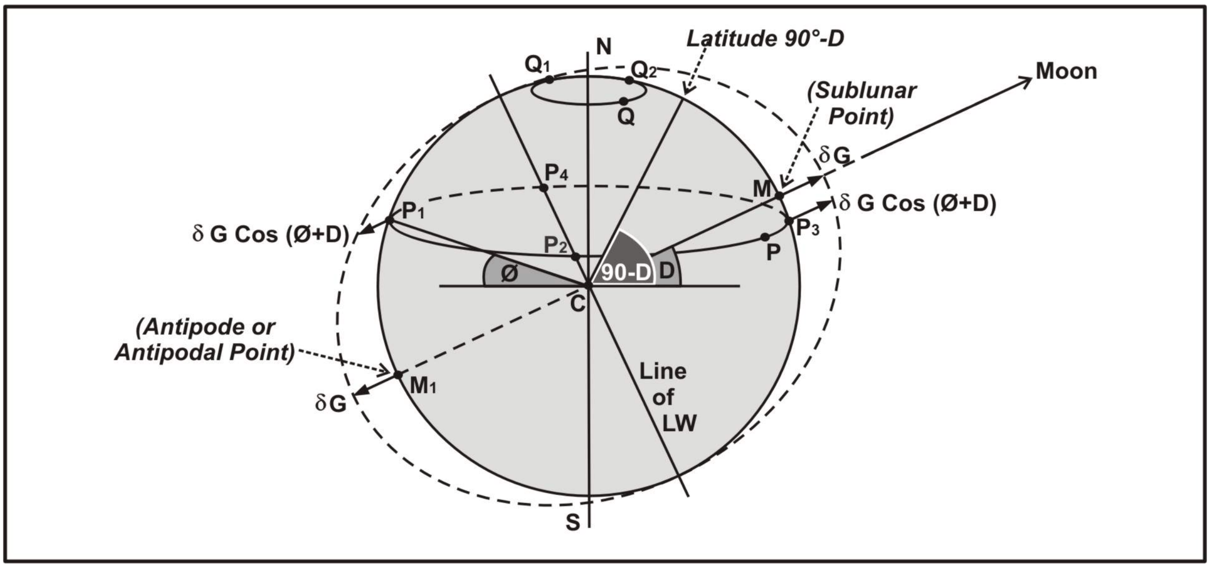


Fig 10-10. Effects of the Moon's Declination (*not to scale*)

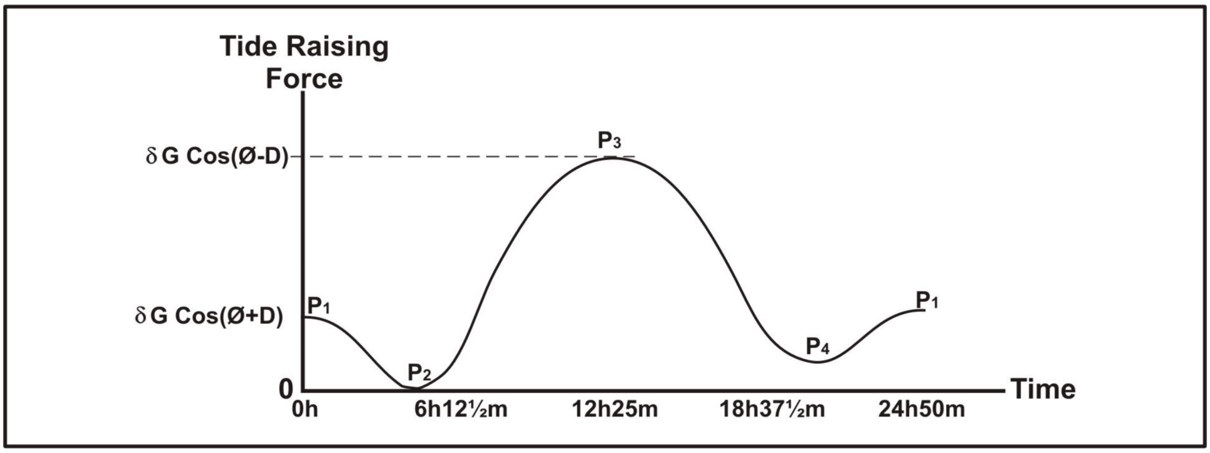


Fig 10-11. Diurnal Inequality (Semi-Diurnal Tides)

b. **Diurnal Tides.** At a point *Q* on the Earth's surface, where its *Latitude* is greater than 90˚ minus the Moon's *Declination* (*90˚-D*) - (see Fig 10-10 above), the *Tide Raising Force* never reaches zero; from Fig 10-12 (opposite), it can be seen that at *Q* there is only one HW and one LW every *Lunar Day*. Such *Tide*s, with 1 cycle per day, are called *Diurnal Tide*s (see Para 1020 for further details).

c. **Declination Cycles.** The Moon's *Declination* changes between North - South maxima and back every 27⅓ days, causing a similar effect on the *Tide* to be experienced roughly every fortnight. In addition, over an 18.6 year cycle, the Moon's maximum monthly *Declination* oscillates between about 18½˚ and 28½˚.

(1015 continued)

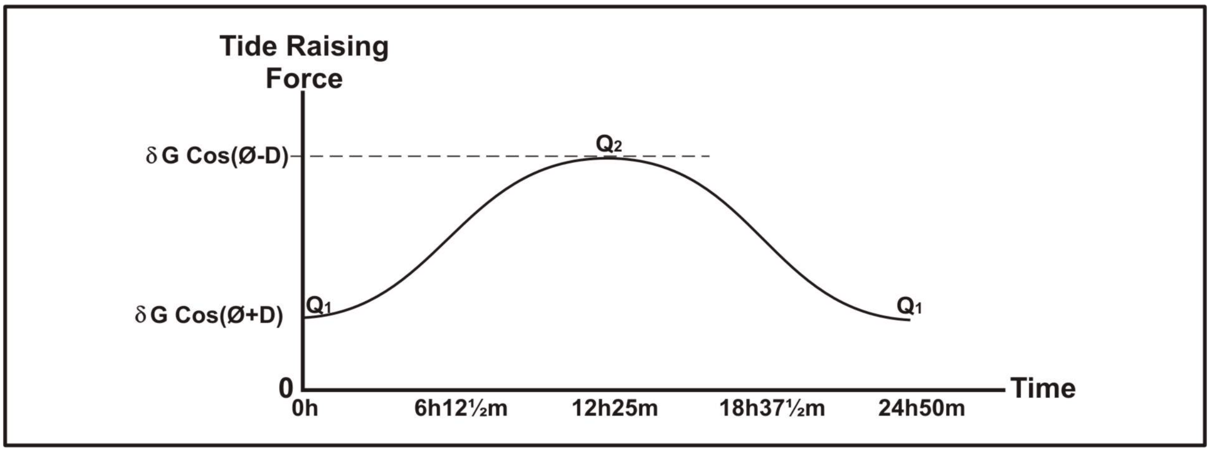


Fig 10-12. Diurnal Tides

(1015) d. **Distance of the Moon.** As the Moon rotates around the Earth approximately once every 27½ days (see Fig 10-1), the *Tide Raising Force* is strongest when the Moon is closest to the Earth (ie *Perigee*), with a *Perigean Tide*. The *Tide Raising Force* is weakest when the Moon is furthest away (ie *Apogee*), with an *Apogean Tide*. Variation in the Moon's distance causes a 15% and 20% difference in the lunar *Tide Raising Force*; thus, *Tides* at *Perigee* are usually appreciably higher than those at *Apogee*.

1016. The Earth-Sun System

a. **Earth-Sun Barycentre.** The Earth and Sun form another independent *Tide Raising* system rotating around the Earth-Sun *Barycentre*. Similarly to the Earth-Moon system (see Para 1011), the Earth-Moon *Barycentre* describes an elliptical orbit around the Earth-Sun *Barycentre* (a point located inside the Sun) - see Fig 10-13 (below). It takes one year (about 365¼ days) for the Earth to complete one orbit around the Sun.

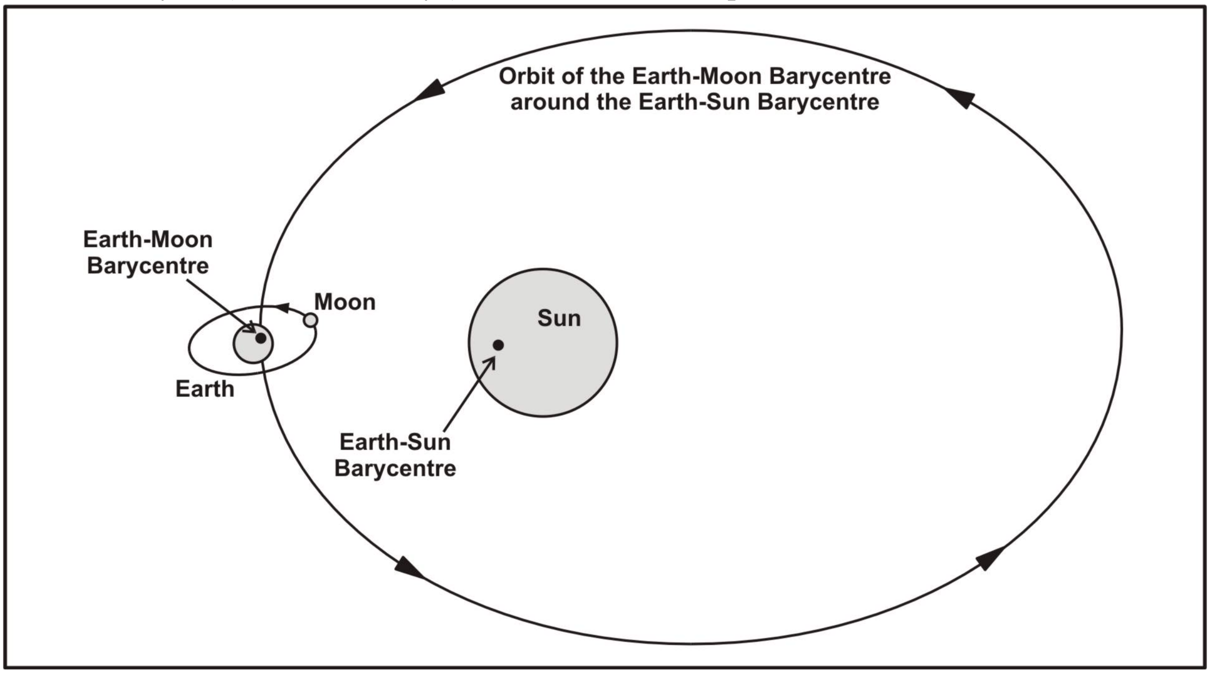


Fig 10-13. The Earth-Sun System (*not to scale*)

(1016) b. **Magnitude of the Sun's Tide Raising Force.** Although the Sun has a much greater mass than the Moon, the Sun's *Tide Raising Force* is only about 45% that of the Moon. This is because the *Tide Raising Force* is inversely proportional to the cube of the distance between the bodies [see formula (10.2)].

c. **Effect of the Sun's Tide Raising Force.** Although of lesser magnitude, the *Tide Raising* effects of the Sun on the Earth are similar to those of the Moon. Thus, *Tides* caused by the Sun will vary according to the following parameters:

- **Earth's Rotation.** The *Solar Day* is approximately 24 hours; thus when the Sun's *Declination* is zero, the *Semi-Diurnal* '*Solar Equilibrium Tide*' will have two HWs 12 hours apart, interspersed with two LWs also 12 hours apart. The time interval between successive HWs and LWs will be 6 hours.

- **Change of Sun's Declination.** The Sun's *Declination* changes much more slowly than that of the Moon and reaches a maximum of about 23½° North and South of the *Equator* on about 22nd June and 22nd December respectively, these dates being known as the *Solstices* (see details at BR 45 Volume 2).

- **Distance of the Sun.** It takes the Earth about 1 year (approx 365¼ days) to complete its elliptical orbit around the Sun. *Perihelion*, when the Earth is closest to the Sun, occurs on about 2nd January, and *Aphelion,* when the Earth is furthest away, occurs on about 1st July. Thus, the Sun's *Tide Raising* Force will be at its maximum in January and at its minimum in July. However, this variation in magnitude is very small, in the order of 3%.

1017. Springs and Neaps

a. **Spring Tides.** Twice every *Lunar Month*, the Earth, Moon and Sun are in line with each other when viewed in the *Ecliptic Plane* (see Fig 10-14 below). At '*New Moon*', the Moon is passing between the Sun and the Earth (which is not visible to an observer on Earth - effectively a 'black' Moon); the Moon's and Sun's *Tide Raising Forces* are thus working in '*Conjunction*'. About 14¾ days later, at '*Full Moon*' (when the Moon is seen as a bright full disc), the Earth is between the Moon and Sun; the Moon's and Sun's *Tide Raising Forces* are thus working in *'Opposition'*. The net result in both cases is a maximum *Tide Raising Force*, producing what are known as *Spring Tides*, when higher HWs and lower LWs than usual will be experienced; *Spring Tides* occur shortly after *New Moons* and *Full Moons* take place (see Para 1017d opposite). Details of the Moon's phases are at BR 45 Volume 2.

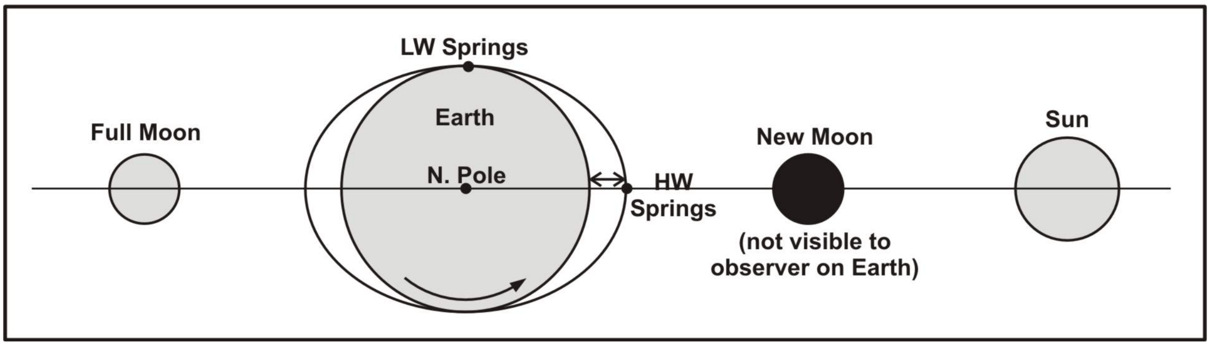


Fig 10-14. Spring Tides (*not to scale*)

(1017) b. **Neap Tides.** Twice every *Lunar Month* (ie about every 14¾ days), the Moon and Sun are at 90˚ to each other (see Fig 10-15 below). At these times the Moon and Sun are said to be in *Quadrature.* This occurs when the Moon is in the *First Quarter* and *Last Quarter*, and at this time the Moon's and Sun's *Tide Raising Forces* are working at 90˚ to each other. The net result in both cases is a minimum *Tide Raising Force*, producing what are known as *Neap Tide*s when lower HWs and higher LWs than usual will be experienced; *Neap Tide*s occur shortly after *First* and *Last Quarters* of the Moon take place (see Para 1017d below). Details of the Moon's phases are at BR 45 Volume 2.

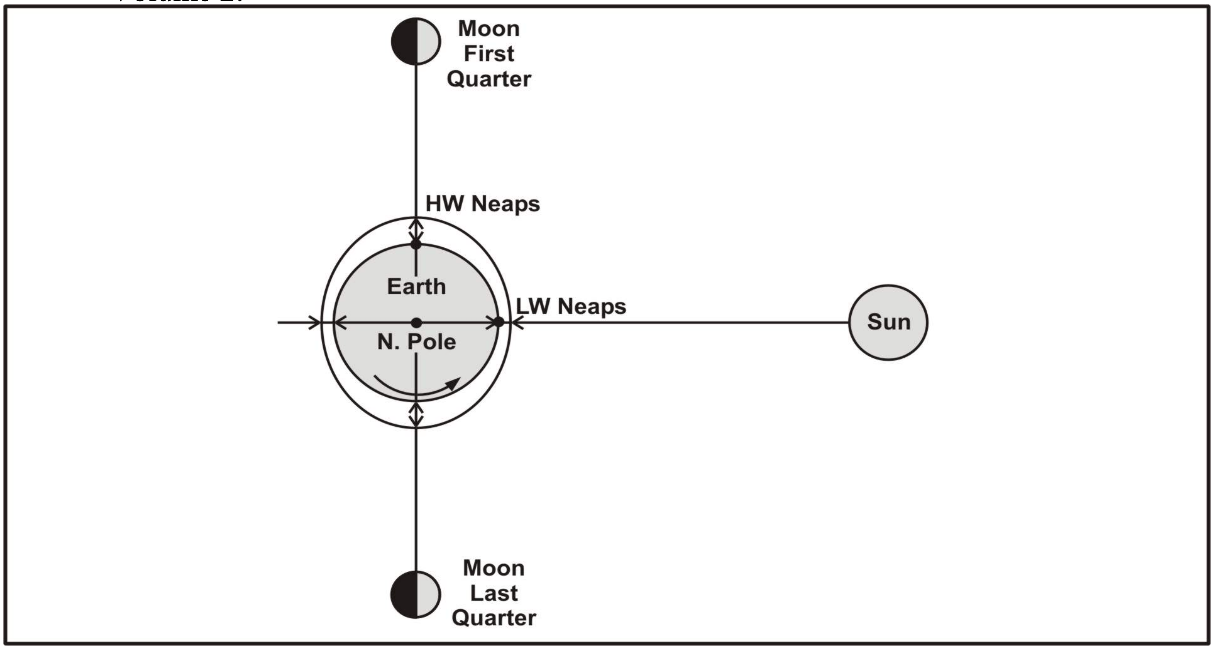


Fig 10-15. Neap Tides (*not to scale*)

c. **Frequency of Springs and Neaps.** Two *Spring Tides* thus occur each *Lunar Month* interspersed with two *Neap Tides*, the interval between successive *Spring* and *Neap Tides* being about 7½ days. This occurs at many places in the world, although other inequalities sometimes alter these timings.

d. **Timing of Springs and Neaps.** *Spring* and *Neap Tides* usually follow the relevant phase of the Moon by 2 or 3 days. This is because there is always a time-lag between the action of the force and the reaction to it, caused by the time taken to overcome the inertia of the water surface and friction. *Spring* and *Neap Tides* will occur at approximately the same time of day at any particular place, since the Moon at that time is in a similar position relative to the Sun.

e. **Equinoctial and Solstitial Declinations.** When the *Declinations* of the Moon and the Sun are the same, their *Tide Raising Forces* act more in concert than when the *Declinations* are different. However, as stated at Para 1015c, the Moon's *Declination* changes rapidly over a 4 week period. It can be at any value at the actual *Equinox* or *Solstice*, although it is bound to reach zero or maximum *Declination* respectively within a few days.

f. **Equinoctial Tides.** At the *Equinoxes* (March and September), when the *Declinations* of Moon and Sun are both zero, the *Semi-Diurnal ‘Luni-Solar’ Tide Raising Force* will be at its maximum, thus causing *Equinoctial Tides*. At these times *Semi-Diurnal Spring Tides* are normally higher than other *Spring Tides*.

(1017) g. **Solstitial Tides.** At the *Solstices* (June and December), when the *Declinations* of Moon and Sun are both maximum, the *Diurnal* *'Luni-Solar' Tide Raising Force* is at its maximum, thus causing the *Solstitial Tides*. At these times *Diurnal Tides* and the *Diurnal Inequality* are at a maximum.

h. **Priming and Lagging.** It was explained at Para 1014 that the effect of the Earth's rotation and that of the Moon relative to each other is to cause a HW at intervals of about 12 hours 25 minutes. The effect of the Earth's rotation and that of the Sun relative to each other is to cause a (smaller) HW at intervals of about 12 hours. Thus, when the effects of both Moon and Sun are taken together, the intervals between successive HWs and LWs will be altered. When the Moon is in a position between *New Moon / Full Moon* and *Quadrature*, the Sun's effect will be to cause the time of HW either to precede the time of the Moon's transit of the *Meridian* or to follow the time of the Moon's transit (see Fig 10-16 below); this is known as *Priming* and *Lagging*.

- **Priming.** The *Tide* is said to *'Prime'* between the *New Moon* and the *First Quarter*, and between *Full Moon* and the *Last Quarter*; HW then occurs before the Moon's transit of the *Meridian*.
- **Lagging.** The *Tide* is said to *'Lag'* between the *First Quarter* and *Full Moon*, and between the *Last Quarter* and *New Moon*; HW then occurs after the Moon's transit of the *Meridian*.

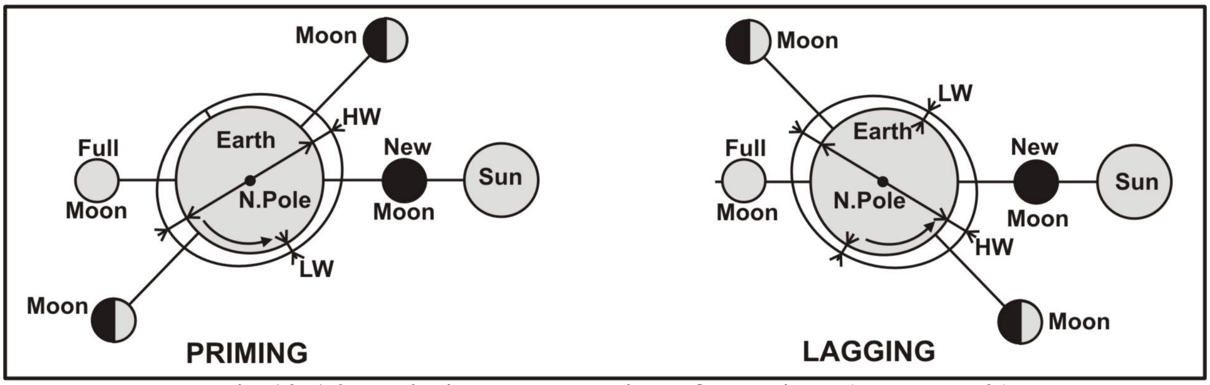


Fig 10-16. Priming and Lagging of the Tides (*not to scale*)

1018. Summary of Tidal Theory

a. **Tide Raising Forces.** *Semi-Diurnal Tide Raising Force* is maximum when the Moon's *Declination* is zero and minimum when it is greatest. *Diurnal Tide Raising Force* is zero when the Moon's *Declination* is zero and maximum when it is greatest. The Sun's *Declination* has a similar effect, but with a different period. Changes in the Sun's and Moon's distances from the Earth each cause variations in the *Tide Raising Force*. The Sun's *Tide Raising Force* is about 45% of the Moon's.

b. **Spring and Neap Tides.** *Spring* and *Neap Tides* occur at intervals of about 14 days, caused by the Moon and Sun either working together at *Full Moon* / *New Moon* (*Springs*) or against each other at *First* and *Last Quarters* (*Neaps*).

1019. Spare

SECTION 2 - THE TIDES IN PRACTICE

1020. Diurnal and Semi-Diurnal Tides

In practice, actual *Tides* may differ considerably from theoretical *Luni-Solar Equilibrium Tides* (see Paras 1013-1018). This is because of the size, depth and configuration of the ocean basins, land masses, the friction and inertia to be overcome in any particular body of water, together with other complicating factors.

a. **Tidal Waves.** For an appreciable *Tide* to be raised in a body of water, it is essential to generate a large enough *Tide Raising Force*; to achieve this, the body of water must be large. The great oceans of the world - the Pacific, Atlantic and Indian Oceans - are large enough to permit *Tides* to be generated, although *Tides* do not appear as a single *Tidal Wave* form but rather as the sum of a number of oscillating *Tidal Wave* forms.

b. **Diurnal, Semi-Diurnal and Mixed Tides.** The natural period of *Tidal Wave* oscillation is the decisive factor in determining whether the body of water responds to *Diurnal* or *Semi-Diurnal Tide Raising Forces,* or a mixture of the two. Hence, *Tides* in practice are often referred to as being *Semi-Diurnal*, *Diurnal* or a mixture of both.

- **Atlantic Ocean.** The Atlantic is more responsive to *Semi-Diurnal Tide Raising Forces*; thus *Tides* on the Atlantic coast and around the British Isles tend to be *Semi-Diurnal* in character (ie two HWs and two LWs per day) and are more influenced by the phases of the Moon than by its *Declination*. ‘Large’ *Springs Tides* occur near *Full* or *New Moons* with ‘small’ *Neap Tides* near the *First* and *Last Quarters*. The largest *Tides* of the year occur at *Springs* near the *Equinoxes* when the *Declinations* of the Sun and Moon are both zero (ie they are over the *Equator*).

- **Pacific Ocean.** The Pacific is generally more responsive to the *Diurnal Tide Raising Forces*, and so *Tides* here tend to have a large *Diurnal* component. In these areas, the largest *Tides* are associated with the greatest *Declination* of Sun and Moon (ie at the *Solstices*). Areas in the West Pacific off New Guinea, Vietnam and in the Java Sea are predominantly *Diurnal* with one single HW and LW per day; on the North / East coasts of Java, *Tides* are purely *Diurnal*.

- **Mixed Tides.** *Mixed Tides*, where *Diurnal* and *Semi-Diurnal Tide Raising Force*s are both important, tend to be characterised by a large *Diurnal Inequality* (see Para 1015, Fig 10-11). This may be apparent in the heights of successive HWs, LWs or both; such *Tides* are common along the Pacific coast of the USA, the East coast of West Malaysia, Borneo, Australia and the waters of South-West Asia. Occasionally, the *Tides* may even be purely *Diurnal*.

- **Mediterranean and Baltic Seas.** The bodies of water of the Mediterranean and Baltic Seas are too small to enable any appreciable *Tide* to be generated.
  - **Strait of Gibraltar.** The Strait of Gibraltar is too restricted to allow the Atlantic *Tides* to have any appreciable effect other than at its extreme Western end.
  - **Adriatic Sea.** The greatest *Tides* are to be found in the Adriatic Sea, where they are predominantly mixed, with a *Diurnal Inequality* at both HW and LW water. The (tidal) *Range* may exceed 0.5 metre in several places in the Adriatic, but is rarely greater than 1 metre.

1021. Shallow Water and Other Special Effects

a. **Oscillating Tidal Wave Distortions.** *Tides* travel as oscillating *Tidal Waves* (see Para 1020a), and as they enter shallow water, they slow down. The trough is retarded more than the crest; thus there is a progressive steepening of the wave front accompanied by a considerable increase in the wave height (*Amplitude*). This distorts the timing, so that the period of rise becomes shorter than the period of fall. These *Shallow Water Effect*s are present to a greater or lesser degree in the *Tides* of all coastal waters.

b. **Estuaries.** The *Amplitude* (height) of the *Tidal Wave* increases even more if it travels up an estuary which narrows from a wide entrance. This may result in very large *Tides* such as those to be found in the Bay of Fundy (Nova Scotia), the Severn Estuary (UK) and around the Channel Islands (UK).

c. **Tidal Bores.** Where a river is fed from an estuary with a large (tidal) *Range*, a phenomenon known as a (tidal) '*Bore*' (Old English - '*Eagre*') may be found. The crest of the rising *Tide* overtakes the trough and tends to break. Should it break, a (tidal) *Bore* occurs in which half or more of the total rise of *Tide* occurs in only a few minutes. Notable (tidal) *Bores* are in the Rivers Severn (UK), Seine (France), Hooghly (India - West Bengal) and Chien Tang Kiang (China - Hangzhou,Yangtze Delta).

d. **Double HWs / LWs.** At certain places, *Shallow Water Effects* are such that more than 2 HWs or 2 LWs may be caused in a day. In UK, at Southampton (see Fig 10-17 below), double HWs occur with an interval of about 2 hours between them; further west, at Portland, double LWs occur (see Fig 10-18 below). Double HWs / LWs also occur on the Dutch coast and at other places. The practical effect of this is to create a longer *'Stand'* at HW / LW (a *'Stand'* is defined as the period at HW or LW between the *Tide* ceasing to rise / fall and starting to fall / rise, respectively).

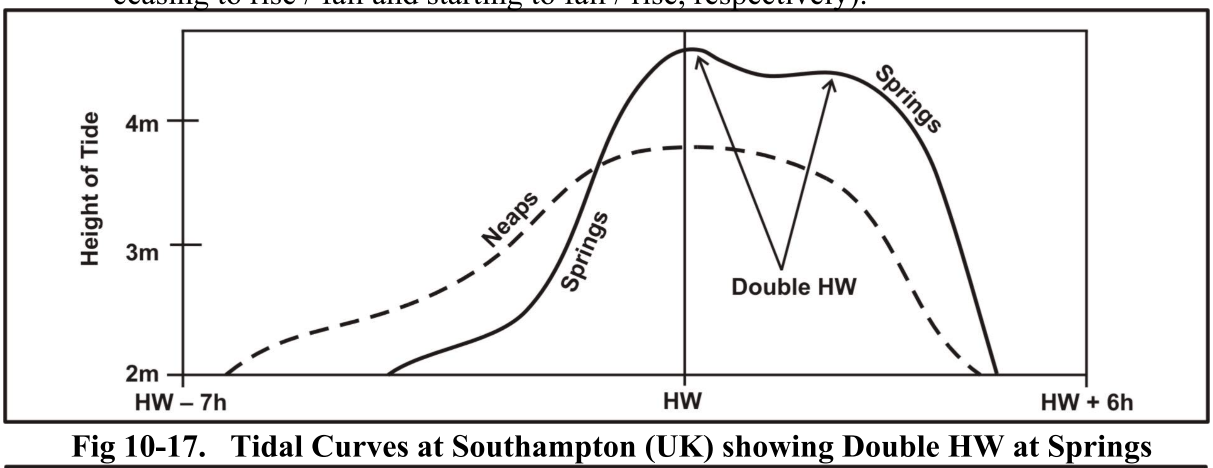


Fig 10-17. Tidal Curves at Southampton (UK) showing Double HW at Springs

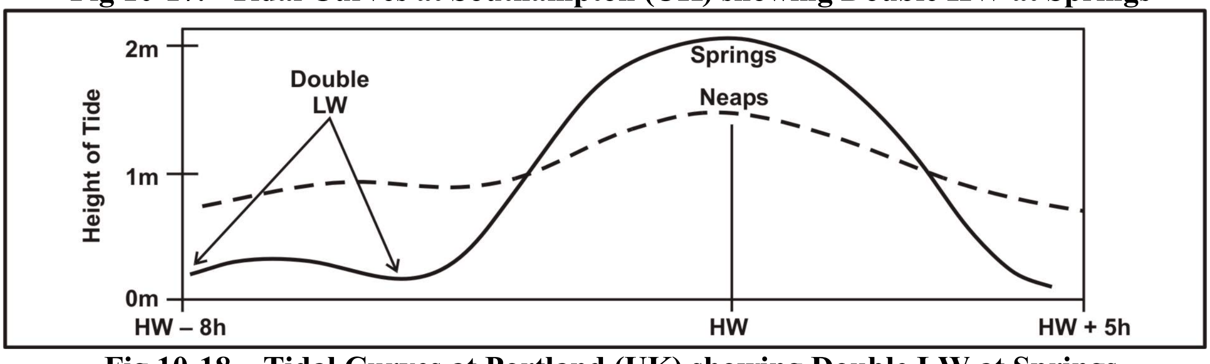


Fig 10-18. Tidal Curves at Portland (UK) showing Double LW at Springs

(1021) e. **Special Tidal Curves - Southern UK.** Because of the distortion of *Tidal Waves* caused by *Shallow Water Effects*, special curves based on LW for determining the *Height of Tide* (*HOT*) on the south coast of UK between Swanage and Selsey are contained in Admiralty Tide Tables Volume 1 (NP 201). The tidal curve at Southampton (a *Standard Port*) is also based on LW because of the complexity of the *Tides* around HW.

1022. Meteorological Effects on Tides

Non-standard meteorological conditions can substantially affect actual *Tides* experienced, as compared to predicted *Tides*. The effect of such conditions is complex and difficult to forecast, even with the resources of *National Hydrographic Offices* (*NHOs*) and Weather Centres. Apparently 'unexplained' changes to *Tides* can occur (eg the effect of a storm-centre hundreds of miles away, in otherwise benign conditions) and mariners should be aware that tidal predictions are indeed only 'predictions'; an adequate margin of safety should always be allowed when planning *Underkeel Clearances*. Some of the main meteorological conditions affecting *Tides* are as follows.

a. **Barometric Pressure.** Tidal predictions are computed for average barometric pressure in the local area; a 34 millibar change can cause a difference in *HOT* of about 0.3 metres. Low barometric pressure will tend to raise sea level and high barometric pressure will tend to depress it. The water level does not adjust itself immediately to a change of pressure and it responds to the average change in pressure over a considerable area. Changes in sea level due to barometric pressure rarely exceed 0.3 metres but when other factors are added, this effect can be important.

b. **Wind.** The effect of wind on *HOT* and HW / LW times is very variable; it depends largely on the topography of the area. In general, wind will raise the sea level in the direction to which it is blowing. A strong onshore wind piles up the water and causes HWs to be higher than predicted; this can have substantial effect in harbours (eg in Portsmouth [UK], where strong onshore winds with low barometric pressure can increase the *HOT* by up to 1.0 metre). Offshore winds have the reverse effect, drawing water away from a coastline, making LWs lower than predicted. Winds blowing along the coast tend to set up long *Tidal Waves* which travel along the coast, raising sea level where the crest of the *Tidal Wave* appears and lowering sea level in the trough.

c. **Seiches.** Abrupt changes in meteorological conditions (eg passage of an intense depression or line squall) may cause an oscillation in the sea level known as a *Seiche*. The period between successive *Tidal Waves* may vary from a few minutes to about 2 hours and the height of the *Tidal Waves* may vary from 1 centimetre to 1 metre.

d. **Positive and Negative Surges - Summary.** A rise in sea level superimposed on the normal tidal cycle, caused by a combination of wind and pressure or other factors, is referred to as a *Positive Surge* and a fall as a *Negative Surge*. Both *Positive* and *Negative Surges* may alter the predicted times of HW / LW, often by as much as 1 hour.

e. **Positive Surges.** *Positive Surges* have the greatest effect when confined to a gulf or bight (eg the North Sea [UK]). Unless they are *Storm Surges* (see Para 1022g), they rarely increase sea level height by more than 1 metre. In a bight (eg the North Sea), northerly winds will raise sea level at the southern end, causing a *Positive Surge*.

(1022) f. **Negative Surges.** *Negative Surges* are of great importance to vessels navigating with small *Underkeel Clearances*. *Negative Surges* are most evident in estuaries and areas of shallow water, and typically occur when strong winds blow water away from shore over a significant *Fetch* of shallow water. Falls in sea level of up to 1 metre are common, while 2 metres have been recorded. In the North Sea (UK), strong Southerly winds will lower sea level at the southern end. *Negative Surges* may also occur due to *Storm Surges*. *Negative Surge* warnings may be given in the Southern North Sea, Thames Estuary and Dover Straits from 6-12 hours (possibly up to 30 hours) ahead.

g. **UK Storm Surges.** *Storm Surges* in the North Sea are wave forms which occur if an intense depression with storm force winds sets up a wave running down the UK coast at a speed similar to that of the *Tidal Wave.* The *Tidal Wave* is reinforced by the storm and increases in amplitude, reaching up to 3 metres in height. If a *Storm Surge* crest coincides with HW *Springs*, a strong *Positive Surge* is created; flooding and damage may be caused along the coastline (see Note 10-3 below). If the trough of a *Storm Surge* coincides with LW *Springs*, a strong *Negative Surge* occurs. Lesser *Negative Surge*s can occur at any part of the tidal cycle, thus reducing *Underkeel Clearances.*

Note 10-3. *In1953 a Storm Surge in the Southern North Sea (UK) raised sea level by 2.7 to 3 metres. This coincided with HW Springs and caused severe flooding in East Anglia, Kent and London, with of loss of over 300 lives; as a result, the 'Thames Barrage' was built across the River Thames east of London and other flood defences were improved. In 2007, a Storm Surge of 2.8 metres occurred in the same area but did not quite coincide with HW Springs; it was held back by the improved flood defences. Storm Surges are also likely in the Bay of Bengal.*

1023. Seismic Waves (Tsunamis)

Tsunamis (often incorrectly called '*Tidal Waves*') are groups of *Seismic Waves* with a very high wave speed (300 to 500 knots) and are entirely unconnected with *Tides*; they are formed by seismic action (earthquake or 'seaquake') on the ocean floor. These eruptions are concentrated at the boundaries of *Tectonic Plates* and Japan is particularly vulnerable to them, although they can affect anywhere with an uninterrupted *Fetch* to a *Tectonic Plate* boundary. They cause great damage and loss of life ashore, and are a serious hazard to coastal shipping and ships in port. See NP 100 (The Mariners Handbook) and BR 45 Volume 6 Chapter 6.

a. **Open Ocean.** At the epicentre, *Tsunamis* have a wave height under 1 metre and a wave length of over 100 miles, so are largely undetectable in the open ocean.

b. **Shallow Water / Shore.** On entering shallow water, the waves become shorter and higher, reaching a wave height of up to 17 metres when they strike the shore. The first indication of a *Tsunami's* approach may be a sudden drop in sea level. A group of waves may then strike at intervals of 10 to 40 minutes ; the second and third waves are usually higher than the first, with the rest gradually decreasing over an extended period.

Note 10-4. *Satellite observation of the 2004 Tsunami which devastated Indian Ocean coastlines with the loss of over 250,000 lives, established that in open ocean the wave height was 0.9 metres with a wave speed of 450 knots. On approaching shallow coastal water, wave height increased to 10 metres while the wave speed reduced to 20 knots.*

1024-1029. Spare

SECTION 3 - TIDAL HARMONICS AND SHM FOR WINDOWS

1030. Harmonic Constituents

a. **Scope of Harmonic Constituents.** The *Tide Raising Force*s comprise a large number of *Harmonic Constituent* cosine curves, the periods and relative *Amplitude*s of which can be calculated from astronomical theory. Some 400 *Harmonic Constituents* have been identified but in practice it is unnecessary to use so many. Up to 160 *Harmonic Constituents* are used for major *Standard Ports* and up to 36 for *Secondary Ports*. The *Harmonic Constituents* are given symbols from which their general significance may be deduced (eg the letter *M* is used for lunar constituents, *S* for solar constituents, the subscript $_1$ for *Diurnal* and the subscript $_2$ for *Semi-Diurnal* components).

b. **Tidal Observations.** To predict with accuracy the *HOT* at any place, extensive tidal observations must be carried out and the results analysed to identify a number of *Harmonic Constituents* making up the *Tide Raising Force*s at that place. Due to the various cycles involved, a period of 18.6 years, equal to the longest cycle, is desirable if all the necessary *Harmonic Constituents* are to be identified. However, adequate predictions may be made over shorter periods, as follows:

- **Admiralty Tide Tables - Standard Ports.** For *Standard Port* predictions in the Admiralty Tide Tables (NPs 201-204), the general rule is for at least 1 complete year's observations to be analysed. This permits the identification of up to 160 *Harmonic Constituents*.

- **Admiralty Tide Tables - Secondary Ports.** For *Secondary Ports*, analysis of at least 1 month's observations is the aim, as this permits the identification of up to 36 *Harmonic Constituents* (see Para 1031 below).

1031. Principles of Harmonic Tidal Analysis

a. **Principal Harmonic Constituents.** The four principal *Harmonic Constituents* with which the user will come into contact are:

- **M_2** - is the principal *Lunar Semi-Diurnal Harmonic Constituent*, which permits calculations of the *Amplitude* caused by a theoretical Moon in circular orbit around the Earth at the average speed of the real Moon, halfway between *Apogee* and *Perigee* and at an average Northerly or Southerly *Declination*.

- **S_2 -** is the principal *Solar Semi-Diurnal Harmonic Constituent*, which permits calculation of the *Amplitude* caused by a theoretical Sun in similar circumstances to that for the Moon (at M_2 above).

- **K_1** - is the *Luni-Solar Declinational Diurnal Harmonic Constituent*, which allows for part of the Moon's and Sun's *Declinations*.

- **O_1** - is the Lunar *Declinational Diurnal Harmonic Constituent*, which allows for the remainder of the Moon's *Declination*.

(1031) b. **Components of Harmonic Constituents.** Each *Harmonic Constituent* has a speed, (°/ hour, when 1 cycle = 360°), an *Amplitude 'H'* and a *Phase Lag 'g'*. Cycle and speed details for the 4 main *Harmonic Constituent* components are at Table 10-1 (below).

Table 10-1. Cycle and Speed Details for the 4 Main Harmonic Constituents

Harmonic Constituent	Cycles per Day	Speed (Degrees per Hour)	Time to Complete 1 Cycle
M_2	2	28°.98	12 h 25 min
S_2	2	30°	12 h 00 min
K_1	1	15°.04	23 h 56 min
O_1	1	13°.94	25 h 50 min

c. **Harmonic Constants.** The *Amplitude 'H'* and *Phase Lag 'g'* of a *Harmonic Constituent* are known as the <u>*Harmonic Constants*</u> of that *Harmonic Constituent*.

- **Amplitude 'H'.** The *Amplitude 'H'* is equal to half the (tidal) *Range* (ie half of HW minus LW heights for each oscillation).

- **Phase Lag 'g'.** The phase of a *Harmonic Constituent* is its position in time, in relation to its theoretical position as deduced from astronomical theory. *Tide Raising Force*s do not act instantaneously (see Para 1017d), thus each *Harmonic Constituent* has a (time) *Phase Lag 'g'*.

d. **Tidal Analysis and Predictions.** The purpose of tidal analysis is to determine the <u>*Harmonic Constants*</u> for a particular location (ie *Amplitude 'H'* and *Phase Lag 'g'*). Tidal predictions are then made using an appropriate number of *Harmonic Constituents*. In many places (eg Portsmouth [UK]), the *Harmonic Constituents* for *Shallow Water Effect*s (see Para 1021) are very complex and extra *Shallow Water Corrections* are applied. The authority for the observations, *Harmonic Constants*, predictions, method of prediction and year of observation are in the Admiralty Tide Tables (NPs 201-204).

1032. Simplified Harmonic Method - 'SHM for Windows®' Software (DP 560)

'*SHM for Windows®*' is a simple Windows-based tidal prediction program using the *UKHO's Simplified Harmonic Method* (*SHM*) of tidal prediction. It is supplied by *UKHO* on a CD-ROM as DP 560. A variation of *SHM* is also published in the Admiralty Tide Tables (NPs 201-204) for those who wish to use it with a pocket calculator.

a **Operation.** '*SHM for Windows®*' is safe, accurate, fast and user-friendly. The user is required to input *Harmonic Constants* and *Shallow Water Corrections*, principally from the 'Admiralty Tide Tables' (NPs 201-204) or 'Tidal Harmonic Constants - European Waters' (NP 160). Predictions are then displayed as a graph of heights against time, for up to 24 hours and up to 7 consecutive days; these results may be printed.

b. **Program Duration and Storage of Data.** The calculation software is everlasting, but does require up to date *Harmonic Constants* to be stored or input. Data for any number of ports or *Tidal Streams* may be stored within the software.

1033-1039. Spare.

SECTION 4 - TIDAL STREAMS AND CURRENTS

1040. Primary Definitions and Types of Tidal Streams

a. **Primary Definitions.** The terms '*Tides*' (vertical), *'Tidal Streams'* (horizontal), *Currents* and the American usage *'Tidal Currents'* are frequently mixed up; their definitions are stated at Para 1002, but are repeated for the convenience of readers.

(Extract from Para 1002):

- **Tides.** '*Tides*' are **periodic vertical reversing** movements of the water on the Earth's surface, caused by *Tide Raising Forces* of the Moon and Sun.
- **Tidal Streams.** *'Tidal Streams'* are the **periodic horizontal reversing** movements of the water accompanying the vertical rising and falling of *Tides* .
- **Currents.** Ocean *Currents* are **non-tidal** movements of water, which may flow **steadily** at all depths in the oceans and may have both horizontal and vertical components; a *Surface Current* can only have a horizontal component. In rivers and estuaries, there is often a permanent *Current* caused by the flow of river water.
- **Tidal Currents.** *'Tidal Currents'* is the American usage for *'Tidal Streams'*. The term *'Tidal Currents'* is NOT used in this book.

b. **Types of Tidal Stream.** *Tidal Stream*s are of two types: *'Rectilinear'* and '*Rotary*'.

- **Rectilinear Tidal Streams.** *Rectilinear Tidal Streams* have only two directions (with small variations), which may be called the *'Flood*' (the incoming *Tidal Stream*) or the *'Ebb*' (the outgoing *Tidal Stream*). Ideally, instead of the terms '*Flood*' and '*Ebb*', the water flow should be described by its direction (eg East-going or 090˚), with its rate (eg 090˚ 2.0 kn).
- **Rotary Tidal Streams.** *Rotary Tidal Streams* continually change in direction; they rotate through 360˚ in a complete cycle. The rate of the *Tidal Stream* usually varies throughout the cycle, with two maxima in approximately opposite directions interspersed with two minima about halfway between the maxima in time and direction.

c. **Occurrences of Tidal Stream Types.**

- **Rectilinear Tidal Streams.** *Rectilinear Tidal Streams* are normally found in port approaches, estuaries, channels and straits, where the direction of the flow of the *Tidal Stream* is constricted by the surrounding land and shoals.
- **Rotary Tidal Streams.** *Rotary Tidal Streams* are normally found offshore where constraints to the water flow are absent.

d. **Semi-Diurnal and Diurnal Components of Tidal Streams.** *Tidal Streams* may have *Semi-Diurnal* and *Diurnal* components, including *Diurnal Inequality*, and can be analysed harmonically or non-harmonically. In European waters (where *Tides* are *Semi-Diurnal*) *Tidal Stream* rates are usually related to the *Range* of the *Tide,* and the times of 'Slack Water' are usually related to (but not necessarily the same as) the times of HW and LW at the nearest *Standard Port* (eg In UK, 'Slack Water' occurs at 'Half Tide' on the East coast but at HW / LW on the South coast). Significant differences in the times of 'Slack Water' may occur between ports / harbours and adjacent offshore areas.

1041. Currents

The causes of *Tides* and *Tidal Streams* due to *Tide Raising Forces* have already been fully explained at Paras 1002-1022 . However, *Currents* (non-tidal movements of water) are caused by a variety of factors quite separate from *Tide Raising Forces*.

a. **Causes of Currents.** *Currents* are caused by meteorological factors (eg wind and barometric pressure), oceanographic factors (eg water of differing sea levels, salinity and/or temperatures) and by topographical factors (eg irregularities in the sea-bed, run-off of water from the land in rivers and estuaries). A detailed explanation of ocean *Current* types (ie *Drift Currents* and *Gradient Currents*) and a summary of the characteristics of the principal ocean *Currents* is at Chapter 11 (Paras 1120-1125).

b. **Causes of River Currents.** In rivers and estuaries, there is often a permanent (but variable) *Current* caused by the flow of river water from the land. The flow of river water in such *Current*s is heavily dependent on rainfall inland.

c. **Assessment of Currents for Chartwork.** The practical assessment of the rate and direction of *Currents* for chartwork is at Chapter 7.

1042. Tidal Stream Data, Atlases and Observations

a. **Semi-Diurnal Tidal Streams.** *Semi-Diurnal Tidal Streams* (eg European waters) may be predicted from *Mean High Water Springs (MHWS) / Mean High Water Neaps (MHWN)* at a *Standard Port*. *Tidal Stream* predictions are displayed in tables on the chart, in *ENC* databases and in *Tidal Stream Atlases* (see Fig 10-19 overleaf), showing the rate and direction at *MHWS / MHWN*, by reference to the time of HW at a suitable *Standard Port*. The rate on occasions other than *MHWS / MHWN* may be found by using the (tidal) *'Range of the Day*' to interpolate or extrapolate from the two mean rates (see Para 1045), thus avoiding the need for date-specific predictions to be published.

b. **Tidal Stream Atlases.** Where the *Tidal Stream* is related to a *Standard Port* (see Para 1042a above), *Tidal Stream Atlases* show *Tidal Streams* in pictorial form (see Fig 10-19 overleaf); they are available from *UKHO* (with instructions for their use) for the waters around UK and the west coast of France. RN / RFA vessels have access to more detailed (classified) *Tidal Stream Atlases* and guidance for HM Naval Bases.

c. **Admiralty Sailing Directions (Pilots).** Limited *Tidal Stream* information is also contained in *Admiralty Sailing Directions* (*Pilots*).

d. **Tidal Streams with Large Diurnal Inequality.** Where the *Diurnal Inequality* of the *Tidal Stream* is large (eg Malacca and Singapore Straits), the procedure at Para 1042a (above) is not possible and individual date-specific predictions are needed.

- **Tidal Stream Tables.** Daily *Tidal Stream* predictions for important areas are published as 'Tidal Stream Tables' in Volumes 3 and 4 of the Admiralty Tide Tables (NPs 203-204).
- **TotalTide®.** *UKHO's TotalTide*® software can predict all *Tidal Streams* with the integrity of the Admiralty Tide Tables; see details at Para 1051.
- **SHM.** *Harmonic Constants* for some *Tidal Streams* are also published in Volumes 2, 3 and 4 of the Admiralty Tide Tables (NPs 202-204) so that predictions may be made using *SHM for Windows*® software; see Para 1032.

(1042) e. **Tidal Stream Observations and Predictions.** *Tidal Stream* predictions for UK waters are generally based on observations extending over a period of 25 hours, which is a far shorter period than the equivalent observations for *Tide* predictions. Permanent *Curren*ts in rivers and estuaries are included, but for coastal predictions any variable *Curren*t is removed before the predictions are compiled. The observation of *Tidal Streams* presents greater difficulties than the observation of *Tides* and a lesser degree of observational accuracy is achievable for the following reasons:

- **Sea-bed Topography.** Because of the rapidly changing effect of sea-bed topography on the direction and rate of the *Tidal Stream*, it is often impossible to give more than an <u>indication</u> of how a vessel will be affected by *Tidal Stream*s.

- **Channels.** In a channel, the *Tidal Stream* may be running strongly (eg 3 kn) in the centre with virtually no *Tidal Stream* (or even a *Tidal Stream* running in the opposite direction) at the edges of the channel. The *Tidal Stream* may vary significantly (eg from zero to 3 kn) in the navigable part of the channel. Thus *Tidal Stream* predictions for any given position in a channel will be correct for that exact position, but may well be incorrect for a position a few metres either side.

- **Complexity.** While the *Tidal Stream* predictions must be accurate enough for navigational purposes, the methods of prediction to achieve an <u>adequate</u> result are not as complex as those for *Tide* predictions.

f. **International Regulations for Charts, Tide Tables and Tidal Stream Data.** Certain countries and ports may make the carriage and use of specified Tide Tables, *Tidal Stream Atlases,* charts and diagrams compulsory for ships proceeding to and from their ports. Mariners are advised to check the necessary regulations in good time.

1043. Tidal Stream at Depth

Published *Tidal Stream* data normally refers to the uppermost 10 metre layer of the sea. <u>The following guidance is of necessity generalised and local conditions may vary from it.</u>

a. **0% to 75% of Depth of Water.** Except for areas fed by river water in addition to the *Tides* (see Para 1043c below), *Tidal Streams* at depths below 10 metres tend to be very similar to those on the surface to a depth of about 75% of the total depth of water. However, the times of Slack Water may be different by as much as 1 hour compared with surface Slack Water times; Slack Water at depth is usually early but sometimes late.

b. **75% to 100% of Depth of Water.** At depths greater than 75% of the total depth of water, until about 1 metre above the sea-bed, *Tidal Streams* fall away in strength to a value which may be about 50% to 60% of the surface rate, and also change direction slightly by about 10˚ to 20˚. In the bottom metre to the sea-bed, *Tidal Streams* may undergo a marked change from those on the surface.

c. **Effect of River Water.** The situation at Para 1043a may be quite different in ports which are fed by river water in addition to the *Tides* (eg Devonport [UK]). The strength and direction of the *Tidal Stream* may vary considerably with depth, dependent on the amount of fresh water flowing down-river, and the depth to which it penetrates.

1044. Eddies, Races and Overfalls

a. **Causes of Eddies, Races and Overfalls.** *Eddies, Tide-Rips, Overfalls* and *Races* are different forms of water turbulence caused by any of the following:

- Abruptly changing topography of the sea-bed.
- Configuration of the coastline.
- Constriction of channels.
- Sudden changes in *Tide* or *Tidal Stream* characteristics.

b. **Eddies.** An *Eddy* is a circular movement of water, the diameter of which may extend from a few inches to a few miles (eg at Portland [UK] there is an anti-clockwise *Eddy* of the *Tidal Stream* South East of Portland between 1 and 5 hours after HW Devonport [UK] - see Fig 10-19 below). Where the effect of *Eddies* are of a permanent nature, they are taken into account when predicting *Tidal Streams*.

c. **Overfalls.** An *Overfall* is another name for a *Tide-Rip* and is caused by a strong *Tidal Stream* near the sea-bed being deflected upwards by obstructions on the bottom, thus causing a confused sea on the surface.

d. **Races.** A *Race* is an exceptionally turbulent *Tidal Stream*, usually caused by a strong water flow round a headland or where *Tidal Streams* converge from different directions. The *Tidal Stream Atlas* for Portland [UK] shows an almost permanent *Race* (for 9 out of 12 hours), south of Portland Bill (see Fig 10-19 below).

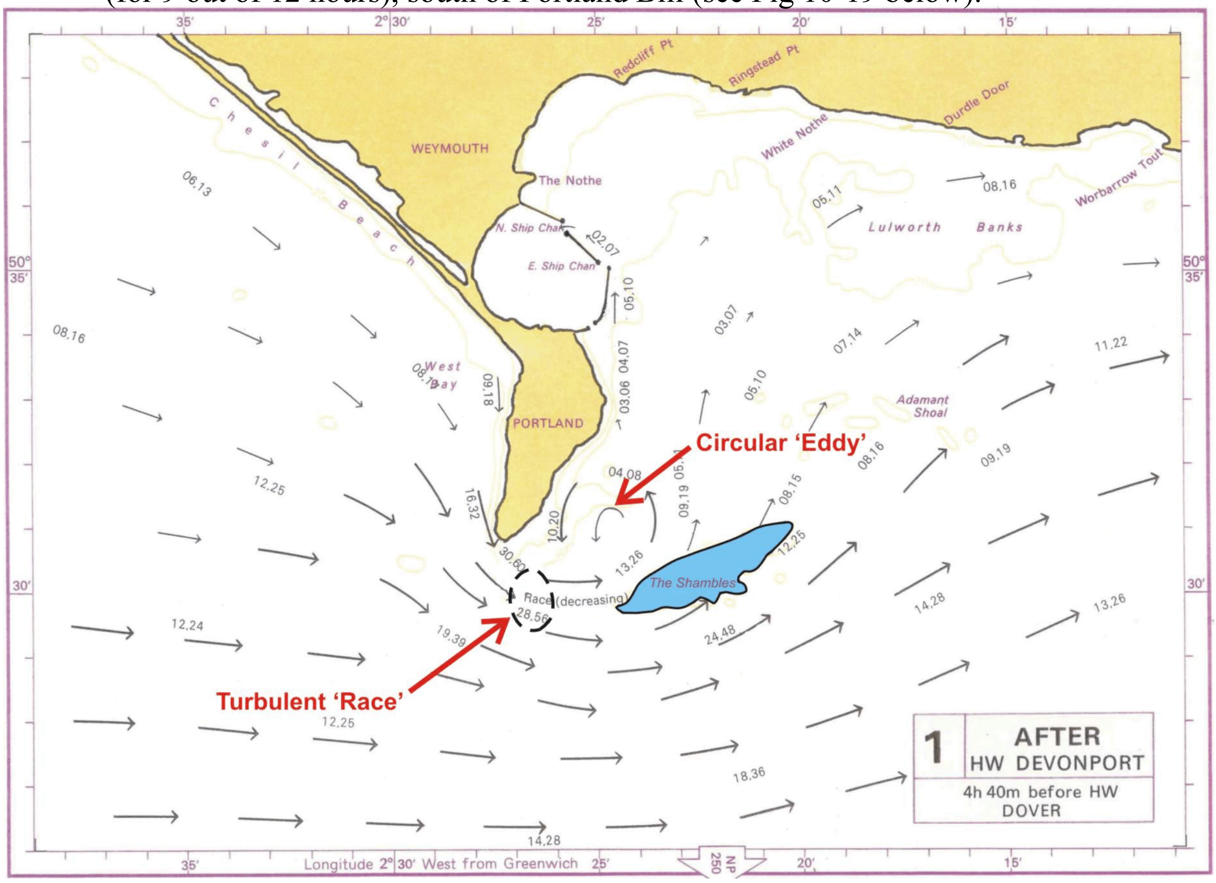


Fig 10-19. Tidal Stream Atlas - showing Circular 'Eddy' (to South East) and Turbulent 'Race' (to South) off Portland - both 1 Hour after HW Devonport (UK)

1045. 'Percentage Springs' (Tides and Tidal Streams) - Calculation and Use

This method applies to _Semi-Diurnal Tidal Streams_ only, but can be adapted for use with reasonable accuracy if moderate *Diurnal Inequality* is present. If large *Diurnal Inequality* is present, *Tidal Stream* predictions must be made iaw Para 1042d. See also the 'Computation of Rates' diagram / worked example in each *Tidal Stream Atlas* (NPs 209-266,337 & 628-636).

a. **Definitions.** *Mean Spring Range (MSR)* and *Mean Neap Range* (*MNR*) are:

- **MSR.** *MSR* is the difference between *MHWS* and *MLWS. Tide Level* definitions are at Para 1062.
- **MNR.** *MNR* is the difference between *MHWN* and *MLWN. Tide Level* definitions are at Para 1062.

b. **Reason for Calculation.** *Semi-Diurnal Tidal Stream* predictions are linked to the *MSR* / *MNR*. In order to calculate the *Tidal Stream* at any particular time and date, it is necessary to establish the tidal '*Range of the Day*' between successive HWs / LWs at the port to which the data is referenced, compare it to the *MSR / MNR* and interpolate or extrapolate appropriately to give a *'Percentage Springs'* (*% Springs*) figure.

c. **Nomenclature.** It is convenient to refer to *MSR* as 100% *Springs* and to *MNR* as 0% *Springs*. The *Range of the Day* may thus be given a '*% Springs'* figure (ie *Percentage of the Day*) by interpolation / extrapolation, and this may be used to interpolate / extrapolate from the *MSR / MNR Tidal Stream* rates given at the *Tidal Stream* diamond or in the *Tidal Stream Atlas*. When the *Range of the Day* exceeds the *MSR* at the port to which the data is referenced, '% *Springs*' may be greater than 100%. Similarly, when the *Range of the Day* is less than *MNR,* '% *Springs*' will be negative (and *Tidal Streams* will be less than the lower figure given on the chart / atlas).

Example 10-1. MSR and MNR for Portsmouth are 3.9m and 1.9m respectively. At Portsmouth the heights of HW and LW are as follows:

18 February 1446 HW 4.2m 2008 LW 1.3
In this case, the '*Range of the Day*' is 2.9m; interpolation by inspection gives *50% Springs*.

27 February 1112 HW 4.7m 1653 LW 0.5
In this case, the '*Range of the Day*' is 4.2m; interpolation by inspection gives *115% Springs*.

2 September 0618 HW 3.7m 1206 LW 2.2
In this case, the '*Range of the Day*' is 1.5m; interpolation by inspection gives *-20% Springs*.

d. **Formula.** Although accurate interpolation or extrapolation by inspection is usually straightforward, '% *Springs*' may be calculated precisely, using the following formula:

$$\% \; Springs = \left(\frac{Range \; of \; Day - MNR}{MSR - MNR} \right) \text{ x } 100 \qquad \textbf{. . . 10.3}$$

Thus in Example 10-1 for 18 February (above), formula (10.3) gives**:**

$$\% \; Springs = \left(\frac{2.9 - 1.9}{3.9 - 1.9} \right) \text{ x } 100 = \left(\frac{1.0}{2.0} \right) \text{ x } 100 = 50\% \qquad \text{. . . (formula 10.3)}$$

Similar treatment may be applied to the other calculations for 27 Feb and 2 Sep.

e. **Use.** Care must be taken to use *% Springs* correctly when extrapolating (ie *50% Springs* indicates a result half-way between the *MNR* and *MSR* rates).

1046. 'Tidal Nurdle' - Construction and Use

This method applies to *Semi-Diurnal Tidal Stream* only, but can be adapted for use with reasonable accuracy if moderate *Diurnal Inequality* is present. If large *Diurnal Inequality* is present, *Tidal Stream* predictions must be made by the methods outlined at Para 1042d.

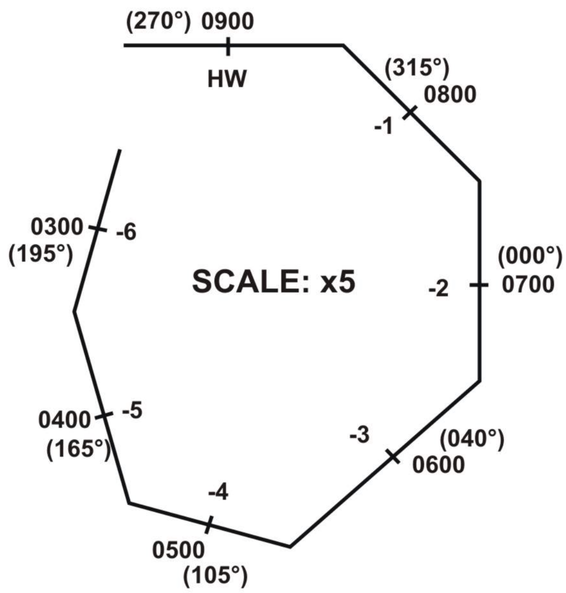
a. **Purpose.** A '*Tidal Nurdle*' diagram may be shown on the chart (at suitable *Scale*) as a representation of the *Tidal Stream* vectors applicable for a particular position over a period of several hours. It is based on *Tidal Stream* diamond or *Tidal Stream Atlas* data, and may be applied to any chartwork to allow instant assessment of future *Tidal Streams* or comparison of the predicted *Tidal Streams* to those actually experienced.

b. **Construction.** A '*Tidal Nurdle*' diagram is constructed as follows (see Figs 10-20 and 10-21 (opposite):

- **Step 1.** Select the *Tidal Stream* diamond on the chart (or *Tidal Stream Atlas* pages) appropriate to the area for which tidal information is required.
- **Step 2.** From the Admiralty Tide Tables or *TotalTide*® software, obtain the time of HW at the *Standard Port* to which the chart refers. From this, determine whether tidal information is required before or after HW. Calculate the '*Percentage Springs'* figure (see Para 1045 - previous page).
- **Step 3.** For the appropriate *Tidal Stream* diamond (or appropriate *Tidal Stream Atlas* data), note the *MSR / MNR* rates and direction:
    - **Interpolate.** Using the '*Percentage Springs*' figure from Step 2, interpolate between *MSR / MNR* rates to obtain the *Rates for the Day*.
    - **Vector Diagram.** In a convenient and suitable place on the chart draw a vector diagram in the direction of the *Tidal Stream*, **using a suitable *Scale Factor* with the *Rates for the Day*.** Annotate the diagram (boldly) with the selected *Scale Factor*. **The length of each vector will be the result of multiplying the *Tidal Stream* rate by the *Scale Factor*, plotted using the distance *Scale* of the chart in use.**
- **Step 4.** At the mid-point of the vector produced in Step 1, annotate the vector with the interval from HW and the direction to which the *Tidal Stream* is flowing. Add a small table summarising the *Rates for the Day*.
- **Step 5.** Repeat Steps 3 and 4 for times before and after HW, noting that:
    - **Times Before HW.** If the diagram is for times before HW, then vectors are plotted backwards from the previous one (see Fig 10-20 opposite).
    - **Times After HW.** If the diagram is for times after HW, then vectors are plotted on from the last one produced (see Fig 10-21 opposite).

c. **Precautions before Use.** The *Scale Factor* of the *Tidal Nurdle* diagram (see Step 3 above) must be taken into account when it is used.

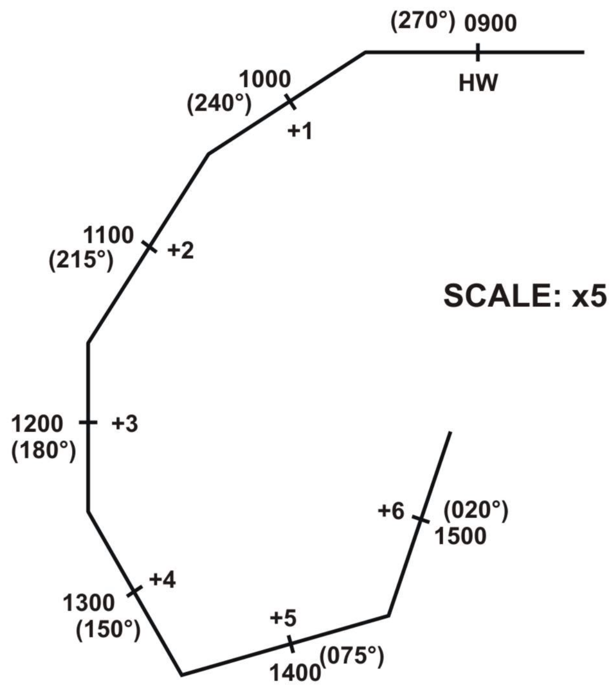
(1046 continued)



Rates for the Day (Interpolated)

HW	270	0.8
-1	315	0.7
-2	000	0.7
-3	040	0.9
-4	105	0.6
-5	165	0.7
-6	195	0.6

Fig 10-20. 'Tidal Nurdle' Vector Diagram - Times Before HW



Rates for the Day (Interpolated)

HW	270	0.8
+1	240	0.7
+2	215	0.8
+3	180	0.6
+4	150	0.7
+5	075	0.8
+6	020	0.7

Fig 10-21. 'Tidal Nurdle' Vector Diagram - Times After HW

Notes:

10-5. Annotation at Mid Point of Vector. *All references to intervals before or after HW, are at the mid-point of each vector produced (see Para 1046b - Step 4 opposite).*

10-6. Precautions before Use - Scale Factor. *Attention is drawn to Para 1046c (opposite).*

1047-1049. Spare

SECTION 5 - ADMIRALTY TIDE TABLES AND ADMIRALTY TOTALTIDE

1050. Using Admiralty Tide Tables

Full instructions for use are included in each Volume of the Admiralty Tide Tables (NP 201-204). The following information provides a summary of these procedures.

a. **Intermediate HOT / Times at Standard Ports.** To find the *HOT* at a *Standard Port* at a given time (see Example 10-2 / Fig 10-22 below), or vice versa:

- **HW / LW Heights.** Plot the heights of HW and LW on the appropriate side of the required time and join by a sloping line.
- **HW Time.** Enter HW time (or LW in certain cases in UK [eg Southampton]) and other times before / after HW to cover the required time / event.
- **Answers.**
  - **For Heights.** From the required time, go vertically to the curves, interpolating between *MSR* and *MNR* curves (but not extrapolating outside them). Then go horizontally to sloping line, thence vertically to height scale and read off the height required.
  - **For Times.** From the required height, go vertically to the sloping line, thence horizontally to the curves, interpolating between *MSR* and *MNR* curves (but not extrapolating outside them). Then go vertically to time scale and read off the time required.

Example 10-2. Find the *HOT* at Ullapool at 1900. HW / LW times and heights are: HW 0420 4.6m, LW 1033 1.6m, HW 1641 4.6m, LW 2308 1.2m. (Answer: Height 3.7m)

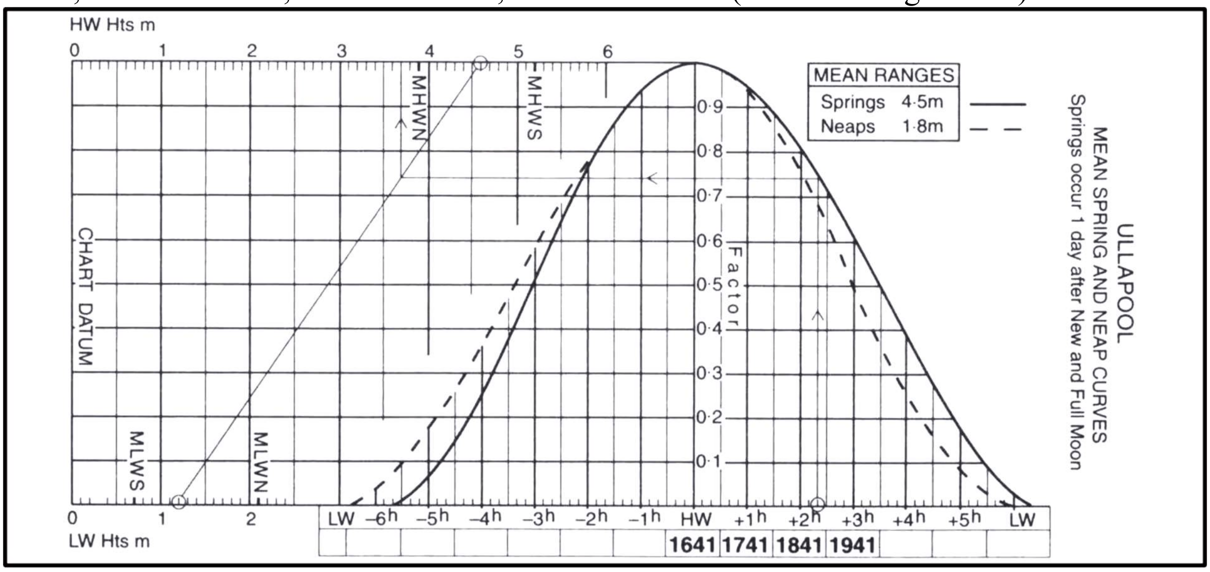


Fig 10-22. Example 10-2: Finding the Height of Tide (HOT) at a Standard Port

b. **Standard Tidal Curves.** Where an individual tidal curve is not provided for a *Standard Port*, the standard tidal curve (at front of tables) may be used provided that:

- The duration of the rise or fall of the *Tide* is between 5 and 7 hours.
- There is no shallow water correction.

If either of these criteria is not met, *TotalTide*® or *SHM for Windows*® should be used.

c. **Special Tidal Curves - Southern UK.** Special curves based on LW are used for some ports on the south coast of UK. See details at Para 1021e.

(1050) d. **HW and LW at Secondary Ports.** For calculating heights at *Secondary Ports*, use data from the Admiralty Tide Tables (NPs 201-204) Parts I and II as follows:

- **Step 1.** Establish the predicted HW and LW heights at the *Standard Port*
- **Step 2.** Algebraically SUBTRACT *Seasonal Variation* for *Mean Sea Level* (*MSL*) (see Para 1060c/d) at the *Standard Port*.
- **Step 3.** Establish the *Secondary Port* height differences (interpolating or extrapolating as necessary) and apply them
- **Step 4.** Algebraically ADD *Seasonal Variation* for *MSL* (see Paras 1060c/d) at the *Secondary Port.*

e. **Intermediate HOT / Times at Secondary Ports.** Once having established the correct *HOT* / times of HW and LW at a *Secondary Port* (Para 1050d above), use this data with the procedures at Para 1050a/b/c (opposite), to calculate the intermediate *HOT* / times required.

f. **Tidal Prediction Form - Secondary Ports HW / LW.** Copies of *UKHO's* 'Tidal Prediction Form' are at the back of each volume of Admiralty Tide Tables to assist users in correctly applying the tabulated differences and *MSL Seasonal Variations* for HW / LW at *Secondary Ports*.

g. **Tabulation of Secondary Port Differences.**

- **Semi-Diurnal Tides.** Where the *Tide* is mainly *Semi-Diurnal* in character, the differences are tabulated for *MHWS, MLWS, MHWN, MLWN* at the Standard Port. *Tide Level* definitions are at Para 1062.
- **Large Diurnal Inequality.** When the *Diurnal Inequality* is large, the differences are tabulated for *Mean Higher High Water* (*MHHW*), *Mean Lower High Water* (*MLHW*), *Mean Higher Low Water* (*MHLW*) and *Mean Lower Low Water* (*MLLW*). *Tide Level* definitions are at Para 1062.

h. **SHM / SHM for Windows® / TotalTide®.** Details of the *Simplified Harmonic Method* (*SHM*) and *SHM for Windows®* software are at Para 1032. Copies of a *UKHO* form to assist users in correctly utilising the *Harmonic Constants*, to compute a tidal prediction by calculator or spreadsheet, are at the back of each volume of Admiralty Tide Tables. For certain *Secondary Ports* (which are annotated accordingly in the Admiralty Tide Tables) no suitable *Standard Port* is available and thus simple time / height differences for these *Secondary Ports* cannot be applied; in such cases predictions must be made with *TotalTide®* or by using *Harmonic Constants* with *SHM / SHM for Windows®*.

i. **Co-Tidal / Co-Range Charts or Atlases.** *Co-Tidal* and *Co-Range* charts or atlases should be used for tidal predictions for offshore areas and coastlines between *Secondary Ports*. See Para 1052 (overleaf).

1051. Admiralty TotalTide® Software

The Admiralty *TotalTide®* software program provides predictions for all ports and *Tidal Stream* data currently available in *UKHO* publications. The accuracy of its predictions has the integrity of the Admiralty Tide Tables, provided the latest software edition is used. *HOTs* for multiple ports may be calculated for up to 7 consecutive days; results may be printed. Outputs include periods of daylight, *Nautical Twilight*, phases of the Moon, *Springs / Neaps* indicators and *Underkeel / Vertical Clearances*. *TotalTide®* runs under MS Windows and may be interfaced to certain *WECDIS / ECDIS* equipments for the display of *HOTs / Tidal Streams*.

1052. Co-Tidal and Co-Range Charts / Atlases

Co-Tidal and *Co-Range* charts / atlases show lines of equal time and *Range* of *Tides* in the open sea (see Figs 10-23a/b below). They are only available for limited areas, but are of great importance to vessels with small *Underkeel Clearances* in those areas. However, as it is difficult to obtain data offshore, interpolation from inshore stations is also used; thus the data on *Co-Tidal* and *Co-Range* charts / atlases must be used with caution.

a. **Amphidromic Points.** At *Amphidromic Point*s the (tidal) *Range* is zero or very small. *Co-Tidal* lines radiate outwards from them, with HW / LW times progressing clockwise or anti-clockwise (see Fig 10-23a below); *Co-Range* lines surround *Amphidromic Point*s (see Fig 10-23b opposite). Near *Amphidromic Point*s, the *Range* of the *Tide* may alter considerably within a short distance.

b. **Co-Tidal Lines.** *Co-Tidal* lines (12h, 11h, 10h etc) are drawn through points of equal *Mean High Water Interval* (*MHWI*). *MHWI* is the mean time interval between the passing of the Moon over the *Meridian* of Greenwich and the time of the next HW at the place concerned. See Fig10-23a (below).

c. **Co-Range Lines.** *Co-Range* lines (5m, 4m, 3m etc) are drawn through positions of equal *MSRs*). See Fig 10-23b (opposite).

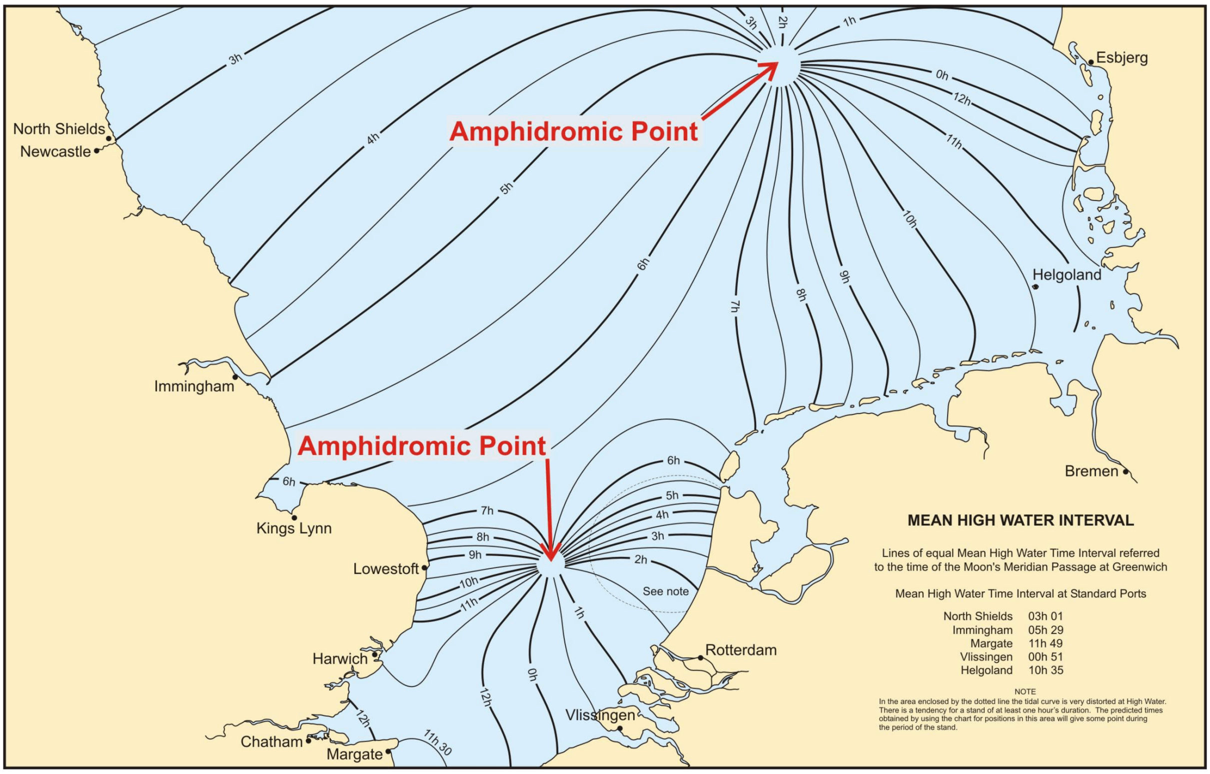


Fig 10-23a. Co-Tidal Chart (Southern North Sea [UK])

(1052c continued)

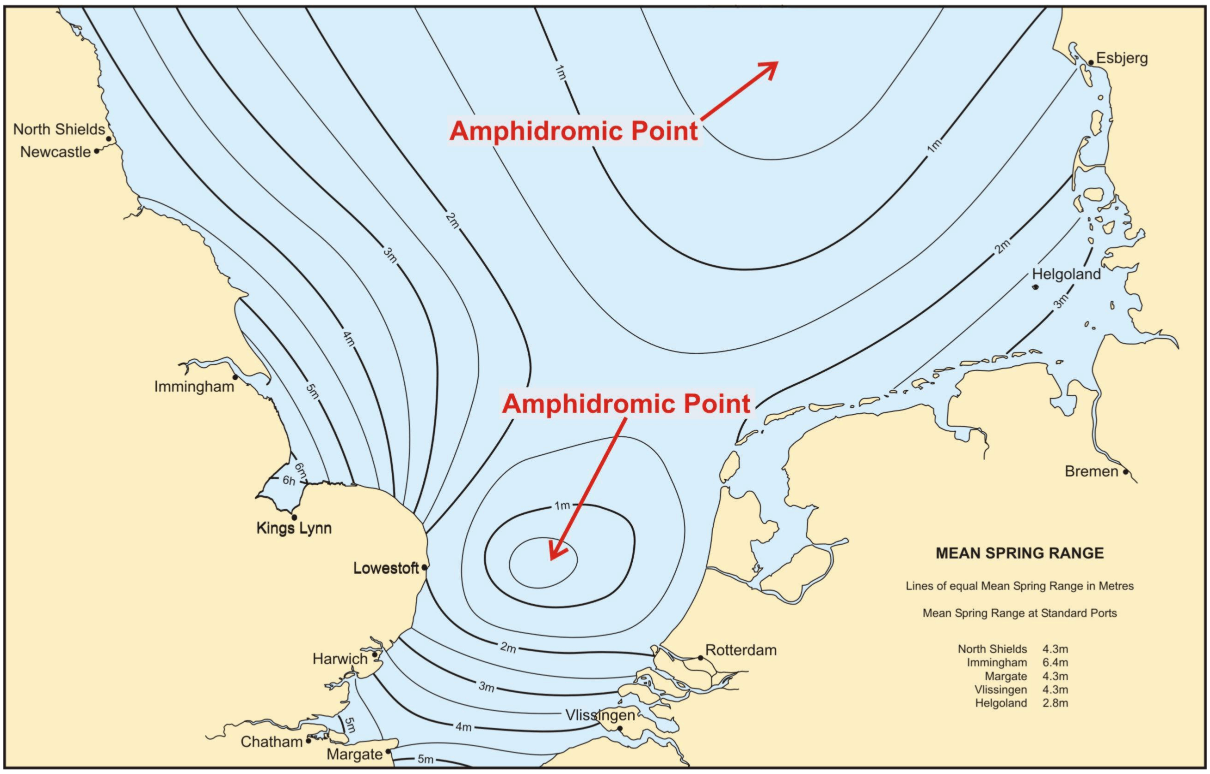


Fig 10-23b. Co-Range Chart (Southern North Sea [UK])

1053. Scope of the Admiralty Tide Tables

The Admiralty Tide Tables (NPs 201-204) are set out in 4 Volumes and cover the whole world. Information in this paragraph is based on the Admiralty Tide Tables dated 2008.

a. **Standard and Secondary Ports.** The Admiralty Tide Tables cover about 250 *Standard Ports,* nearly 7,000 *Secondary Ports* and 125 *Tidal Stream* systems (see Paras 1030b / 1042e for tidal observation criteria). The authority for the observations, method of prediction and year of observation are stated in each volume.

b. **Content of the Admiralty Tide Tables.** There are slight differences in the content of the 4 volumes to reflect the tidal characteristics of different parts of the world.

- **Part I.** Part I gives daily predictions of the times and heights of HW and LW at a selected number of *Standard Ports*.
- **Part Ia.** Part Ia gives hourly height predictions for a few *Standard Ports* in Vol 1, but, it gives daily predictions of *Tidal Streams* in Vols 3 & 4.
- **Part II.** Part II gives data for prediction at a large number of *Secondary Ports*, as height and time differences from a *Standard Port*.
- **Part III.** Part III lists *Harmonic Constants* for use with *SHM* predictions.
- **Part IIIa.** Part IIIa lists *Harmonic Constants* for *Tidal Stream* prediction in Vols 2, 3 and 4.
- **Tables.** Up to 8 Supplementary Tables are included in each Volume.

1054-1059. Spare

SECTION 6 - LEVELS AND DATUMS

1060. Tidal Levels

a. **Tidal Levels for Standard and Secondary Ports.** A list of *Tidal Levels* for *Standard Ports* is given at Supplementary Table V in the Admiralty Tide Tables (NP 201-204); *Tidal Levels* (*Vertical Datums*) for a large number of *Secondary Ports* are at Part II of the Admiralty Tide Tables.

b. **Tidal Levels for Tidal Prediction.** The *Tidal Level* for tidal predictions must be the same as *Chart Datum* (see Para 1062c), to ensure that the depth of water is equal to the charted depth plus the *HOT*. *Tidal Levels* established at *Standard Ports* vary widely; they do not conform to any uniform standard. Modern charting practice is to establish *Tidal Levels* at or near the level of *Lowest Astronomical Tide* (*LAT*), but when planning passages, the *Tidal Levels* printed on the chart and in Supplementary Table V in the Admiralty Tide Tables should be checked for agreement. Where the *UKHO* is the surveying authority, *Tidal Level*s have been adjusted to approximate to *LAT*.

c. **Mean Sea Level (MSL).** *Tidal Levels* for *Standard Ports* do not necessarily remain constant, due to changes in *Mean Sea Level (MSL)*; a number of *MSLs* have risen by 0.1 metres over the past 40 years. In addition, regular *Seasonal Variations* can occur to *MSL*, due either to established meteorological patterns (barometric pressure, wind strength and direction), or the effect of river water (ports with a temperate climate).

d. **Allowance for Seasonal Variations in Mean Sea Level (MSL).** Regular *Seasonal Variations* in *MSL* greater than 0.1 metres are tabulated in the HW and LW predictions for *Standard Ports* in Part II of the Admiralty Tide Tables. See Para 1050d for details of calculations necessary to allow for *Seasonal Variations* in *MSL.*

1061. Chart Datum and Land Survey Datums

Tidal Levels are referred to *Chart Datum* (see Para 1062c) of the largest *Scale* Admiralty chart. A variety of '*Land Survey Datums*' are in use worldwide; in UK, *Ordnance Datum* is used for this purpose.

a. **Tide Tables.** The connection between *Chart Datum* and *Ordnance Datum* in UK is given at Supplementary Table III. Where known, the equivalent information for other *Tidal Levels* outside UK is given at Supplementary Table IV.

b. **Charts.** On large and medium *Scale* charts for which *UKHO* is the primary authority, the panel giving tidal height may also tabulate the difference between *Chart Datum* and *Ordnance Datum* for the area.

c. **Lack of Data.** If absolute heights are required at a point on the coast where no tidal data is given, or where there is no connection to the *Land Survey Datum*, they may be obtained by interpolation from heights obtained from places on either side where data is available.

1062. Tidal Levels and Heights - Definitions

Definitions of *Tidal Levels* and heights are defined below and overleaf, and are shown diagrammatically at Fig 10-24 (below).

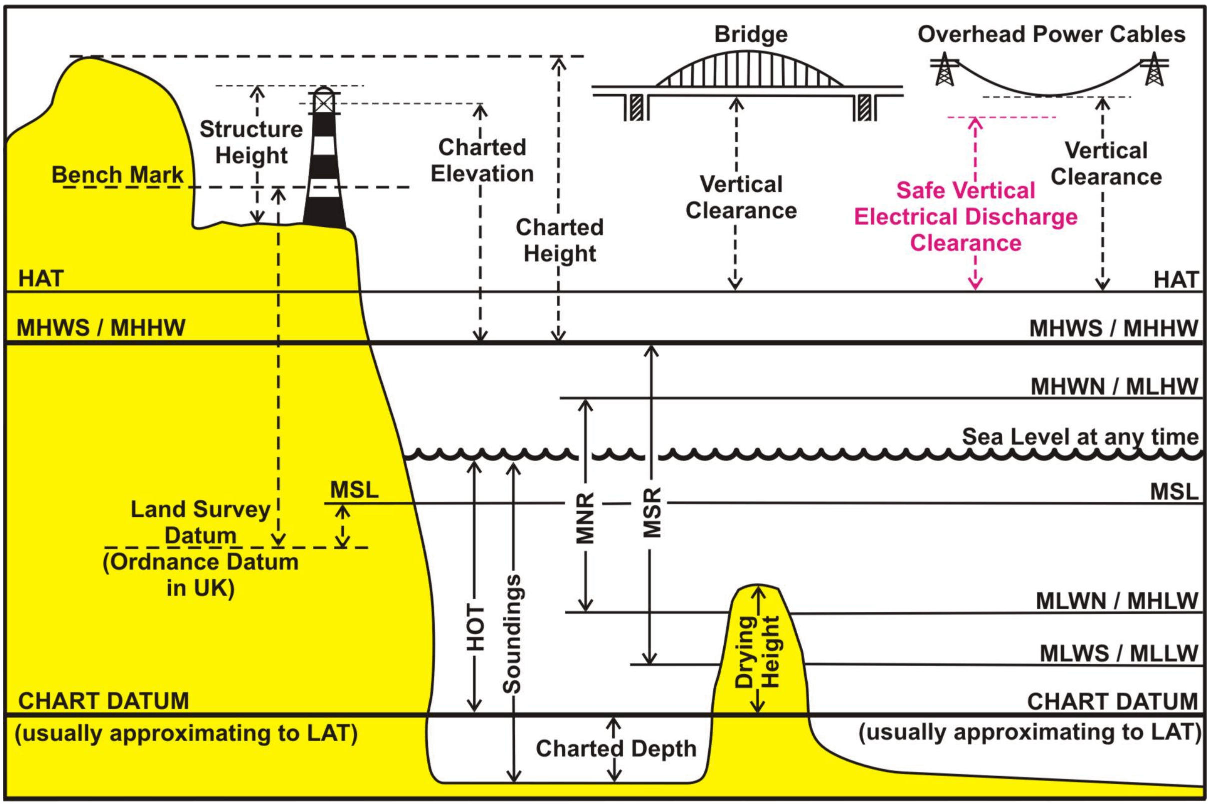


Fig 10-24. Tidal Levels and Heights (*not to scale*)

a. **Heights.** Except for *Vertical Clearances*, heights on Admiralty charts are given above *MHWS* in areas where the *Tides* are *Semi-Diurnal* and *MHHW* where there is a *Diurnal Inequality*. *Mean Sea Level* (*MSL*) is used where there is negligible *Tide*.

b. **Vertical Clearances.** Since 2004, *Vertical Clearances* have been quoted above *Highest Astronomical Tide* (*HAT*) in all areas where there is an appreciable (tidal) *Range*. *MSL* is still used where there is negligible *Tide*. The '**Heights**' statement beneath the title of each information panel makes it quite clear which *Vertical Datum* (*HAT, MHWS* or *MSL*) is being used for *Vertical Clearances* on that particular chart. Where a 'Safe' *Vertical Clearance* has been obtained to avoid the risk of electrical discharge from overhead power cables, it is shown on charts in magenta; however the deduction used may vary with changes in transmission voltage and should be used with caution.

c. **Chart Datum.** By international agreement, *Chart Datum* is defined as a level so low that the *Tide* will not frequently fall below it. In areas for which *UKHO* is the authority, *Chart Datum* is the approximate level of *Lowest Astronomical Tide* (*LAT*).

d. **Variation in Values.** The average values of *MHWS, MHWN, MLWS, MLWN, MHHW, MLHW, MHLW* and *MLLW* (see definitions at Para 1062e overleaf) vary from year to year in a cycle of approximately 18.6 years (the 'Metonic Cycle'). *Tidal Levels* shown in Supplementary Table V of the Admiralty Tide Tables are average values over the whole cycle.

(1062) e. **Tidal Levels.** *Tidal Levels* are referred to *Chart Datum* (see Para 1062c - previous page). *MSR / MNR* are defined at Para 1045a. Definitions of other *Tidal Levels* are:

- **Highest Astronomical Tide (HAT) and Lowest Astronomical Tide (LAT).** *Highest* and *Lowest Astronomical Tides* (*HAT* and *LAT*) are the highest and lowest levels respectively which can be predicted to occur under average meteorological considerations and any combination of astronomical conditions. *HAT* and *LAT* are not the extreme levels which can be reached; *Storm Surges* (see Para 1022) may cause considerably higher and lower levels to occur.

- **Mean High Water Springs (MHWS).** The height of *Mean High Water Springs* (*MHWS*) is the average throughout a year when the average maximum *Declination* of the Moon is 23½˚, of the heights of two successive HWs during those periods of 24 hours (approximately every fortnight) when the *Range* of the *Tide* is greatest.

- **Mean Low Water Springs (MLWS).** The height of *Mean Low Water Springs* (*MLWS*) is the average height obtained from two successive LWs during the same period as *MHWS.*

- **Mean High Water Neaps (MHWN).** *Mean High Water Neaps* (*MHWN*) is the average throughout a year, as defined for *MHWS,* of the heights of two successive HWs during those periods (approximately every fortnight) when the *Range* of the *Tide* is least.

- **Mean Low Water Neaps (MLWN).** *Mean Low Water Neaps* (*MLWN*) is the is the average height obtained from two successive LWs during the same period as *MHWN.*

- **Mean Tide Level (MTL).** *Mean Tide Level* (*MTL*) is the mean of the heights of *MHWS, MHMW, MLWS* and *MLWN.*

- **Mean Sea Level (MSL).** *Mean Sea Level* (*MSL*) is the average level of the sea surface over a long period, preferably 18.6 years, or the average level which would exist in the absence of *Tides*.

- **Mean Higher High Water (MHHW).** *Mean Higher High Water* (*MHHW*) is the mean of the higher of the two daily HWs over a long period of time. When only one HW occurs in a day, this is taken as the higher HW.

- **Mean Lower High Water (MLHW).** *Mean Lower High Water* (*MLHW*) is the mean of the lower of the two daily HWs over a long period of time. When only one HW occurs on some days, ▵ is printed in the *MLHW* column of the Admiralty Tide Tables to indicate that the *Tide* is usually *Diurnal*.

- **Mean Higher Low Water (MHLW).** *Mean Higher Low Water* (*MHLW*) is the mean of the higher of the two daily LWs over a long period of time. When only one LW occurs on some days, ▵ is printed in the *MHLW* column of the Admiralty Tide Tables to indicate that the *Tide* is usually *Diurnal*.

- **Mean Lower Low Water (MLLW).** *Mean Lower Low Water* (*MLLW*) is the mean of the lower of the two daily LWs over a long period of time. When only one LW occurs on a day, this is taken as the lower LW.

APPENDIX 1

PLANE TRIGONOMETRY

1. Scope of Appendix

Trigonometry is the branch of mathematics dealing with the relations between the angles and sides of a triangle and with the relevant functions of any angles. Appendix 1 contains the following information:

- **Para 2: Units - The Degree, Radian and π**
- **Para 3: Trigonometric Functions 0˚ to 90˚ - Sin, Cos and Tan**
- **Para 4: Trigonometric Functions 0˚ to 360˚ - Sin, Cos and Tan**
- **Para 5: Sin & Cos Curves -180˚ to 540˚ and Tan Curves -180˚ to 270˚**
- **Para 6: Inverse Trigonometric Functions - Sin, Cos and Tan**
- **Para 7: Pythagorean Relationships between Trigonometric Functions**
- **Para 8: Non-Right Angled Triangles - Sine, Cosine and Area Formulae**
- **Para 9: Functions of Sum and Difference of Two Angles**
- **Para 10: Sine and Cosine of Small Angles**

2. Units - The Degree, Radian and π

a. **The Degree.** The angle between two intersecting lines is the inclination of one line to the other, and this inclination is commonly measured in ‘degrees’ and sub-divisions of a degree. In one complete revolution there are 360 degrees; when the two arms of the angle are perpendicular, the angle is said to be a ‘right angle’, in which there are 90 degrees (see Fig A1-1 below). The sub-divisions of the degree are the ‘minute’ and ‘second’, the relation between them is:

$$1^\circ = 60 \text{ minutes } (')$$
$$1' = 60 \text{ seconds } ('')$$

In navigation, angles are measured clockwise from North (000˚), through East (090˚), South (180˚) and West (270˚) to North (see Fig A1-1 below).

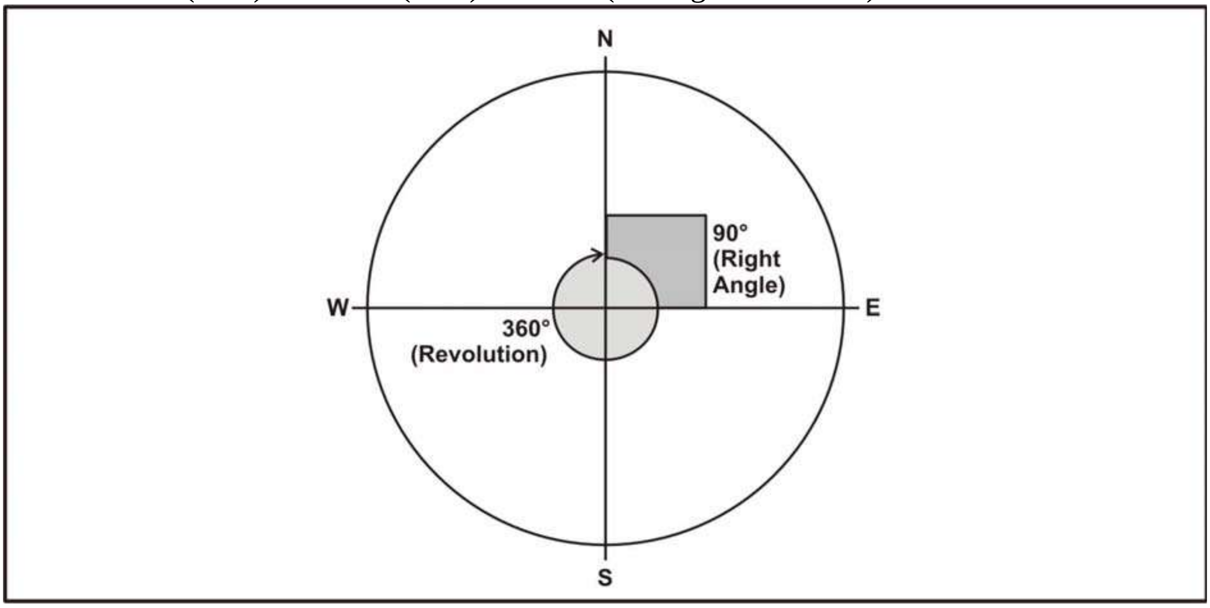


Fig A1-1. Degrees in a Right Angle and in One Revolution

(2) b. **The Radian and π.** The 'Radian' is defined as the angle subtended at the centre of a circle by a length of arc equal to the radius (see Para 0127). 'π' is defined as the ratio of the circumference of a circle to its diameter; this ratio is constant in all cases and π is approximately equal to 3.1415927 (see Para 0127). From this it follows that:

- **Radians to Degrees.** The angle subtended by an arc equal to the radius is also constant and equal to 360˚ ÷ 2π, or approximately 57˚ 17' 45".
- **Radians in a Right Angle.** The number of radians in a right angle is ½π.
- **Length of any Arc of a Circle.** The length of any arc of a circle is equal to the radius multiplied by the angle in radians.

3. Trigonometric Functions 0˚ to 90˚ - Sin, Cos and Tan

a. **The Right-Angled Triangle.** In Fig A1-2 (opposite), the *Plane Triangle ABC* is right-angled at *C*; the sides *BC*, *CA* and *AB* are of length *a*, *b* and *c* respectively; and the angle *CAB* is of size θ. For navigational convenience *AC* is taken as due north so that the (true) bearing of *B* from *A* is θ.

b. **The Six Trigonometric Functions of a Right-Angled Triangle.** There are six trigonometric functions of a right-angled *Plane Triangle*. Two of these, the sine and cosine, are of fundamental importance while the other 4, tangent, cotangent, secant and cosecant are derived from them. The six functions are defined and abbreviated thus:

$$\sin\theta = \frac{\text{side opposite the angle}}{\text{hypotenuse}} = \frac{a}{c} \quad \text{. . . } \mathbf{A1.1}$$

$$\cos\theta = \frac{\text{side adjacent to the angle}}{\text{hypotenuse}} = \frac{b}{c} \quad \text{. . . } \mathbf{A1.2}$$

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b} = \frac{a}{c} \times \frac{c}{b} = \frac{\sin\theta}{\cos\theta} \quad \text{. . . } \mathbf{A1.3}$$

$$\cot\theta = \frac{b}{a} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} \quad \text{. . . } \mathbf{A1.4}$$

$$\sec\theta = \frac{c}{b} = \frac{1}{\cos\theta} \quad \text{. . . } \mathbf{A1.5}$$

$$\operatorname{cosec}\theta = \frac{c}{a} = \frac{1}{\sin\theta} \quad \text{. . . } \mathbf{A1.6}$$

The last four trigonometric functions are defined in terms of sine and/or cosine. The last three functions are reciprocals of the first three.

In Fig A1-2, where angle *BCA* is 90˚ and angle *CAB* equals θ,

$$a = c\sin\theta$$
$$b = c\cos\theta$$

Thus, if *AC* points North, *B* is both '*c sin* θ' East of *A* and '*c cos* θ' North of *A*.

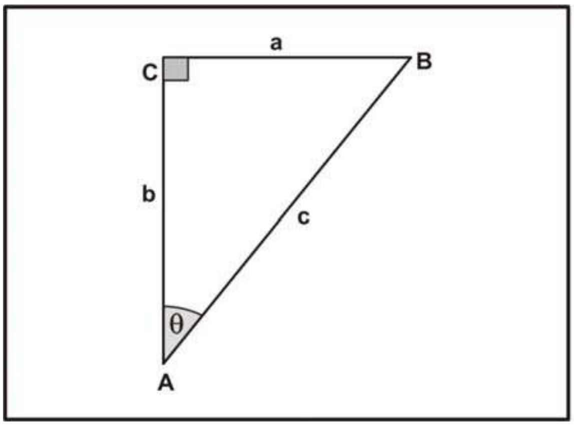


Fig A1-2. The Right-Angled Triangle

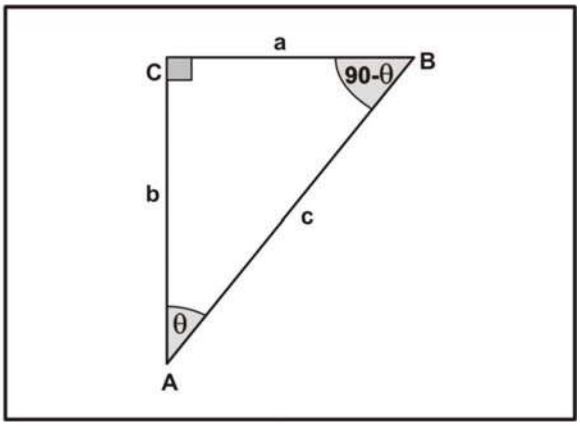


Fig A1-3. Complimentary Angles

(3) c. **Complementary Angles.** Angles that add together to make 90˚ are said to be 'complementary'. Thus, if one angle is 34˚, its complementary angle is 56˚. In any right-angled triangle the two acute angles are complementary, since the sum of the three angles, of which one is 90˚, must be 180˚. It can also be shown from Fig A1-3 that:

$$\sin\theta = \frac{a}{c} = \cos(90°-\theta) \qquad (eg\ \sin 34° = \cos 56°) \qquad \text{. . . A1.7}$$

$$\cos\theta = \frac{b}{c} = \sin(90°-\theta) \qquad (eg\ \cos 34° = \sin 56°) \qquad \text{. . . A1.8}$$

$$\tan\theta = \frac{a}{b} = \cot(90°-\theta) \qquad (eg\ \tan 34° = \cot 56°) \qquad \text{. . . A1.9}$$

d. **Trigonometric Functions of Specific Angles.** Fig A1-4 and Table A1-1 show the relationship between the trigonometric functions of specific angles and the length of sides of right-angled triangles.

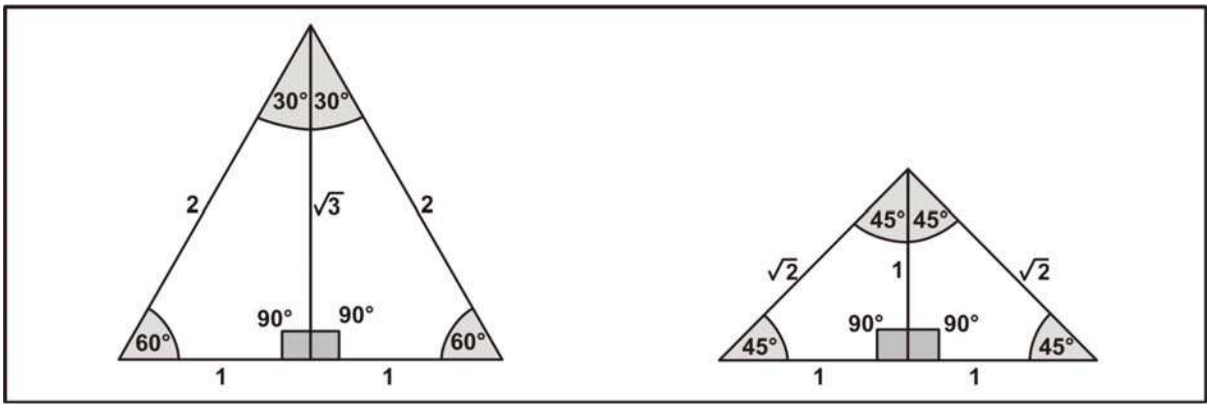


Fig A1-4. Trigonometric Functions of Specific Angles

Table A1-1. Trigonometric Functions of Specific Angles

θ	0˚	30˚	45˚	60˚	90˚
sin θ	0	0.5	$\frac{1}{\sqrt{2}} \simeq 0.707$	$\frac{\sqrt{3}}{2} \simeq 0.866$	1
cos θ	1	$\frac{\sqrt{3}}{2} \simeq 0.866$	$\frac{1}{\sqrt{2}} \simeq 0.707$	0.5	0
tan θ	0	$\frac{1}{\sqrt{3}} \simeq 0.577$	1	$\sqrt{3} \simeq 1.732$	∞

4. Signs and Values of the Trigonometric Functions Between 000˚ and 360˚. The definitions at formulae (A1.1 to A1.6) of the six trigonometric functions for acute angles may be extended to angles up to 360˚.

a. **Sign Conventions between 000˚ and 360˚.** Bearing and direction are measured clockwise from 000˚ to 360˚. In can be shown from Fig A1-5 (below) that Northerly and Easterly directions may be considered as +ve, while Southerly and Westerly are −ve . South may be said to be the equivalent of negative North and west the equivalent of negative East. Tangent, cotangent, secant and cosecant may be defined in terms of sine and/or cosine (see Para 3b).

b. **Signs of Trigonometric Functions - The Four Quadrants.** Bearing and direction 000˚ to 360˚ may be divided into 4 quadrants (see Fig A1-5 below) for the purposes of establishing the signs of functions.

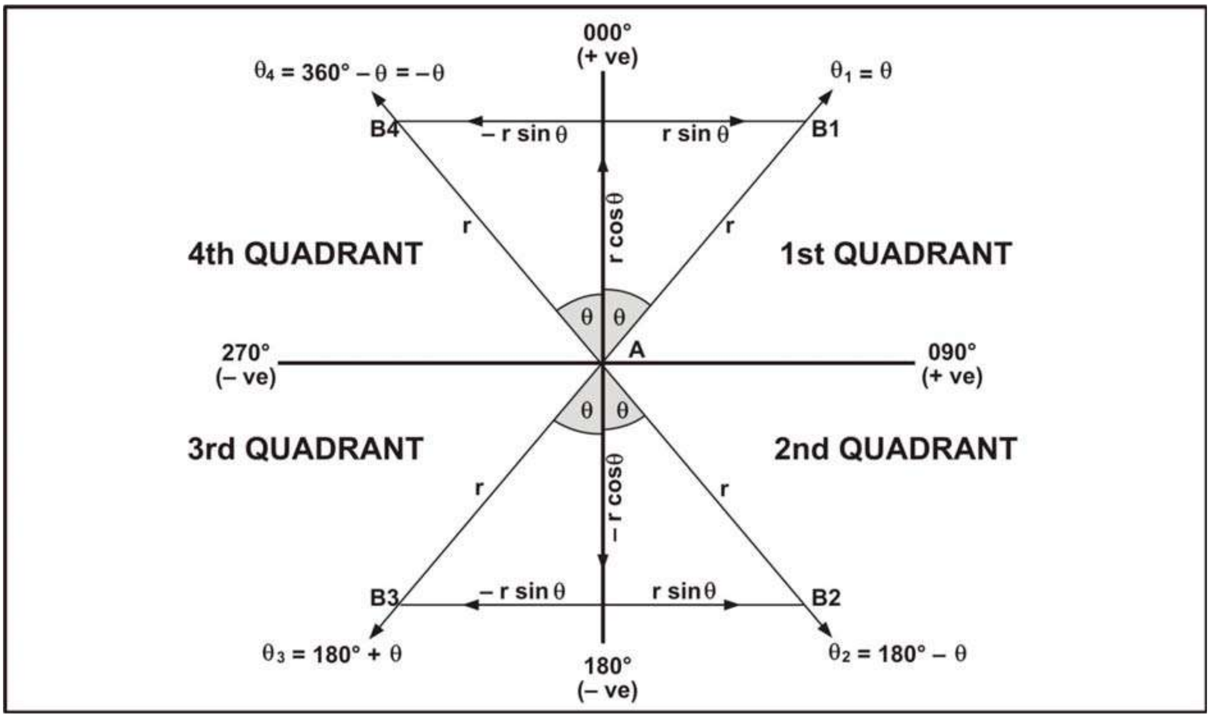


Fig A1-5. Signs of Trigonometric Functions - The Four Quadrants

- **Explanation of Fig A1-5.** B_1 is in the '1st Quadrant', at a distance *r* and on a bearing θ_1 from *A*, where θ_1 equals angle θ; B_1 is $r \sin \theta$ East of *A* and $r \cos \theta$ North of *A*. Equivalent points B_2, B_3, and B_4 are in the 2nd, 3rd and 4th quadrants.

- **Example - 3rd Quadrant.** In Fig A1-5 (above), B_3 is South and West of A at a distance r on a bearing θ_3 (equal to angle 180˚ + θ). Thus:

$$\sin \theta_3 \; (\sin [180° + \theta]) = \frac{-r \sin \theta}{r} = -\sin \theta$$

$$\cos \theta_3 \; (\cos [180° + \theta]) = \frac{-r\cos\theta}{r} = -\cos \theta$$

$$\tan \theta_3 = \frac{\sin\theta_3}{\cos\theta_3} = \frac{-\sin\theta}{-\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

In summary, B_3 is $r \sin \theta$ West of A, which is thus equivalent to $-r \sin \theta$. B_3 is also $r \cos \theta$ South of A, and thus equivalent to $-r \cos \theta$.

(4) c. **Signs in Each Quadrant.** Reciprocals of sine, cosine and tangent (ie cosecant, secant and cotangent respectively) take the same sign as their original function. From Fig A1-5 (opposite) it may be shown that:

- **1st Quadrant.** Bearings between 000˚ and 090˚ lie between North (+ve) and East (+ve). For direction θ_1, sine, cosine and tangent are all +ve.
- **2nd Quadrant.** Bearings between 090˚ and 180˚ lie between East (+ve) and South (−ve). For direction θ_2, sine is (+ve), cosine and tangent are (−ve).
- **3rd Quadrant.** Bearings between 180˚ and 270˚ lie between South (−ve) and West (−ve). For direction θ_3, tangent is (+ve), sine and cosine are (−ve).
- **4th Quadrant.** Bearings between 270˚ and 360˚ lie between West (-ve) and North (+ve). For direction θ_4, cosine is (+ve), sine and tangent are (−ve).

d. **Summary of Signs and Mnemonic "All Stations To Crewe".** The signs of sine, cosine and tangent (plus their reciprocals, which take the same sign as their original functions) are summarised at Fig A1-5a (below) for the four quadrants. The mnemonic ***"All Stations To Crewe"*** provides a reminder of the signs of the trigonometric functions.

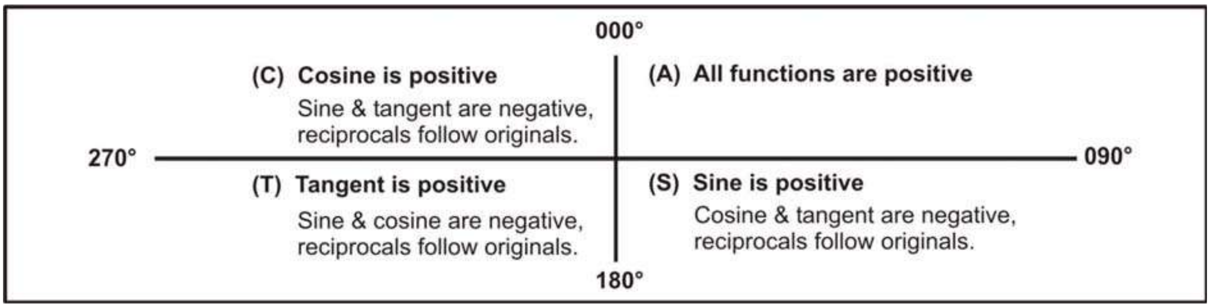


Fig A1-5a. Summary of Signs of Trigonometric Functions 000˚ to 360˚

e. **Values of Trigonometric Functions.** The value (as distinct from the sign) of any trigonometric function of an angle greater than 90˚ is equal to the value of the trigonometric function of the angle made with the North−South axis. For example, the value of sin 127˚ equals sin 53˚ (180˚ − 127˚), while the value of cosine 296˚ equals cosine 64˚ (360˚ − 296˚). The angle (180˚ − θ) is known as the 'Supplement' of θ. 'Supplementary Angles' add together to 180˚. The signs and values of the trigonometric functions of angles in each quadrant are summarised in Table A1-2 (below). See also examples at Fig A1-6 / Table A1-3 (overleaf).

Table A1-2. Values of Trigonometric Functions

Direction	Angle	Sine Angle	Cosine Angle	Tangent Angle
θ_1	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
θ_2	$180^\circ-\theta$	$\sin(180^\circ-\theta)$ $=\sin\theta$	$\cos(180^\circ-\theta)$ $=-\cos\theta$	$\tan(180^\circ-\theta)$ $=-\tan\theta$
θ_3	$180^\circ+\theta$	$\sin(180^\circ+\theta)$ $=-\sin\theta$	$\cos(180^\circ+\theta)$ $=-\cos\theta$	$\tan(180^\circ+\theta)$ $=\tan\theta$
θ_4	$360^\circ-\theta$ $(=-\theta)$	$\sin(360^\circ-\theta)$ $=-\sin\theta$ $=\sin(-\theta)$	$\cos(360^\circ-\theta)$ $=\cos\theta$ $=\cos(-\theta)$	$\tan(360^\circ-\theta)$ $=-\tan\theta$

(4) f. **Examples of Values of Trigonometric Functions.** The signs and values of the trigonometric functions of some (example) angles in each quadrant are and demonstrated at Fig A1-6 / Table A1-3 (below).

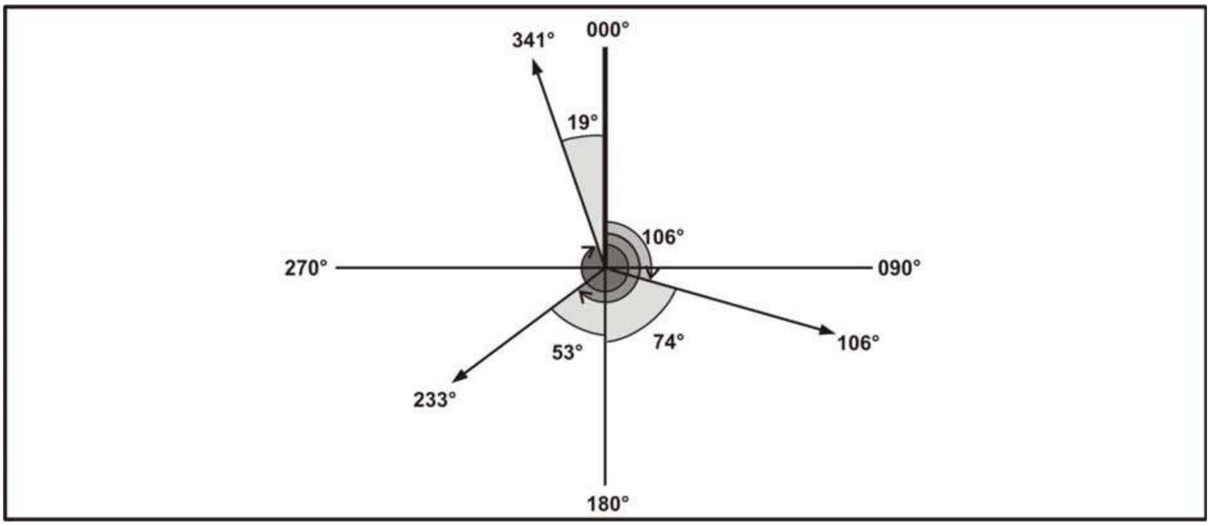


Fig A1-6. Examples of Signs and Values of Trigonometric Functions 000˚ to 360˚

Table A1-3. Examples of Signs and Values of Trigonometric Functions 000˚ to 360˚

Angle	Sign	Cosine	Tangent
106˚	+ sin 74˚	- cos 74˚	- tan 74˚
233˚	- sin 53˚	- cos 53˚	+ tan 53˚
341˚	- sin 19˚	+ cos 19˚	- tan 19˚

5. Sin & Cos Curves -180˚ to 540˚ and Tan Curves -180˚ to 270˚

a. **Angles Greater than 360.** Although, in navigation, angles outside the range 0˚ to 360˚ are rarely encountered, the definitions given earlier may be extended to angles greater than 360˚. The value 360˚ (or multiples of 360˚) may be subtracted from the angle concerned to reduce it to an angle between 0˚ and 360˚.

b. **Negative Angles.** Negative angles may be taken as angles measured anti-clockwise from North and brought to an angle between 0˚ and 360˚ by the addition of 360˚ (or multiples of 360˚).

c. **Sin, Cos and Tan Curves.** The graphs of sin θ, cos θ and tan θ may be deduced for any given range. Fig A1-7 (opposite) shows the graphs of sin θ and cos θ between −180˚ and +540˚, and the graph of tan θ between −180˚ and +270˚. The following points should be noted:

- Both sin θ and cos θ repeat every 360˚.
- Tan θ repeats every 180˚.
- In any 360˚, there are two angles which have the same value for any trigonometric function:

(eg sin 35˚ = sin 145˚, cos 134˚ = cos 226˚, tan 213˚ = tan 33˚, etc).

(5c continued)

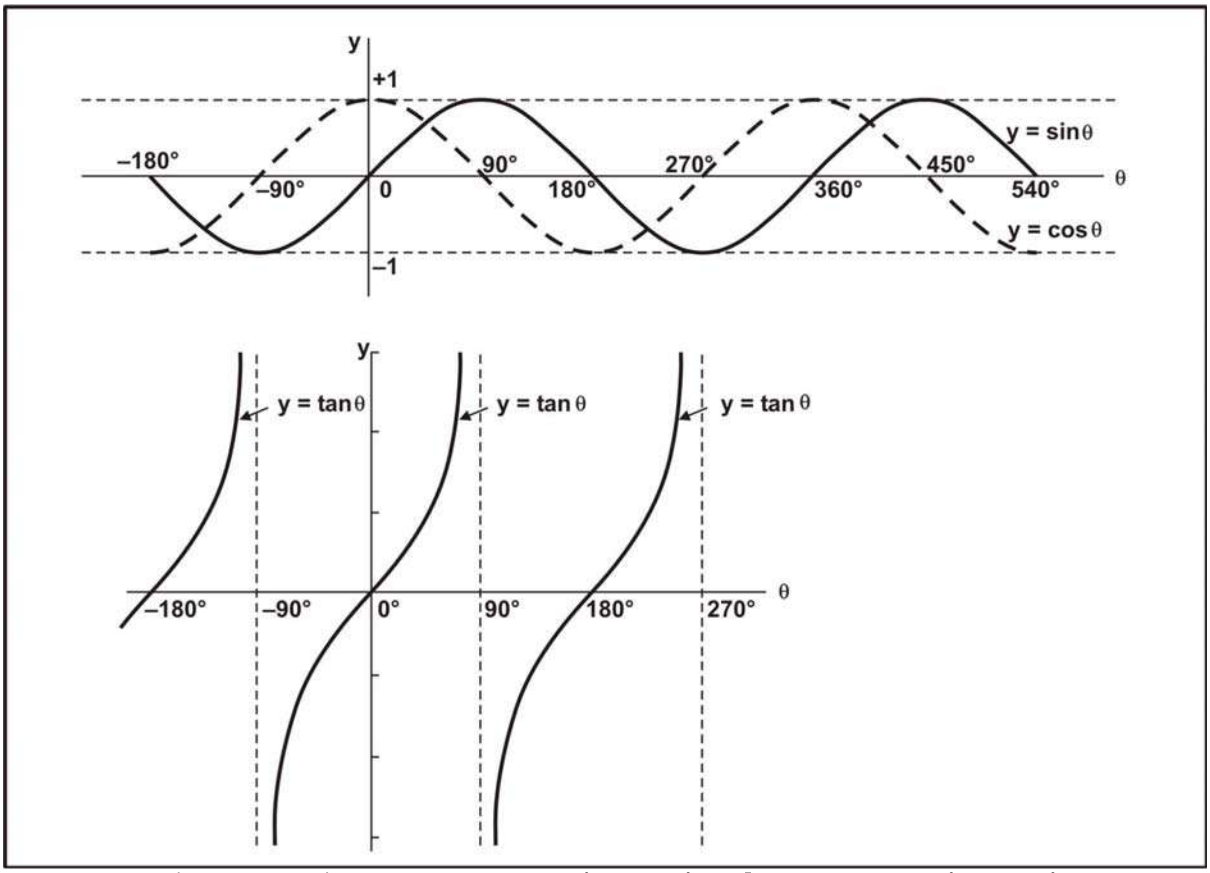


Fig A1-7. Sin & Cos Curves 0˚ to 540˚ and Tan Curves 0˚ to 270˚

6. Inverse Trigonometric Functions - Sin, Cos and Tan

a. **Principal Values.** As there are two angles in any 360˚ which have the same value for any trigonometric function, it follows that the inverse function has more than one value. However, a calculator / spreadsheet can only give what is called the 'Principal Value' of the inverse trigonometric function. The principal value ranges for sine, cosine and tangent are:

$\sin^{-1}$: $-90^\circ \leq \theta \leq 90^\circ$
$\cos^{-1}$: $0^\circ \leq \theta \leq 180^\circ$
$\tan^{-1}$: $-90^\circ < \theta < 90^\circ$

b. **Selection of Appropriate Value.** The principal value may not be the one required in a particular problem, and the graph of the appropriate trigonometric function should be used to determine other values. For example:

$\sin^{-1} +0.5$	=	30˚ (the angle could be 030˚ or 150˚.)
$\sin^{-1} -0.5$	=	−30˚ (the angle could be 210˚ or 330˚.)
$\cos^{-1} +0.866$	≃	30˚ (the angle could be 030˚ or 330˚.)
$\cos^{-1} -0.866$	≃	150˚ (the angle could be 150˚ or 210˚.)
$\tan^{-1} +0.577$	≃	30˚ (the angle could be 030˚ or 210˚.)
$\tan^{-1} -0.577$	≃	−30˚ (the angle could be 150˚ or 330˚.)

(6) c. **Assessment of Appropriate Value by Inspection and/or Comparison.** When assessing values of trigonometric functions, care must be taken to ensure that, if necessary, the displayed angle reading is adjusted to the correct value.

- **Inspection.** Assessment may often be achieved by inspection. For example, if a bearing θ is such that tan $\theta \simeq -0.577$, but it is also known that the bearing is in the 4^{th} (North-West) quadrant, then the angle required must be 330˚ (and not 150˚, nor the −30˚ given by a calculator).
- **Comparison.** The two trigonometric functions corresponding to the displayed value may also be compared. For example, if the sine of the displayed value is −ve, while the cosine is also −ve, the angle corresponding to both values can only be in the 3^{rd} (South-West) quadrant, where sine and cosine are both negative.

7. Pythagorean Relationships between Trigonometric Functions

By the theorem of Pythagoras, the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. Using the notation from Fig A1-3:

$$a^2 + b^2 = c^2$$

thus:

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

or

$$sin^2\,\theta + cos^2\theta = 1 \qquad \text{. . . }\mathbf{A1.10}$$

Further division by $\cos^2\theta$ gives:

$$tan^2\,\theta + 1 = sec^2\,\theta \qquad \text{. . . }\mathbf{A1.11}$$

or, if (**A1.10**) is divided by $sin^2\,\theta$:

$$1 + cot^2\,\theta = cosec^2\,\theta \qquad \text{. . . }\mathbf{A1.12}$$

Formulae (A1.10 to A1.12) hold for all values of θ, because the square of any quantity is always positive, although the quantity itself may be negative.

8. Non-Right Angled Triangles - Sine, Cosine and Area Formulae

a. **Non-Right Angled Triangles.** There are several formulae connecting the sides and angles of acute and obtuse triangles (Fig A1-8) and the choice of formula is governed, as a rule, by the data available and the requirements of the problem to be solved.

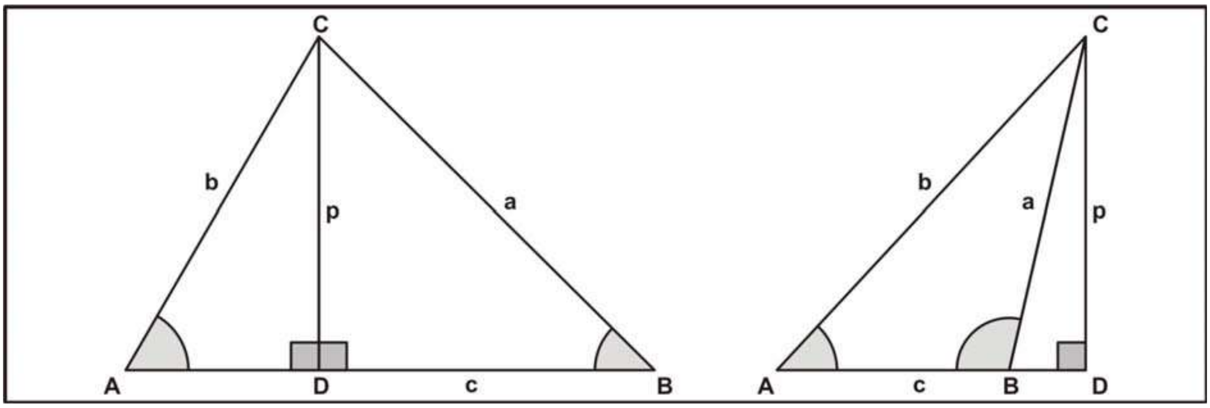


Fig A1-8. Non-Right Angled Acute and Obtuse Triangles ABC

b. **The Sine Formula.** The *Sine Formula* is established by dropping a perpendicular from any vertex on to the opposite side (see Fig A1-8 above). In Fig A1-8, the perpendicular is *CD*, denoted by ‘p’. Then:

$$\sin A = \frac{p}{b}$$

also $\frac{p}{a} = \sin B$ (acute triangle) or $\sin(180^\circ - B)$(obtuse triangle) $= \sin B$

ie $p = b \sin A$ and $p = a \sin B$

$$\therefore \; b \sin A = a \sin B$$

or
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Similarly, if a perpendicular is dropped from *A* to *BC*, or *BC* produced:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Hence
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \text{. . . } \mathbf{A1.13}$$

- **Summary.** Thus, in a triangle, if two angles *A* and *B* and one side are given, by simple arithmetic the third angle is $180^\circ - (A + B)$; the *Sine Formula* may be used to calculate the remaining sides.

(8) c. **Ambiguity in the Sine Formula.** Ambiguity arises if the *Sine Formula* is used for solving the triangle when two sides and an angle other than the included angle are given, when the given angle is opposite the smaller side (see Fig A1-9 below). If, in Fig A1-10, the sides b and c and the angle C are given, the angle found from the formula is either ABC or its supplement AB_1C, because the sine of an angle is equal to the sine of its supplement.

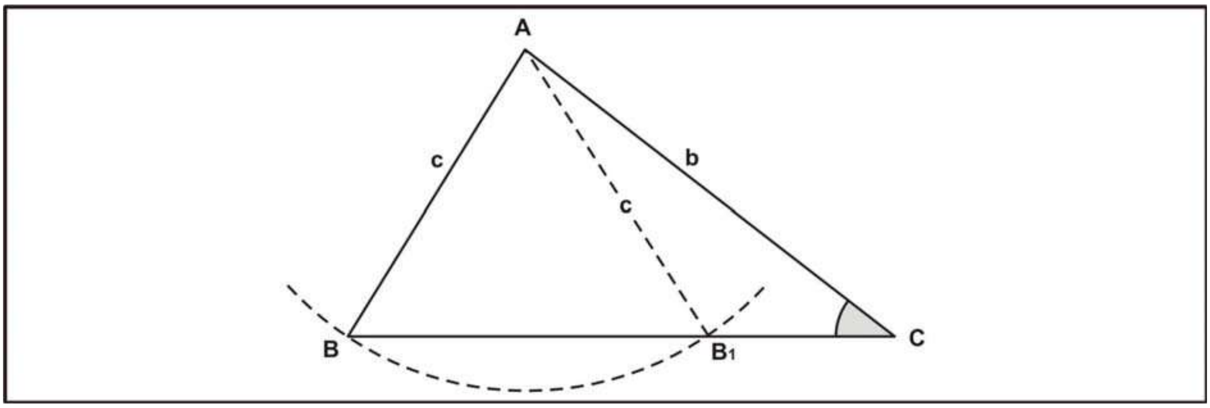


Fig A1-9. Ambiguity in the Sine Formula

d. **The Cosine Formula.** The *Cosine Formula* is established by applying the theorem of Pythagoras to the right-angled triangles ADC and BDC in Fig A1-8. Thus:

$$\begin{aligned} a^2 &= p^2 + BD^2 \\ \mathrm{b}^2 &= \mathrm{p}^2 + AD^2 \\ \therefore a^2 &= (b^2 - AD^2) + BD^2 \\ &= b^2 - AD^2 + (c - AD)^2 \\ &= b^2 - AD^2 + c^2 - 2cAD + AD^2 \\ &= b^2 + c^2 - 2cAD \\ &= b^2 + c^2 - 2bc \cos A \end{aligned} \qquad \text{. . . }\mathbf{A1.14}$$

In the same way it can be established that:

$$b^2 = c^2 + a^2 - 2ca \cos B \qquad \text{. . . }\mathbf{A1.15}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \qquad \text{. . . }\mathbf{A1.16}$$

- **Negative Cosines.** The *Cosine Formula* is true for any triangle, but if the angle A, B or C is greater than 90˚, the angle lies in the 2nd quadrant and its cosine is negative.

- **Summary.** Thus, in a triangle, the *Cosine Formula* gives the third side when two sides and the included angle are known, or any angle when the three sides are known.

e. **Area of a Triangle.** The area of a triangle is equal to half the base multiplied by the perpendicular height. The area of the triangle ABC (see Fig A1-8, previous page) may also be found by transposing this value using the *Sine Formula* to give:

$$\text{Area} = \tfrac{1}{2} ab \sin C \qquad \text{. . . }\mathbf{A1.17}$$

$$\text{Area} = \tfrac{1}{2} bc \sin A \qquad \text{. . . }\mathbf{A1.18}$$

$$\text{Area} = \tfrac{1}{2} ca \sin B \qquad \text{. . . }\mathbf{A1.19}$$

9. Functions of Sum and Difference of Two Angles

a. **Trigonometric Functions of Combined Angles.** The trigonometric functions of combined angles may be determined. For example, the sine, cosine and tangent of the angles *A* + *B* in Fig A1-10 (below) may be found as follows:

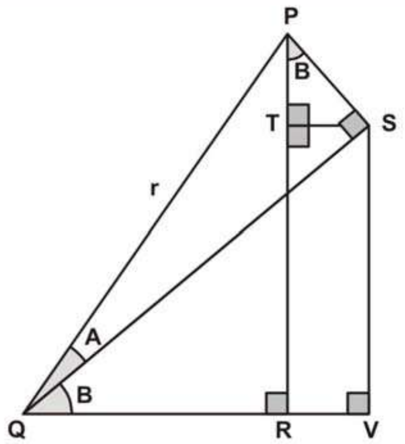


Fig A1-10. Trigonometric Functions of Combined Angles.

In Fig A1-10 (above), triangle PQR is right-angled at *R*. The line *QS* divides the angle *Q* into the angles *A* and *B*. *PS* is a perpendicular from *P* to *QS* and *SV* is a perpendicular from *S* to *QV*. *ST* is the perpendicular from *S* to *PT*. The angle *SPT* equals the angle *B*.

Thus:

$$\begin{aligned} r \sin(A+B) &= PR = PT + TR = PS \cos B + SV \\ &= r \sin A \cos B + QS \sin B \\ &= r \sin A \cos B + r \cos A \sin B \\ \therefore \sin(A+B) &= \sin A \cos B + \cos A \sin B \end{aligned} \qquad \text{. . . \textbf{A1.20}}$$

and:

$$\begin{aligned} r \cos(A+B) &= QR = QV - RV = QS \cos B - TS \\ &= r \cos A \cos B - PS \sin B \\ &= r \cos A \cos B - r \sin A \sin B \\ \therefore \cos(A+B) &= \cos A \cos B - \sin A \sin B \end{aligned} \qquad \text{. . . \textbf{A1.21}}$$

and:

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

Dividing top and bottom by (cos *A* cos *B*):

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \text{. . . \textbf{A1.22}}$$

Sin (A − B), cos (A − B) and *tan (A − B)* may be found from formulae A1.20 to A.122, as *sin (−B) = −sin B, cos (−B) = cos B* and *tan (−B) = −tan B*, and substituting these values.

Thus:

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \qquad \text{. . . \textbf{A1.23}}$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \qquad \text{. . . \textbf{A1.24}}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \qquad \text{. . . \textbf{A1.25}}$$

(9) b. **Double and Half-Angle Formulae.** If A is equal to B, it follows from formulae A1.20 to A1.22, that:

$$\sin 2A = 2 \sin A \cos A \quad \text{. . . A1.26}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \quad \text{. . . A1.27} \\ &= 1 - 2\sin^2 A \\ &= 2\cos^2 A - 1 \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad \text{. . . A1.28}$$

In terms of the half-angle these formulae become:

$$\sin A = 2 \sin ½A \cos ½A \quad \text{. . . A1.29}$$

$$\begin{aligned} \cos A &= \cos^2 ½A - \sin^2 ½A \quad \text{. . . A1.30} \\ &= 1 - 2\sin^2 ½A \\ &= 2\cos^2 ½A - 1 \end{aligned}$$

$$\tan A = \frac{2 \tan ½A}{1 - \tan^2 ½A} \quad \text{. . . A1.31}$$

c. **Sum and Difference of Functions.** The above formulae, relating to the sines and cosines of sums and differences, may be combined to give other formulae which relate to the sums and differences of sines and cosines.

- **Adding Formulae (A1.20) and (A1.23).** By adding formulae (A1.20) and (A1.23), and writing P for $(A + B)$ and Q for $(A - B)$ so that A is equal to $½(P + Q)$ and B to $½(P - Q)$:

$$\begin{aligned} \sin (A + B) + \sin (A - B) &= 2 \sin A \cos B \\ \sin P + \sin Q &= 2 \sin ½(P + Q) \cos ½(P - Q) \quad \text{. . . A1.32} \end{aligned}$$

- **Subtracting Formula (A1.23) from Formula (A1.20).** By subtracting formula (A1.23) from formula (A.1.20):

$$\begin{aligned} \sin (A + B) - \sin (A - B) &= 2 \cos A \sin B \\ \sin P - \sin Q &= 2 \cos ½(P + Q) \sin ½(P - Q) \quad \text{. . . A1.33} \end{aligned}$$

- **Use of Formulae (A1.21) and (A1.24).** By using formulae (A1.21) and (A1.24), it can be shown that:

$$\cos P + \cos Q = 2 \cos ½(P + Q) \cos ½(P - Q) \quad \text{. . . A1.34}$$

$$\cos P - \cos Q = -2 \sin ½(P + Q) \sin ½(P - Q) \quad \text{. . . A1.35}$$

10. Sine and Cosine of Small Angles

Certain approximations are possible in the sine and cosine of angles, provided the angle is small.

a. **Sine of Small Angles.** In Fig A1-11 (below), *AOB* is a small angle θ, measured in radians. *AB* is the arc of a circle which subtends this small angle. The radius of a circle is *r*, and *BC* is perpendicular to *OA* at *C*. The length of arc of a circle is equal to the radius multiplied by the angle subtended in radians (see Appendix 1, Para 2b). Thus:

$$AB = r \times \theta$$

or

$$\theta = \frac{AB}{r}$$

but

$$\sin\theta = \frac{BC}{r}$$

Therefore, when θ is sufficiently small for *AB* to approximate to *BC*:

$$\sin\theta = \theta$$

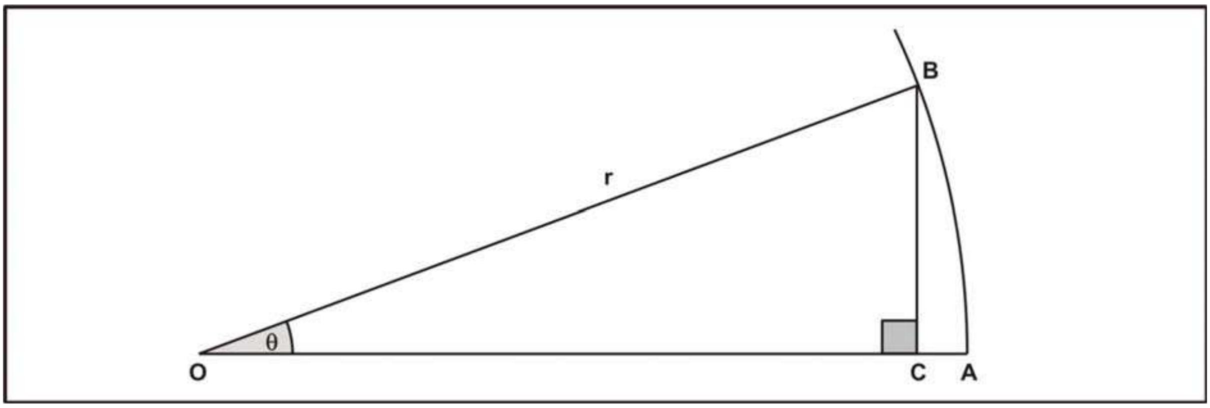


Fig A1-11. The Sine of a Small Angle

If there are *x* minutes in this small angle of θ radians, then there must be $\frac{x}{\theta}$ minutes in 1 radian. But 1 radian is equal to $\frac{360°}{2\pi}$ or 3437.7468 minutes of arc.

hence

$$\frac{x}{\theta} = 3437'.7468$$

or

$$\theta = \frac{x}{3437'.7468}$$

The relation $\sin\theta = \theta$ therefore becomes:

$$\sin x' = \frac{x}{3437'.7468}$$

Since this relation holds for any value of *x* that is small:

$$\sin 1' = \frac{1}{3437'.7468}$$

$$\therefore \quad \sin x' = x \sin 1' \qquad \text{. . . A1.36}$$

(10) b. **Cosine of a Small Angles.** In Fig A1-12 (below), when θ is small, *OC* approximates to *OA*, which is the same as *OB*.

But: $$\cos\theta = \frac{OC}{OB}$$

Therefore, when θ is small, $\cos\theta$ is equal to 1.

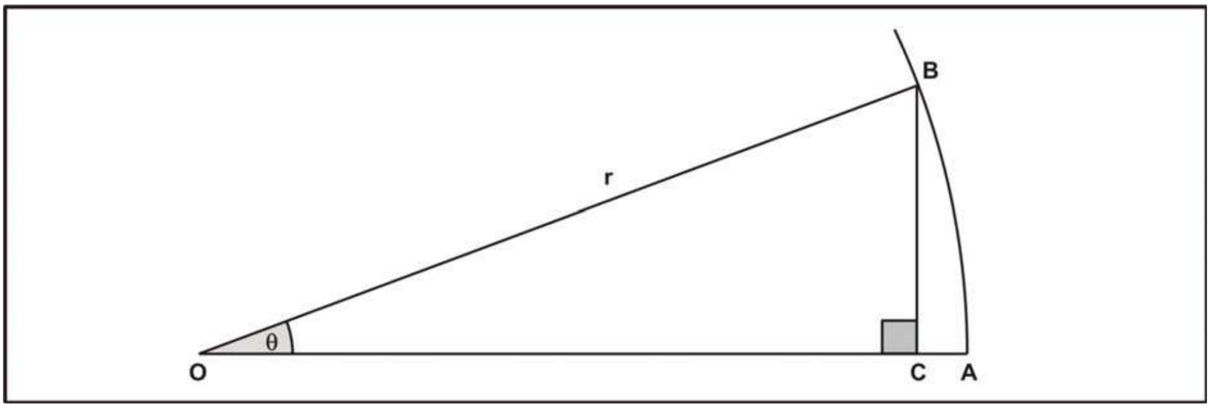


Fig A1-12. The Cosine of a Small Angle

A second approximation can be obtained if $\cos\theta$ is expressed in terms of the half-angle (in radians), for then:

$$\cos\theta = 1 - 2\sin^2 \tfrac{1}{2}\theta$$

$$\cos\theta = 1 - 2(\tfrac{1}{2}\theta)^2$$

$$\therefore \quad \cos\theta = 1 - \tfrac{1}{2}\theta^2 \qquad \text{. . . } \mathbf{A1.37}$$

APPENDIX 2

SPHERICAL TRIGONOMETRY

1. Scope of Appendix

Trigonometry is the branch of mathematics dealing with the relations between the angles and sides of a triangle and with the relevant functions of any angles. Appendix 2 contains the following information:

- **Para 2: Spherical Trigonometry Definitions**
- **Para 3: Properties of a Spherical Triangle**
- **Para 4: The Solution of a Spherical Triangle**
- **Para 5: The Cosine Formula**
- **Para 6: The Sine Formula**
- **Para 7: Polar Spherical Triangles and the Polar Cosine Rule**
- **Para 8: The Four-Part Formula**
- **Para 9: Right-Angled Spherical Triangles**
- **Para 10: Napier's Mnemonic Rules for Right-Angled Spherical Triangles**
- **Para 11: Quadrantal Spherical Triangles**
- **Para 12: The Versine and Haversine**
- **Para 13: The Haversine Formula**
- **Para 14: The Half-Log Haversine Formula**
- **Para 15: Haversine / Half Log Haversine Solution - Example**

2. Spherical Trigonometry Definitions

a. **Spherical Trigonometry**. Spherical trigonometry is the science of trigonometry (see Appendix 1) when applied to triangles marked on the surface of a *Sphere* by planes through its centre.

b. **Axis (of the Earth).** The Earth's *Axis* is its shortest diameter (*PP*'), about which it rotates in space (defined at Para 0110c).

c. **Sphere.** A *Sphere* is defined as a surface, every point on which is equidistant from one and the same point, called the 'centre'. The distance of the surface from the centre is called the 'radius' of the *Sphere*.

d. **Great Circle.** A *Great Circle* is the intersection of a *Spherical* surface and a plane which passes through the centre of the *Sphere*. It is the shortest distance between two points on the surface of a *Sphere* (defined at Para 0110c).

e. **Small Circle.** A *Small Circle* is the intersection of a *Spherical* surface and a plane which does NOT pass through the centre of the *Sphere* (defined at Para 0110c).

f. **Spherical Triangle.** Any three-sided figure *ABC* in Fig A2-1 (overleaf), formed by the minor arcs of three *Great Circle*s on the *Spherical* surface is known as a *Spherical Triangle.* The side of a *Spherical Triangle* is the angle it subtends at the centre of the *Sphere* and may be measured in degrees and minutes, or radians. In Fig A2-1, *ABC* is a *Spherical Triangle* formed by the minor arcs of three *Great Circle*s, *AB*, *AC* and *BC*. The length *a* of the side *BC* is equal to the angle subtended at the centre of the *Sphere*, that is, *BOC*. Similarly, *b* and *c* are equal to the angles *AOC* and *AOB*.

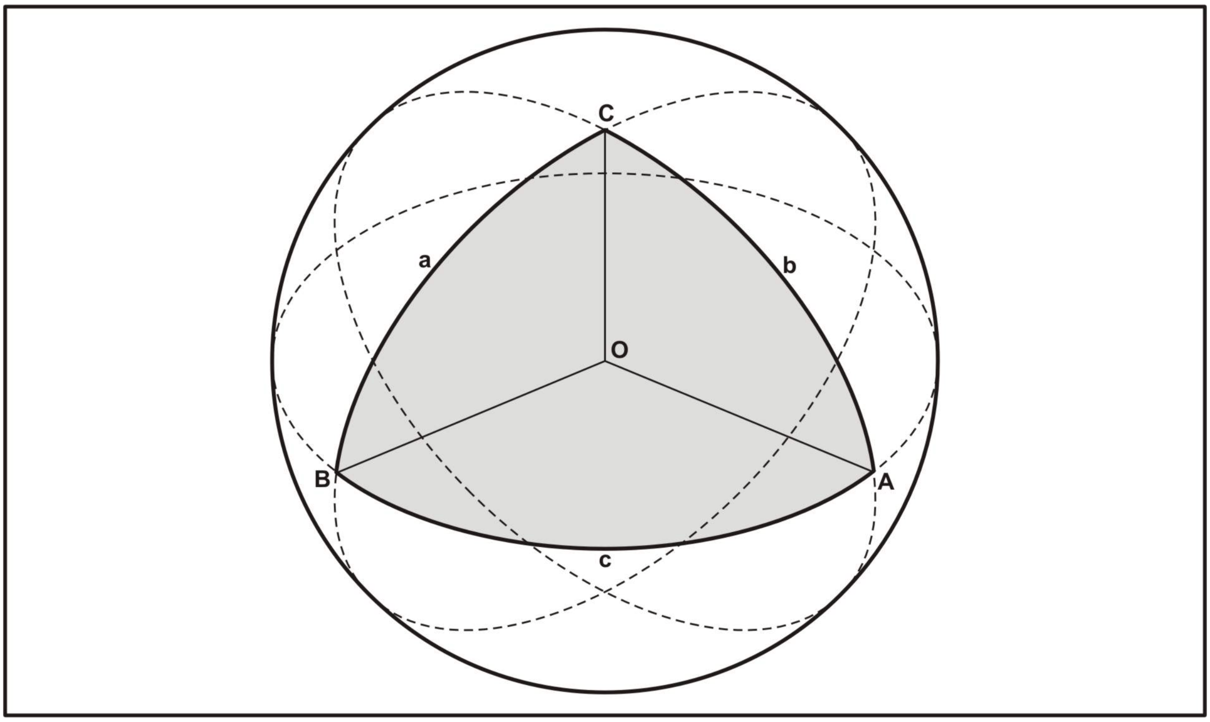


Fig A2-1. The Spherical Triangle

(2) g. **Spherical Angles.** In a *Spherical Triangle* (see Fig A2-1), the angle *A* is the angle between the planes containing the *Great Circle*s *AB* and *AC*, that is, the angle between the plane *AOB* and the plane *AOC*. Similarly, the angle *B* is the angle between the planes *AOB* and *COB*, and the angle *C* is the angle between the planes *AOC* and *COB*. In a *Spherical Triangle ABC*, it is customary to refer to its angles as *A, B* and C, and to the sides opposite these angles as *a, b* and *c*. This is analogous to the conventions adopted in a *Plane Triangle* set out in Appendix 1.

3. Properties of a Spherical Triangle

Certain properties of a *Spherical Triangle* are equivalent to those of a *Plane Triangle*: the largest angle is always opposite the largest side, the smallest angle is always opposite the smallest side and one side is always less than the sum of the other two sides (eg $c < a + b$ in Fig A2-1). However, there are two very important differences between *Spherical Triangle*s and *Plane Triangle*s.

a. **The Sum of the Three Angles of the Spherical Triangle.** The sum of the three angles of the *Spherical Triangle* $A + B + C$ is <u>always greater</u> than 180˚ (π radians) and <u>always less</u> than 540˚ (3π).

b. **The Sum of the Three Sides of the Spherical Triangle.** The sum of the three sides of the *Spherical Triangle* $a + b + c$ is always less than 360˚ (2π).

4. The Solution of a Spherical Triangle

A *Spherical Triangle* has six dimensions: the sizes of its three angles and the lengths of its three sides. Various formulae connect these angles and sides so that, if sufficient of them are given, the rest can be found. The common calculations are those of finding the third side when two sides and their included angle are known, and finding a particular angle when the three sides are known.

5. The Cosine Formula

In Fig A2-2 (below), *O* is the centre of the *Sphere* of radius *R*. *AB, BC* and *CA* are the minor arcs of three *Great Circle*s forming the *Spherical Triangle ABC* on the surface of the *Sphere*. *OA* = *OB* = *OC* = *R*. Angle *BOC* = *a*, angle *AOC* = *b* and angle *BOA* = c .

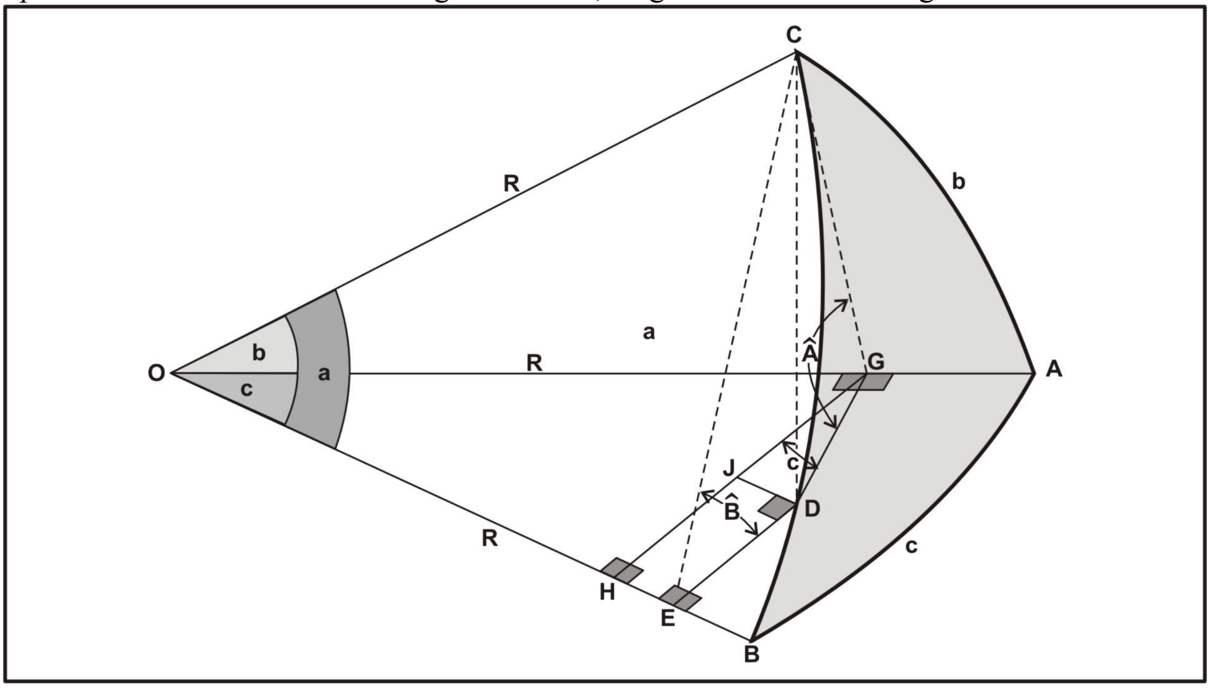


Fig A2-2. Spherical Trigonometry - The Cosine and Sine Formulae

CD is the perpendicular from *C* to the plane *OAB* and *CE* is the perpendicular from *C* to the line *OB*. ∴ *DE* is perpendicular to OB and angle *CED* = *Spherical* angle at *B*.

Similarly, *CG*, is the perpendicular from *C* to the line OA. ∴ *DG* is perpendicular to *OA*, and angle *CGD* = *Spherical* angle at *A*.

GH is the perpendicular from *G* on to *OB*, and *DJ* is the perpendicular from *D* on to *GH*. *JD* is parallel and equal to HE. Angle *JGD* = *c*.

In the triangle *COE*, which is right-angled at *E*:

$$\frac{OE}{OC} = \cos a \qquad \therefore \quad OE = R \cos a$$

$$\begin{aligned} \text{but } OE &= OH + HE \\ &= OG \cos c + GD \sin c \\ &= R \cos b \cos c + CG \cos A \sin c \\ &= R \cos b \cos c + R \sin b \cos A \sin c \end{aligned}$$

∴ Cosine Rule: $\cos a = \cos b \cos c + \sin b \sin c \cos A$ . . . **A2.1**

and Similarly: $\cos b = \cos c \cos a + \sin c \sin a \cos B$ . . . **A2.2**

$\cos c = \cos a \cos b + \sin a \sin b \cos C$ . . . **A2.3**

Thus, if any two sides and their included angle are given, the third side may be found, this side being the one opposite the only *Spherical* angle in the formula. Such formulae are analagous to the *Cosine Formulae* for the *Plane Triangle* set out in Appendix 1. When all three sides of the *Spherical Triangle* are known, the angle may be found by transposing the relevant *Cosine Formula*. For example, from formula (A2.1):

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \qquad \text{. . . A2.4}$$

6. The Sine Formula

In Fig A2-2 (previous page), in the triangles *CED* and *CGD*, which are both right-angled at *D*:

$$\frac{CD}{CE} = \sin B \quad \text{and} \quad \frac{CD}{CG} = \sin A$$

$$\text{thus: } CE \sin B = CD = CG \sin A$$

$$\therefore \quad R \sin a \sin B = R \sin b \sin A$$

$$\text{ie} \quad \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} \quad \text{and, by symmetry:}$$

$\therefore$ **Sine Rule:**

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad \text{. . . A2.5}$$

The *Sine Formula* for the *Spherical Triangle* is analogous to the *Sine Formula* for the *Plane Triangle* set out in Appendix 1, and has the same limitation in that ambiguity arises if it is used to solve the triangle when two sides and one angle are given. It must be remembered that as sin $\theta = \sin(180^\circ - \theta)$, there is no way of knowing from the formula alone whether the quantity found is greater or less than 90˚.

In the example at Fig A2-3 (below), *a* is 66˚, *b* is 50˚ and *B* is 40˚.

$$\sin A = \frac{\sin a}{\sin b} \text{ x } \sin B$$

$$= \sin a \sin B \operatorname{cosec} b$$

$$= \sin 66^\circ \sin 40^\circ \operatorname{cosec} 50^\circ = 0.76657$$

$$A = 50^\circ 02'.8 \text{ or } 129^\circ 57'.2$$

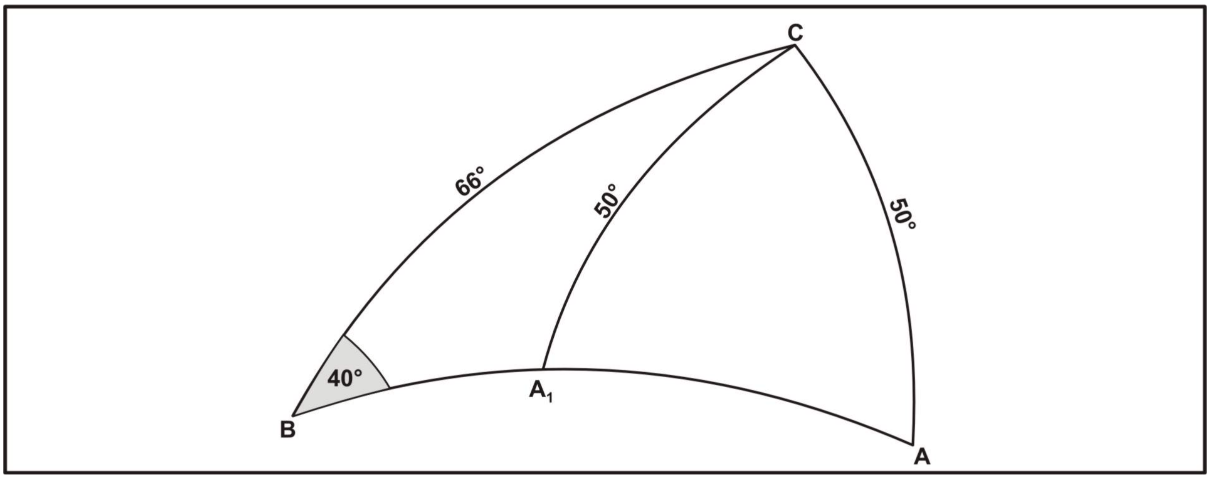


Fig A2-3. Spherical Trigonometry - Ambiguity in Sine Formula

Fig A2-3 (above) shows that *ABC* and A_1BC are possible triangles. The ambiguity, however, may often be resolved in practice and the formula is easier and quicker to use on a calculator than the *Cosine Formula*. The *Sine Formula* may therefore often be used to find the *Great Circle* course, having first found the *Distance*.

When the complete solution of the *Spherical Triangle* is found using the *Cosine Formula*, the sine rule is a useful cross-check against the accuracy of its workings:

$$\text{ie:} \quad \frac{\sin a}{\sin A}, \ \frac{\sin b}{\sin B} \ \text{and} \ \frac{\sin c}{\sin C}$$

7. Polar Spherical Triangles and the Polar Cosine Rule

The *Polar Cosine Rule* may be used to calculate a side of the *Spherical Triangle* given all three angles. It may also be used to calculate an angle given the other two angles and the opposite side.

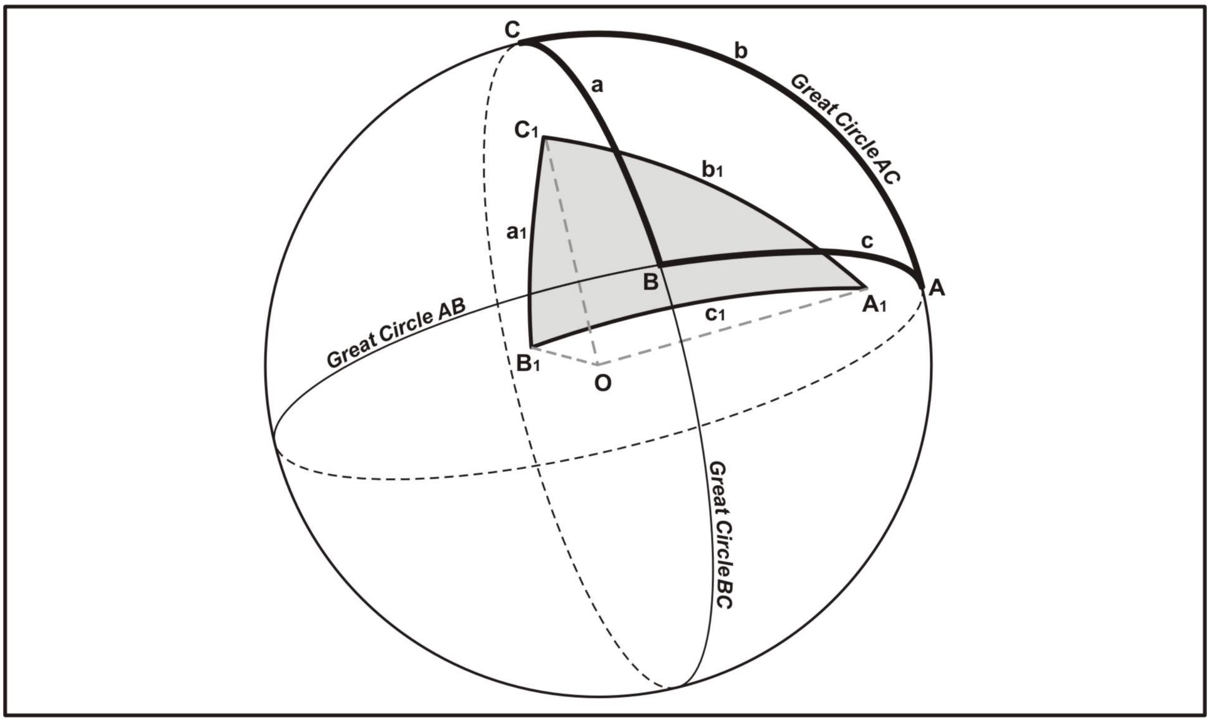


Fig A2-4. Polar Triangles

In the same way as the *Equator* is related to the Earth's *Axis* (which cuts the Earth's surface at the North and South *Poles* - see definition at Para 2b), so every *Great Circle* has an *'Axis'* and two '*Poles'*. *Polar Triangle* $A_1B_1C_1$ of the *Spherical Triangle ABC* (see Fig A2-4 above) is formed as follows:

- A_1 is the '*Pole*' of the *Great Circle BC* on the same side of *BC* as *A*. OA_1 is perpendicular to the plane of the *Great Circle* through *BC*.
- B_1 is the '*Pole*' of the *Great Circle AC*. OB_1 is perpendicular to the plane of the *Great Circle* through *AC*.
- C_1 is the '*Pole*' of the *Great Circle AB*. OC_1 is perpendicular to the plane of the *Great Circle* through *AB*.

The two triangles *ABC* and $A_1B_1C_1$ are mutually *Polar*. In the *Polar Triangle* $A_1B_1C_1$:

$$a_1 = \pi - A \qquad A_1 = \pi - a$$
$$b_1 = \pi - B \qquad B_1 = \pi - b$$
$$c_1 = \pi - C \qquad C_1 = \pi - c$$

If these values are substituted in the *Cosine Rule* (formula A2.1), the *Polar Cosine Rule* formula is obtained:

Polar Cosine Rule: $$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C} \qquad \text{. . . A2.6}$$

Formula (A2.6) may be used to calculate a side of the *Spherical Triangle* given all three angles. It may also be used to calculate an angle given the other two angles and the opposite side.

8. The Four-Part Formula

The *Four-Part Formula* is one in which the terms are four consecutive angles and sides of any *Spherical Triangle* (see Fig A2-5 below). It may be used to find the initial *Course* or final *Course* direct from *Latitude* and *Longitude* without first finding the *Great Circle Distance*.

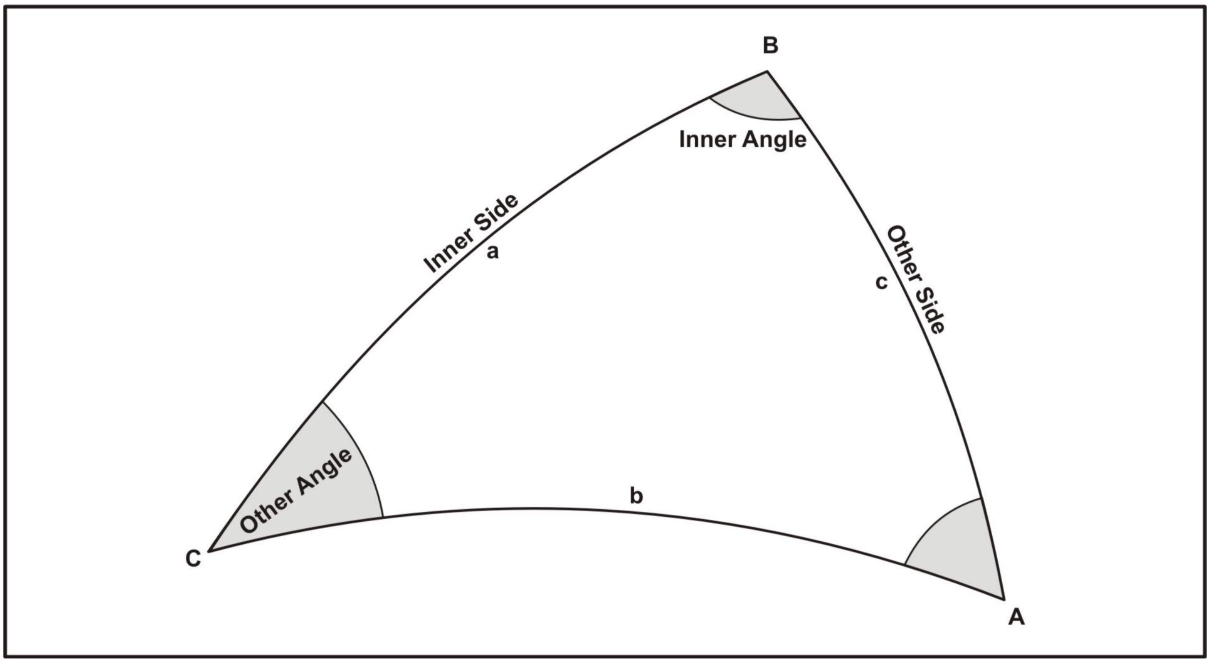


Fig A2-5. The Four-Part Spherical Triangle

In Fig A2-5 (above), the four parts to be considered are *C*, *a*, *B* and *c*. The angle *B*, contained by the two sides *a* and *c*, is called the 'inner angle' or 'IA'. The side *a*, common to the angles *B* and *C*, is called the 'inner side' or 'IS'. The others are the 'other angle' *C*, denoted by '*OA*', and the 'other side' *c*, denoted by 'OS'.

The *Four-Part Formula* (A2.7) states that:

$$\cos(IS)\cos(IA) = \sin(IS)\cot(OS) - \sin(IA)\cot(OA) \qquad \ldots \textbf{A2.7}$$

It may be proved thus:

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

By substituting for cos *b*:

$$\cos c = \cos a\,(\cos c \cos a + \sin c \sin a \cos B) + \sin a \sin b \cos C$$
$$\cos c = \cos c\,(1 - \sin^2 a) + \sin a \cos a \sin c \cos B + \sin a \sin b \cos C$$

Therefore, since cos *c* cancels out and sin *a* is common to the remaining terms:

$$\sin a \cos c = \cos a \sin c \cos B + \sin b \cos C$$

$$\sin a \frac{\cos c}{\sin c} = \cos a \cos B + \frac{\sin b \cos C}{\sin c}$$

Hence, by the *Sine Formula*:

$$\sin a \cot c = \cos a \cos B + \frac{\sin B \cos C}{\sin C}$$

$$\cos a \cos B = \sin a \cot c - \sin B \cot C$$

9. Right-Angled Spherical Triangles

If one angle of a *Spherical Triangle* is a right angle, the formulae for solving the triangle are greatly simplified.

Thus, if the angle *C* in the triangle *ABC* is a right angle (see Fig A2-6 below), the *Cosine Formula* (A2.3) becomes:

$$\cos c = \cos a \cos b \qquad \text{. . . A2.8}$$

And, the *Sine Formula* (A2.5) becomes:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \sin c \qquad \text{. . . A2.9}$$

The numerous formulae thus obtainable are best summarised by *Napier's Rules* (see Para 10 below).

10. Napier's Mnemonic Rules for Right-Angled Spherical Triangles

Right-angled *Spherical Triangle*s may be used:

- To find the position of the *Vertex* on a *Great Circle*.
- To solve an isosceles triangle where two points are in the same *Latitude*, by bisecting the triangle.
- To find where a *Great Circle* cuts the *Equator*.
- To solve the *Composite Track*.

Triangle *ABC* (see Fig A2-6) is 'extended' to form symbolic five quantities [displayed clockwise: *a, b,* ($90° - A$), ($90° - C$) and ($90° - B$)]. These quantities may also be shown as the sectors of a circle having a vertical radius that represents the right angle at *C*.

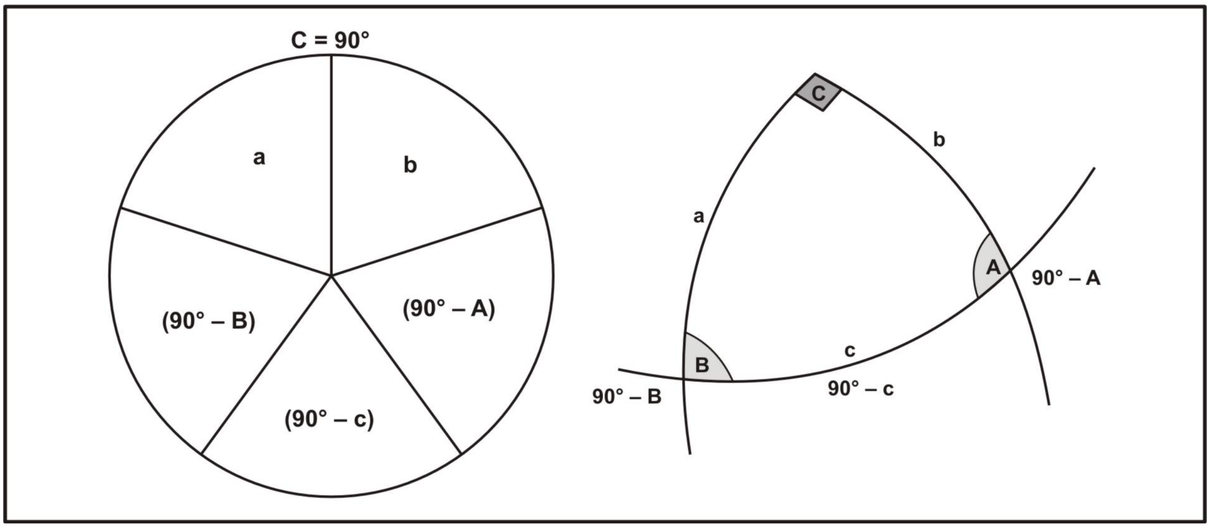


Fig A2-6. Napier's Right-Angled Triangles

If any one of the quantities at Fig A2-6 (above) is taken as the 'middle' quantity, two of the other four quantities become 'adjacent' and the remaining two quantities become 'opposite', *Napier's Rules* thus become:

Napier's Rules: *sin middle = products of tans of adjacents* . . . **A2.10**
Napier's Rules: *sin middle = products of cosines of opposites* . . . **A2.11**

(10 - Napiers' Rules continued)

In triangle *ABC* (see Fig A2-6 previous page), which is right-angled at *C*, *Napier's Rules* give ten formulae (see Table A2-1 below), which may also be derived from the various formulae described earlier.

Table A2-1. Napier's Ten Rules

MIDDLE	FORMULA	ALSO DERIVED FROM
a	$\sin a = \tan b \cot B$ $\sin a = \sin c \sin A$	Four-Part Formula Sine Rule
b	$\sin b = \tan a \cot A$ $\sin b = \sin c \sin B$	Four-Part Formula Sine Rule
$(90^\circ - A)$	$\cos A = \tan b \cot c$ $\cos A = \cos a \sin B$	Four-Part Formula Polar Cosine Rule
$(90^\circ - c)$	$\cos c = \cot A \cot B$ $\cos c = \cos a \cos b$	Polar Cosine Rule Cosine Rule
$(90^\circ - B)$	$\cos B = \tan a \cot c$ $\cos B = \cos b \sin A$	Four-Part Formula Polar Cosine Rule

11. Quadrantal Spherical Triangles

A *Quadrantal Spherical Triangle* is a *Spherical Triangle* where one side is equal to 90˚ (eg side *c* in Fig A2-7 below). As with the right-angled *Spherical Triangle*, the *Quadrantal Spherical Triangle* may be 'extended' and a five-part figure constructed. The symbolic five quantities are now [A, B, $(90^\circ - a)$, $(C - 90^\circ)$, $(90^\circ - b)$]. These quantities may also be combined in accordance with *Napier's Rules* (formulae A2.10 and A2.11), as follows:

$$\sin A = \tan B \cot b$$
$$\sin A = \sin a \sin C$$
$$\cos a = \cos A \sin b$$
(etc)

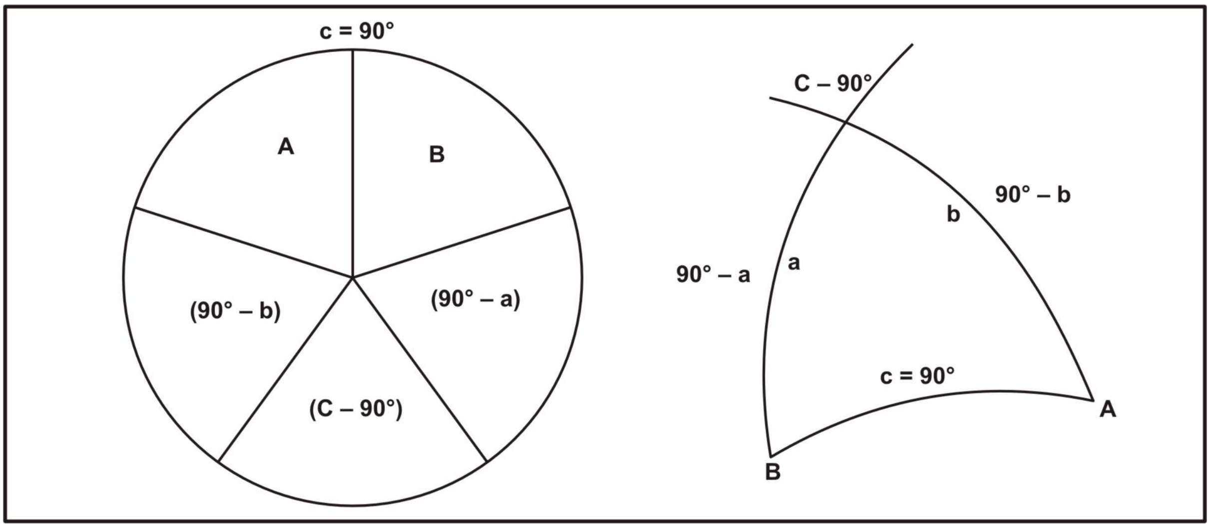


Fig A2-7. Napier's Quadrantal Spherical Triangles

12. The Versine and Haversine

If obliged to use logarithmic tables (instead of a calculator, when the *Cosine / Sine Rules* are more useful), it is more convenient to solve a *Spherical Triangle* using a function called the '*Haversine*' of the angle. This function is half the 'versine' − hence the name *Haversine* − and the versine of an angle is defined as the difference between its *Cosine* and unity, that is:

Versine: $versine\ \theta = 1 - \cos\theta$ . . . **A2.12**

and it follows that:

Haversine: $haversine\ \theta = \tfrac{1}{2}(1 - \cos\theta)$ . . . **A2.13**

The *Haversine* of an angle is thus always positive, and increases from 0 to 1 as angles increase from 0˚ to 180˚. Fig A2-8 shows the *Haversine* curve in relation to the *Cosine* curve from which it is derived. Norie's Tables give the values of *Haversines* between 0˚ and 360˚.

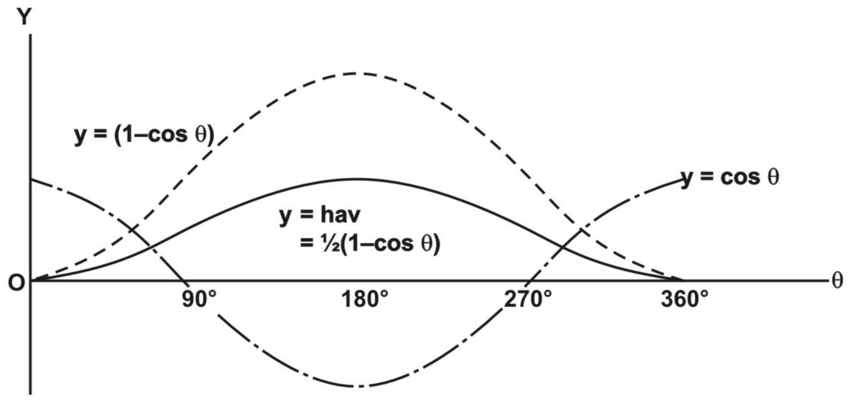


Fig A2-8. The Haversine Curve

13. The Haversine Formula

To express the *Cosine Rule* in terms of *Haversines* instead of *Cosines*, substitute for the appropriate *Cosines* their values in terms of the *Haversines*.

Thus *cos A* can be written ($1 - 2\ hav\ A$), and the formula becomes:

$$\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c\,(1 - 2\ hav\ A) \\ \cos a &= \cos b \cos c + \sin b \sin c - 2 \sin b \sin c\ hav\ A \\ \cos a &= \cos(b \sim c) - 2 \sin b \sin c\ hav\ A \end{aligned}$$

Similar substitutions for *cos a* and *cos (b ~ c)* give:

$$1 - 2\ hav\ a = 1 - 2\ hav\,(b \sim c) - 2 \sin b \sin c\ hav\ A$$

Haversine: $hav\ a = hav\,(b \sim c) + \sin b \sin c\ hav\ A$ . . . **A2.14**

Great Circle Distance.

When calculating the *Great Circle Distance* between two points, *F* and *T*, with known *Latitudes* and *Longitudes*, the *Haversine Formula* (A2.14) becomes:

hav *FT* = hav *FPT* sin *PF* sin *PT* + hav (*PF* ~ *PT*)

hav *FT* = hav (d.long) sin (90˚ − lat F) sin (90˚ ± lat T) + hav [(90˚ − lat F) ~ (90˚ ± lat *T*)]

F may be in either North or South *Latitudes*, thus:

GC Distance: hav dist = hav d.long cos lat *F* cos lat *T* + hav (co-lat *F* ~ co-lat *T*) . . . **A2.16**

14. The Half-Log Haversine Formula

The *Half-Log Haversine Formula*, which gives one of the angles when the three sides are known, is derived from the *Cosine Rule* by making substitutions similar to those used in building the *Haversine Formula*.

As before, the first substitution gives:

$$\cos a = \cos (b \sim c) - 2 \sin b \sin c \text{ hav } A$$
$$2 \sin b \sin c \text{ hav } A = \cos (b \sim c) - \cos a$$

By the rule for the subtraction of two *Cosines*, this equation becomes:

$$2 \sin b \sin c \text{ hav } A = 2 \sin \tfrac{1}{2} [a + (b \sim c)] \sin \tfrac{1}{2} [a - (b \sim c)]$$

Therefore, by division:

$$\text{hav } A = \text{cosec } b \text{ cosec } c \sin \tfrac{1}{2} [a + (b \sim c)] \sin \tfrac{1}{2} [a - (b \sim c)]$$

But, from the definition of the *Haversine*:

$$\text{hav } x = \tfrac{1}{2} (1 - \cos x) = \tfrac{1}{2} [1 - (1 - 2 \sin^2 \tfrac{1}{2}x)]$$
$$\text{hav } x = \sin^2 \tfrac{1}{2}x$$

Therefore, by analogy:

$$\sin \tfrac{1}{2} [a + (b \sim c)] = \sqrt{\text{hav } [a + (b \sim c)]}$$
$$\sin \tfrac{1}{2} [a - (b \sim c)] = \sqrt{\text{hav } [a - (b \sim c)]}$$

By substitution:

$$\text{hav } A = \text{cosec } b \ \text{cosec } c \sqrt{\text{hav } [a + (b \sim c)] \text{ hav } [a - (b \sim c)]}$$

In logarithmic form the *Half-Log Haversine Formula* becomes:

Half-Log Haversine Formula:

$$\log \text{hav } A = \log \text{cosec } b + \log \text{cosec } c + \tfrac{1}{2} \log \text{hav } [a + (b \sim c)] + \tfrac{1}{2} \log \text{hav } [a - (b \sim c)] \quad \ldots \textbf{A2.15}$$

Initial Great Circle Course / Great Circle Bearing.

When calculating the *Great Circle* initial *Course* when sailing on a *Great Circle* track from one point to another (or the bearing of one point on the Earth's surface from another), the *Half-Log Haversine Formula* (A2.15) is applied. Thus, the bearing of *T* from *F* is given by:

$$\text{hav } PFT = \text{cosec} PF \ \text{cosec} FT \sqrt{\text{hav } [PT + (PF \sim FT)] \text{ hav } [PT - (PF \sim FT)]}$$

$$\log \text{hav } PFT = \log \text{cosec} PF \ \log \text{cosec} FT + \tfrac{1}{2} \log \text{hav } [PT + (PF \sim FT)] + \tfrac{1}{2} \log \text{hav } [PT - (PF \sim FT)]$$

Initial GC Course:

$$\begin{aligned} \log \text{hav initial course} &= \log \text{cosec co-lat } F + \log \text{cosec distance} \\ &\quad + \tfrac{1}{2} \log \text{hav}[\text{co-lat } T + (\text{co-lat } F \sim \text{distance})] \\ &\quad + \tfrac{1}{2} \log \text{hav}[\text{co-lat } T - (\text{co-lat } F \sim \text{distance})] \end{aligned} \quad \ldots \textbf{A2.17}$$

The *haversine* / *Half-Log Haversine* solution to Example 2-6 from Para 0211 is set out opposite.

15. Haversine / Half Log Haversine Solution to Example 2-6 from Para 0211

A ship proceeds from position F (45˚N, 140˚E) to T (65˚N, 110˚W). Find the *Great Circle Distance* and the initial *Course*.

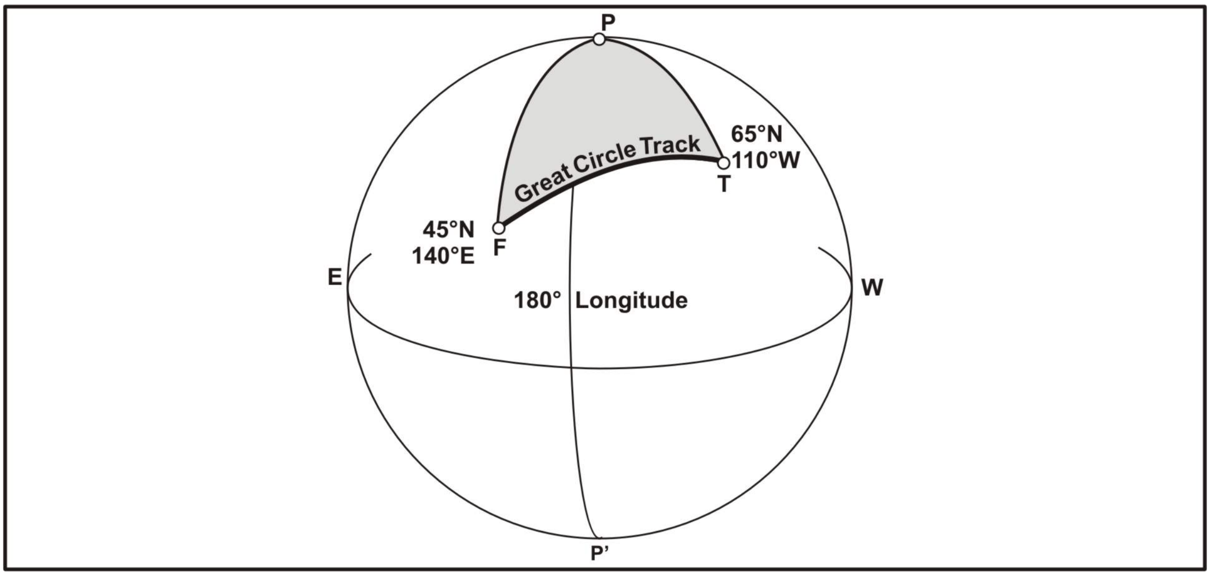


Fig A2-9 (Copy of Fig 2-14). Ship's Great Circle Track from Example 2-6

Great Circle Distance:

From formula (A2.16):

$$\text{hav dist} = \text{hav d.long} \cos \text{lat F} \cos \text{lat T} + \text{hav (co-lat F} \sim \text{co-lat T)}$$

Thus:

$$\begin{aligned} \text{hav dist} &= \text{hav } 110^\circ \cos 45^\circ \cos 65^\circ + \text{hav } (45^\circ \sim 25^\circ) \\ &= \text{hav } 110^\circ \cos 45^\circ \cos 65^\circ + \text{hav } 20^\circ \\ &= 0.67101 \times 0707107 \times 0.422618 + 0.030154 \\ &= 0.230676 \\ &= \text{hav } 57^\circ 24.5' \end{aligned}$$

$\therefore$ ***Great Circle Distance = 3444.5'***

Initial Great Circle Course:

From formula (A2.17):

$$\begin{aligned} \text{log hav initial course} &= \text{log cosec co-lat F} + \text{log cosec distance} \\ &\quad + \tfrac{1}{2} \text{ log hav[co-lat T} + \text{(co-lat F} \sim \text{distance)]} \\ &\quad + \tfrac{1}{2} \text{ log hav[co-lat T} - \text{(co-lat F} \sim \text{distance)]} \end{aligned}$$

Thus:

$$\begin{aligned} \text{log hav initial Course} &= \text{log cosec } 45^\circ + \text{log cosec } 57^\circ 24.5' \\ &\quad + \tfrac{1}{2} \text{ log hav}[25^\circ - 12^\circ 24.5'] \\ &\quad + \tfrac{1}{2} \text{ log hav}[25^\circ + 12^\circ 24.5'] \\ &= \text{log cosec } 45^\circ + \text{log cosec } 57^\circ 24.5' \\ &\quad + \tfrac{1}{2} \text{ log hav } 12^\circ 35.5' + \tfrac{1}{2} \text{ log hav } 37^\circ 24.5' \\ &= 0.150515 + 0.074414 + \bar{1}.040056 + \bar{1}.506074 \\ &= \bar{2}.771059 \end{aligned}$$

$\therefore$ ***Initial GC Course = N 28˚07.3'E = 028˚***

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APPENDIX 3

THE SPHERICAL EARTH

1. Scope of Appendix

Appendix 3 contains the following information:

- **Para 2: Meridional Parts for the Sphere**
- **Para 3: Construction of the 'Mer Parts' Formula for the Sphere**
- **Para 4: Evaluation of the 'Mer Parts' Formula for the Sphere**
- **Para 5: Corrected Mean Latitude for the Sphere**

2. Meridional Parts for the Sphere

As stated at Para 0422b:

The Meridional Parts of any Latitude are the number of 'Longitude Units' in the length of a Meridian between the Parallel of that Latitude and the Equator. A 'Longitude Unit' is the length on the chart representing one minute of arc in Longitude.

3. Construction of the 'Mer Parts' Formula for the Sphere

In Fig A3-1 (below), *X* is any point on the Earth in *Latitude* ϕ, and *Y* is a neighbouring point differing from it in *Latitude* by the small amount $\Delta\phi$. X_1 and Y_1 are the corresponding points on the *Mercator Projection* chart. As all *Meridians* are straight lines at right angles to the *Equator*, A_1B_1 is equal to X_1Z_1.

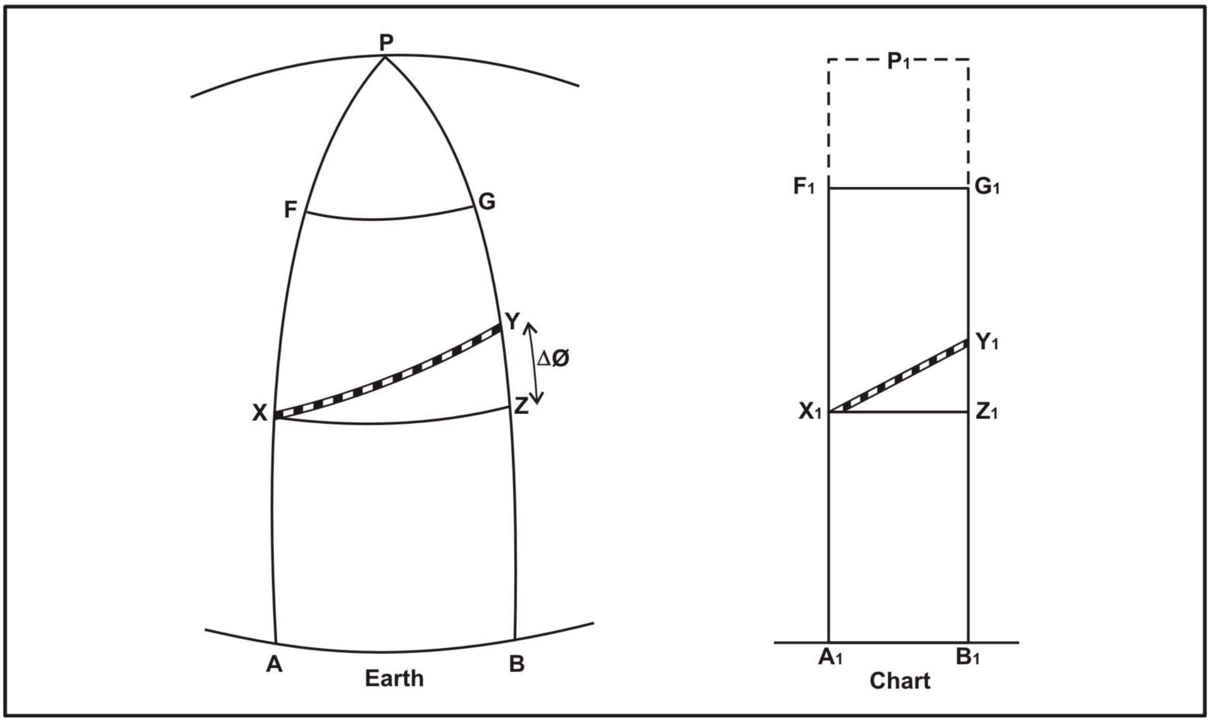

Fig A3-1. Construction of the 'Mer Part' Formula

a. **Scale of the Chart.** The ratio that the chart length A_1B_1 bears to the geographical distance *AB* decides the *Longitude Scale* of the chart. That is, when *AB* and A_1B_1 are expressed in the same units, A_1B_1 is some fraction of *AB* (ie *AB* is equal to kA_1B_1 where *k* is some constant).

(3) **Example A3-1.** If *AB* is 1 minute of arc and A_1B_1 is 1 mm, 1 mm on the chart is equivalent to 1 minute of arc or approximately 1,853,300 mm on the Earth, and *k* is 1,853,300. The value of *k* thus determines the size of the chart.

b. **Actual Measurement of Meridional Parts.** However, for the actual measurement of *Meridional Parts*, it is sufficient to know the chart unit that represents 1 minute of arc along the *Equator*. In the Example A3-1 (above), where 1 mm represents 1 minute, the *Meridional Parts* of X_1 are simply the number of millimetres in X_1A_1.

c. **Calculation of Chart Length.** To calculate this chart length and so determine the number of minutes of arc along the *Equator* to which it is equivalent, consider the distortion that occurs away from the *Equator*:

XZ is the *Parallel* through *X*, and *XY* the *Rhumb Line* joining *X* to *Y*. On the chart X_1Z_1 is the *Parallel* and X_1Y_1 the *Rhumb Line*, both lines being straight. Then, if all lengths *XZ, AB,* A_1B_1 ... are measured in the same units:

$$XZ = AB\cos\phi \qquad \text{. . . (formula 2.1)}$$

$$= kA_1B_1\cos\phi$$

$$= kX_1Z_1\cos\phi$$

$$\text{ie}\quad X_1Z_1 = \frac{1}{k}XZ\sec\phi$$

d. **Stretching of Distance Scale.** Any arc of a *Parallel*, the *Latitude* of which is ϕ, is thus represented on the chart by a line proportional to the actual length of the arc multiplied by $\sec\phi$, a quantity greater than unity. The distance *Scale* along the *Parallel* is therefore stretched, as follows:

If *Y* is taken sufficiently close to *X* for *XYZ* to be considered a plane triangle right-angled at *Z*:

$$\frac{Z_1Y_1}{ZY} = \frac{X_1Z_1}{XZ}$$

$$= \frac{1}{k}\sec\phi$$

$$\text{Thus:}\quad Z_1Y_1 = \frac{1}{k}ZY\sec\phi$$

Any small element of a *Meridian* in near *Latitude* ϕ is thus represented on the chart by a line proportional to the actual length of the element multiplied by $\sec\phi$; the distance *Scale* along the *Meridian* is therefore stretched.

As the actual distance between *Z* and *Y* on the Earth is $\Delta\phi$ in circular measure:

$$\text{Actual Distance} = \frac{180 \times 60}{\pi}\Delta\phi = 3437.747\,\Delta\phi \quad \text{(minutes of arc)}$$

$$\text{Hence:}\quad Z_1Y_1 = \frac{1}{k}3437.747\sec\phi\,\Delta\phi \quad \text{(minutes of arc)}$$

(3d) But, 1 minute of arc is equal to *k* millimetres (or whatever the *Scale* units are).

Therefore: $Z_1Y_1 = \left(\frac{1}{k} 3437.747 \ \sec \phi \Delta \phi\right) k$ (in millimetres or *Scale* units)

The actual chart length of Z_1Y_1 in millimetres, or whatever the *Scale* units are, is thus:

$$Z_1Y_1 = 3437.747 \ \sec \phi \Delta \phi$$

The chart length of any particular *Parallel* from the *Equator*, measured along a *Meridian* is clearly the sum of all the component elements of which the expression just found is typical. If the *Latitude* of the *Parallel* is ϕ <u>measured in radians</u>, this sum, in the chosen units, is given by:

$$3437.747 \int_0^{\phi} \sec\phi \, d\phi \quad (radians)$$

The number of *Meridional Parts* or *Longitude Units* (a *Longitude Unit* is the length on the chart that represents 1 minute of arc in *Longitude*) in the length of a *Meridian* between ***Latitude*** $\boldsymbol{L_F}$ <u>measured in degrees</u> and the *Equator* is thus:

$$3437.747 \ \log_e \ \tan\left(45° + ½L_F°\right) \qquad \textbf{. . . A3.1}$$

e. **Evaluation of the 'Mer Parts' Formula for the Sphere.** From formula (A3.1), the actual evaluation of the '*Mer Parts'* formula for the *Sphere* may be accomplished more easily if the logarithm is expressed to base 10. Thus, if *y* is the number of *Meridional Parts*:

$$= 7915.7045 \ \log_{10} \tan\left(45° + ½L_F°\right) \qquad \text{. . . (formula 4.1)}$$

Suppose the *Latitude* is 40˚. Then:

$$y = 7915.7045 \ \log_{10} \ \tan \ 65°$$

$$= 7915.7045 \text{ x } 0.33132745$$

$$= 2622.69$$

......................................(continued overleaf)

4. Corrected Mean Latitude for the Sphere

At Paras 0205a/b it was established that in some circumstances, the *Mean Latitude* must NOT be used to determine *d.long* by means of formula (2.5), but that a correction to the *Mean Latitude* must first be applied.

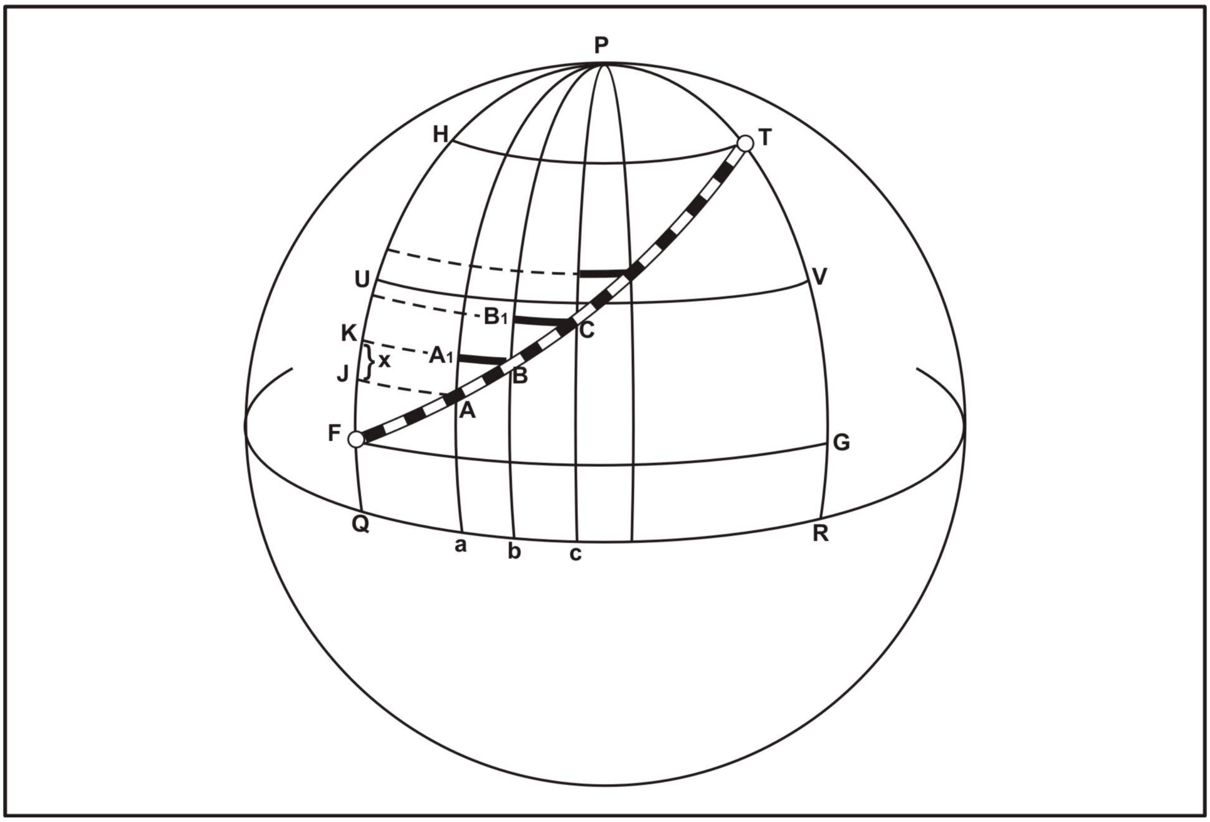

Fig A3-2. The Corrected Mean Latitude

In Fig A3-2 (above), a ship travels from *F* to *T*. Since the *Departure* is greater than *HT* and less than *FG*, it must be exactly equal to the arc of some *Parallel UV*. The *Latitude* of this *Parallel* is called the *Corrected Mean Latitude*, and if it is denoted by *L*, then:

$$QR = UV \sec L$$
$$d.long = Departure \sec L$$

This is an accurate formula, but *L* must be known before it can be used. The problem is therefore to find *L*.

The *Latitude*s of *F* and *T* may be denoted by L_F and L_T, and the difference of *Latitude* between them, *FH*, divided into *n* equal parts of length *x*. *JK* is one of these parts. Then:

$$d.lat = nx = L_T - L_F$$

If *Parallel*s of *Latitude* are now drawn through the points *J, K . . .* , intersecting the *Rhumb Line FT* in *A, B,* etc and the *Meridians* through these points of intersection in A_1, B_1, etc, *n* small triangles are formed. These triangles are equal because in each the side of which AA_1 is typical is *x*, the angle at A_1 is 90˚, and the angle at *A* is the course (which is constant between *F* and *T*). The length of the arc of which A_1B is typical is thus the same for each triangle and if the triangles are made sufficiently small (that is, if *n* if made sufficiently large) for the conditions for evaluating an accurate *Departure* to be realised, the *Departure* between *F* and *T* is the sum of the elements A_1B.

(4) Thus:

$$Departure = ny \quad \text{(where } y \text{ is the length of } A_1B.)$$

Also, the *d.long* corresponding to the element A_1B is *ab* and:

$$ab = A_1B \ sec \ (Latitude \ B)$$
$$ab = y \ sec \ (Latitude \ K)$$

By adding all these elements *ab, bc* etc, the *d.long* is obtained, the formula being:

$$d.long = y[sec \ (L_F + x) + sec \ (L_F + 2x) + \ldots + sec \ L_T]$$

Or, since the *Departure* is equal to *ny*:

$$d.long = Departure \frac{sec \left(L_F + x\right) + sec \left(L_F + 2x\right) + \ldots + sec \ L_T}{n}$$

But the *Corrected Mean Latitude L* is given by:

$$d.long = Departure \ sec \ L$$

Hence, by equating these two values of the *d.long*:

$$sec \ L = \frac{1}{n}\left[sec\left(L_F + x\right) + sec\left(L_F + 2x\right) + \ldots + sec L_T\right]$$

The quantity *sec L* is thus the mean of the secants of the *Latitude*s of the successive *Parallel*s. Written in the integral form in order that the value of *sec L* may be found, the equation is:

$$sec \ L = \frac{1}{nx}\left[sec \ \left(L_F + x\right) + sec\left(L_F + 2x\right) + \ldots + sec \ L_T\right]x$$
$$= \frac{1}{\left(L_T - L_F\right)}\left[sec\left(L_F + x\right) + sec\left(L_F + 2x\right) + \ldots + sec \ L_T\right]x$$

Then, as *n* becomes larger, *x* grows progressively smaller and, in the limit:

$$sec \ L = \frac{1}{d.lat}\int_{LF}^{LT} sec \ L \ dL \ldots$$

And, if *d.lat* is expressed in radians:

$$\text{sec } L = \frac{1}{d.lat}\left[log_e \ tan \ \left(\frac{\pi}{4} + \frac{L}{2}\right)\right]_{L_F}^{L_T}$$

Or, if *d.lat* is expressed in minutes of arc:

$$\text{sec } L = \frac{1}{d.lat} \times \frac{180 \times 60}{\pi} \times log_e 10[log_{10} tan(45° + ½L_T°) - log_{10} tan(45° + ½L_F°)]$$

Thus it may be seen that:

$$\frac{180 \times 60}{\pi} \times log_e 10[log_{10} tan(45° + ½L_T°) - log_{10} tan(45° + ½L_F°)]$$

corresponds to the *Meridional Parts* formula (4.1) and is equal to the *Difference of Meridional Parts* (*DMP*). Thus:

$$sec \ L = \frac{DMP}{d.lat \ (minutes \ of \ arc)} \qquad \text{. . . (formula 2.7)}$$

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APPENDIX 4

PROJECTIONS

1. Scope of Appendix

Appendix 4 contains the following information:

- **Para 2: Conical Orthomorphic Projection on the Sphere.**
- **Para 3: Deduction of the 'Mer Part' Formula for the Sphere.**
- **Para 4: Mercator Projection Chart - Position Circles.**
- **Para 5: Modified Polyconic Projection.**
- **Para 6: Polar Stereographic Projection.**
- **Para 7: Gnomonic Projection.**
- **Para 8: Transverse Mercator Projection - Conversion of Geographical / Grid Coordinates.**

2. Conical Orthomorphic Projection on the Sphere

On *Projections* of the simple *Conical* type, all *Meridians* are equally spaced straight lines meeting in a common point beyond the limits of the chart or map; the *Parallels* (of *Latitude*) are concentric circles, the common centre of which is the point of intersection of the *Meridians* (see Fig A4-1 below). In Fig A4-1:

- The *Cone AVG* is tangential to the *Sphere* along the *Standard Parallel AFGH.*
- *AV* is the radius r_o of the *Standard Parallel* at *Latitude* ϕ_0 (*Co-Latitude* Z_o) on the *Projection*.
- Radius *AV* is equal to $R \tan Z_o$ where *R* is the radius of the *Sphere*.
- The angles *EVF* and *WVF* on the *Projection* are equal, each representing 180˚ of *Longitude* on the *Sphere*.

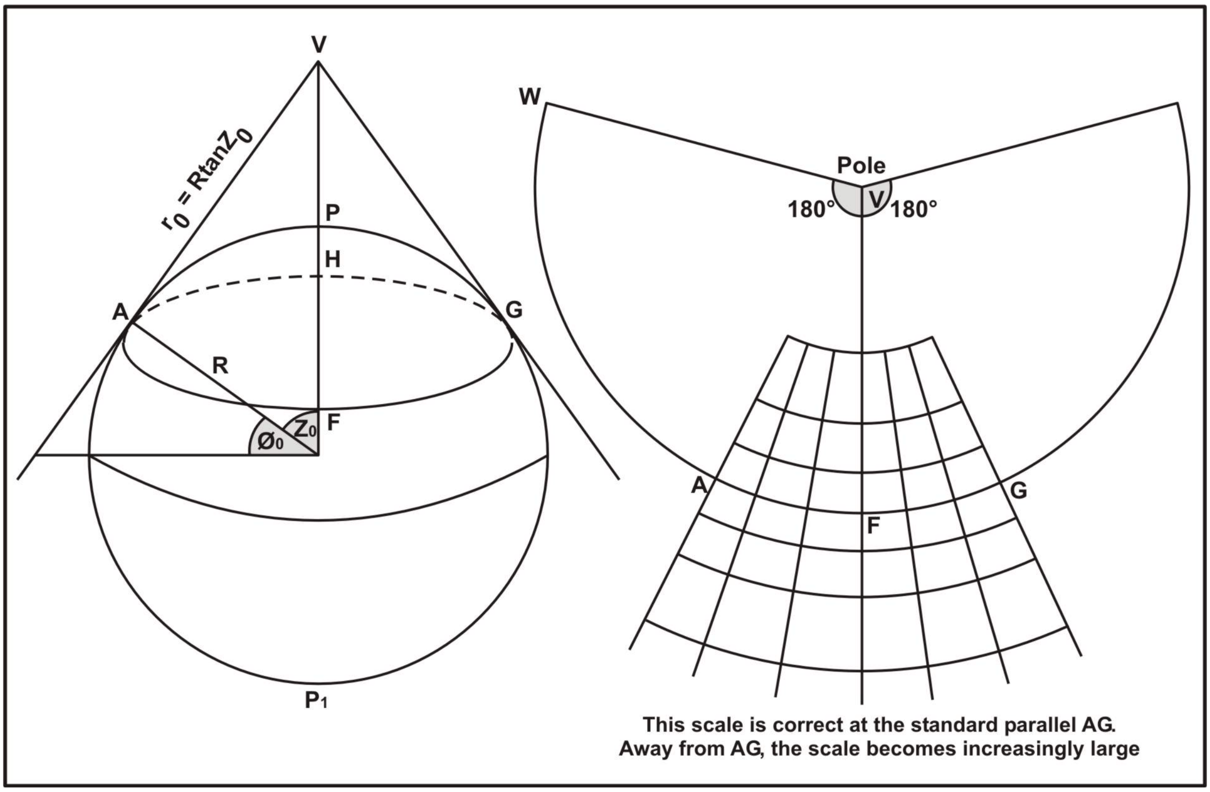


Fig A4-1. The Simple Conical Orthomorphic Projection

(2) a. **Simple Conical Orthomorphic Projection.** The *Meridians* and *Parallels* of the *Simple Conical Orthomorphic Projection* intersect at right angles and thus angles are preserved. Although this is necessary for *Orthomorphism*, it is not sufficient to make the *Projection Orthomorphic*; for this, the *Scale* along the *Meridian* must be equal to the *Scale* along the *Parallel* at any point on the *Projection*.

- **Fig A4-2.** In Fig A4-2 (opposite), *ABCD* is an infinitely small quadrilateral on the *Sphere*, while $A_1B_1C_1D_1$ is its plane representation on the *Conical Projection*.

- **Constant of the Cone.** The small change in the *Meridian* on the *Projection* (dθ) is only a fraction of the equivalent change in the *Meridian* on the *Sphere* (dλ) and this fraction may be referred to as *n* (the *Constant of the Cone*).

  where: $d\theta = n d\lambda$ . . . **A4.1**

  See also further details at Para 2b (opposite).

- **Scale Along the Meridian at A_1.** The *Scale* along the *Meridian* at A_1 is the relationship:

  $$\frac{A_1B_1}{AB} = \frac{-\,dr}{Rd\phi} = \frac{dr}{RdZ} \qquad \text{. . . A4.2}$$

  The negative sign must be allocated to *dr* if ϕ is used, because *r* increases as ϕ decreases. The positive sign must be allocated if *Z* is used, as *r* increases as *Z* increases.

- **Scale along the Parallel at A_1.** The *Scale* along the *Parallel* at A_1 is the relationship:

  $$\frac{A_1D_1}{AD} = \frac{rd\theta}{R\ \cos\ \phi\ d\lambda}$$

  $$\frac{A_1D_1}{AD} = \frac{rd\theta}{R\ \sin\ Z\ d\lambda} = \frac{nr}{R\ \sin\ Z} \qquad \text{. . . A4.3}$$

- **Orthomorphism.** To be *Orthomorphic*, the *Scales* along the *Parallel* and the *Meridian* must be equal, and where *k* is a constant defining the *Scale*:

  ie: $$\frac{dr}{RdZ} = \frac{nr}{R\ \sin\ Z}$$

  $$\frac{1}{r}dr = n\,cosec\ Z\ dZ$$

  ie: $$\int \frac{1}{r}dr = \int n\ cosec\ Z\ dZ$$

  $$log_e r = n\ log_e\ tan\ \frac{Z}{2} + C\ = n\ log_e\ tan\ \frac{Z}{2} + log_e\ k$$

  $$r = k\left(tan\ \frac{Z}{2}\right)^n \qquad \text{. . . A4.4}$$

- **General Properties.** The general properties of a system of conformal *Conical Projections* may be defined by formula (A4.4).

(2a continued)

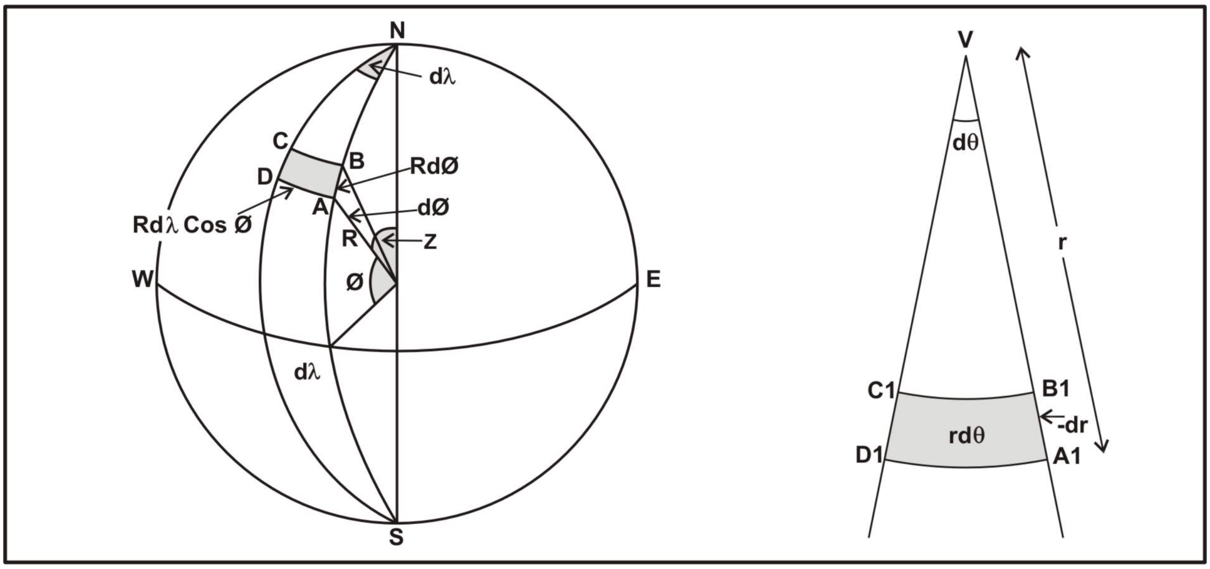


Fig A4-2. Scale on the Conical Orthomorphic Projection

(2) b. **The Constant of the Cone.** From Fig A4-1, it may be seen that the length of the *Standard Parallel AFGH* is $2\pi R\cos\phi_0$, whilst the radius of the *Parallel* on the *Projection* is $R\tan Z_o$ or $R\cot\phi_0$. When this *Conical* shape is displayed for the whole 360˚ of *Longitude* for the Earth, the angle on the *Projection* represents 2π. Thus:

$$R\cot\phi_0\,d\theta = 2\pi R\cos\phi_0$$
$$d\theta = 2\pi\sin\phi_0$$

But, in this case,

$$d\lambda = 2\pi$$

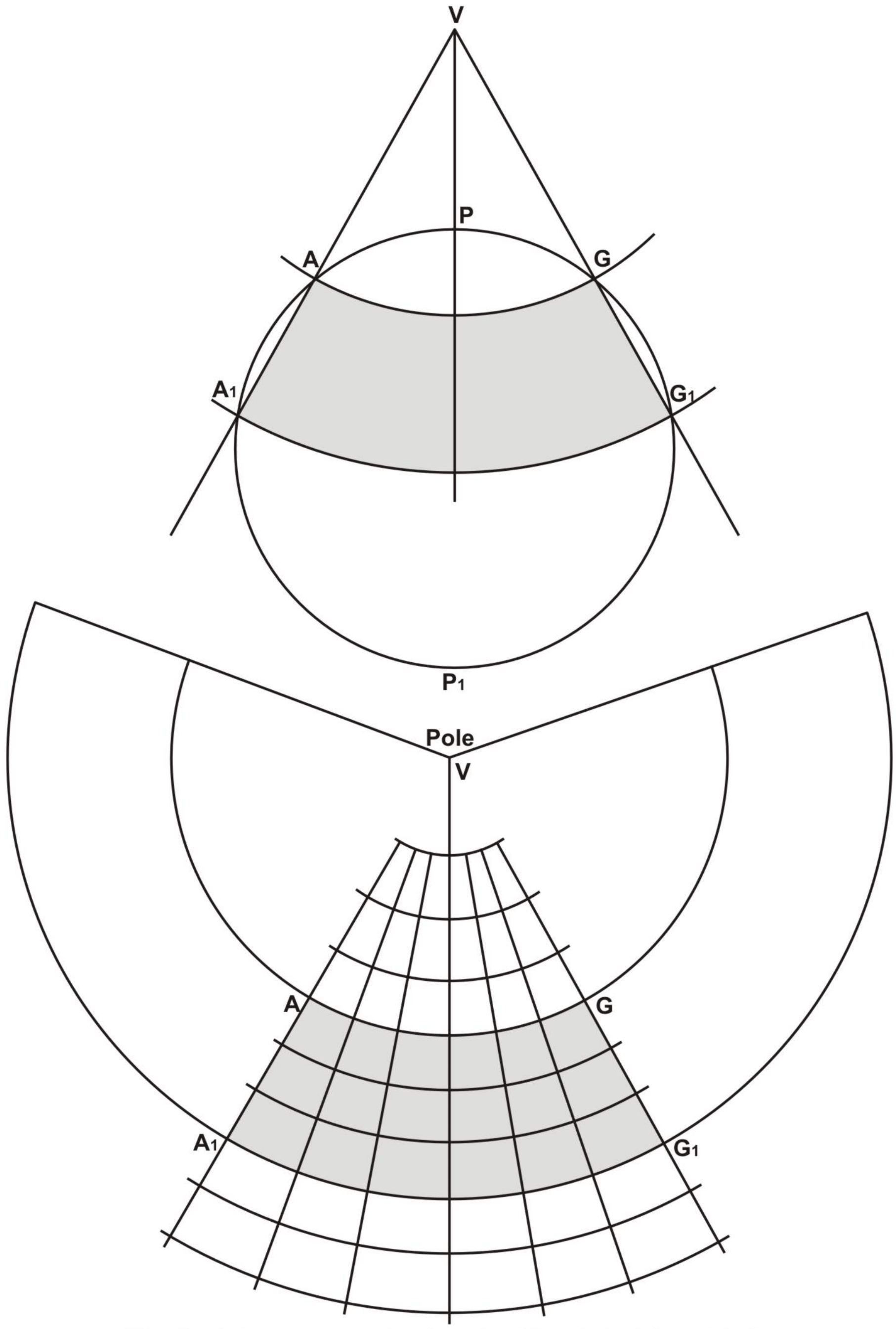
So, from formula (A4.1),

$$2\pi n = 2\pi\sin\phi_0$$
$$\therefore \quad n = \sin\phi_0 = \cos Z_o \qquad \ldots \textbf{A4.5}$$

Thus for the simple *Conical Projection*, the *Constant of the Cone* (n), equals $\sin\phi_0$, which is the sine of the *Standard Parallel*.

.... (continued overleaf)

(2) c. **Conical Orthomorphic Projection with Two Standard Parallels.** Since the *Scale* at any point not on the *Standard Parallel* is too large on a *Simple Conical Orthomorphic Projection*, two *Standard Parallels* may be chosen where the *Scale* is correct (see Fig A4-3 below). However, between the two *Parallels* (AG and A_1G_1 at Fig A4-3 below), the *Scale* of the chart is too small, while beyond them the *Scale* is too large. This *Projection* is known as *Lambert's Conical Orthomorphic Projection*.



The *Scale* is correct at the *Standard Parallels AG and A_1G_1*.
Between them, the *Scale* is too small; outside them it becomes increasingly large.

Fig A4-3. Lambert's Conical Orthomorphic Projection, two Standard Parallels

3. Deduction of the Meridional Parts ('Mer Parts') Formula for the Sphere

The formula giving the *Meridional Parts* ('*Mer Parts*') of any *Latitude* may be derived from the general formula (A4.4) for conformal *Conical Projection*s.

If Z_o is the *Co-Latitude* of the *Standard Parallel*, the radius of the *Parallel* on the *Projection* is given from formula (A4.4), thus:

$$r_o = k\left(tan\ \frac{Z_o}{2}\right)^n$$

The distance between the *Standard Parallel* and any other *Parallel* is given by:

$$r_o - r = k\left[\left(tan\ \frac{Z_o}{2}\right)^n - \left(tan\ \frac{Z}{2}\right)^n\right]$$

And approximating by expanding the right-hand side in its exponential form, given that *n* ultimately tends to zero, becomes:

$$r_o - r \simeq kn\left(log_e\ tan\ \frac{Z_o}{2} - log_e\ tan\ \frac{Z}{2}\right)$$

The value of *k* follows at once from the fact that $r_o\ cos\ Z_o$ is equal to $R\ sin\ Z_o$ and is given by:

$$k = \frac{R\ sin\ Z_o}{cos\ Z_o} \times \frac{1}{\left(tan\ \frac{Z_o}{2}\right)^n}$$

and since: $n = cos\ Z_o$

then: $$kn = \frac{R\ sin\ Z_o}{\left(tan\ \frac{Z_o}{2}\right)^n}$$

When the *Conical Projection* becomes a *Cylindrical Projection*, the *Standard Parallel* becomes the *Equator*; this being the *Mercator Projection,* Z_o becomes 90˚.

thus: $kn = R$

The value of $(r_o - r)$, which is now the chart length of a *Parallel* in *Latitude* ϕ from the *Equator*, measured along a *Meridian*, is therefore given by:

$$r_o - r = -R\ log_e\ tan\ \frac{Z}{2} = R\ log_e\ cot\ \frac{Z}{2}$$

$$= \frac{10800}{p}\ log_e\ tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \text{ radians}$$

$$= 3437.747\ log_e\ tan\left(45° + \frac{1}{2}\phi°\right) \qquad \text{. . . (formula A3.1)}$$

$$= 7915.7045\ log_{10}\ tan\left(45° + \frac{1}{2}\phi°\right) \qquad \text{. . . (formula 4.1)}$$

Note A4.1. *Symbol* ϕ *is used for Latitude at formulae (A3.1) and (4.1) above, although elsewhere it may be represented by T or* L_F*, depending on the context.*

4. Mercator Projection Chart - Position Circles

If a *Position Circle* is drawn on the Earth's surface as a circle with the *Geographic Position* of the heavenly body as centre, it will be *Small Circle* (see Fig A4-4 below). When plotted on a *Mercator Projection* chart, the shape of the *'Position Circle'* will no longer be a circle but a curve (see Figs 4-5 and 4-6 opposite); the problem is to find formula (A4.7) to establish the resulting curve path:

Fig A4-4 (below) shows the relative positions of the *Pole P*, the observer Z, and the *Geographic Position* of the heavenly body *U*, when the *True Altitude* (obtained from a *Sextant*) is *a*, and the *Declination* is *d*. The *Latitude* of Z is ϕ. Then, if X and x are the easterly *Longitude*s of Z and *U*, the *Hour Angle* of the heavenly body is $(x - X)$.

The *Cosine Formula* applied to the *Spherical* triangle *PZU* gives:

$$\cos UZ = \cos PU \cos PZ + \sin PU \sin PZ \cos UPZ \qquad \text{. . . (formula A2.1)}$$

ie:

$$\sin a = \sin d \sin \phi + \cos d \cos \phi \cos (x - X)$$

or:

$$\cos (x - X) = \sin a \sec d \sec \phi - \tan d \tan \phi \qquad \text{. . . } \mathbf{A4.6}$$

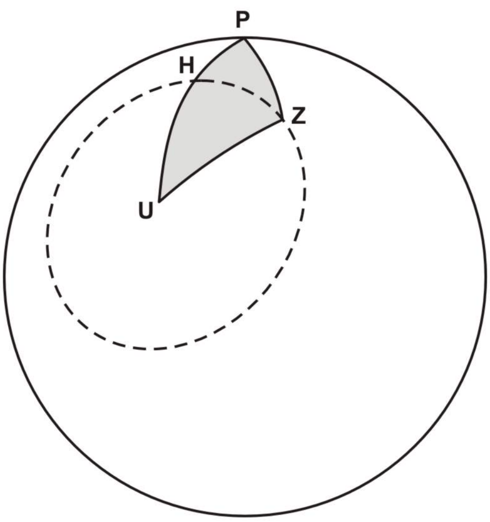


Fig A4-4. A Position Circle Drawn on the Earth's Surface

If the coordinates of Z on the chart are *x* and *y*, then:

$$\sec \phi = \tfrac{1}{2}(e^{y} + e^{-y})$$

and:

$$\tan \phi = \tfrac{1}{2}(e^{y} - e^{-y})$$

Hence, by substitution:

$$2 \cos (x - X) = e^{y} (\sin a \sec d - \tan d) + e^{-y} (\sin a \sec d + \tan d) \qquad \text{. . . } \mathbf{A4.7}$$

Formula (A4.7) is the general equation of the curve on the chart that represents the *Position Circle*; the curve itself is defined by the values of *a, d* and *X*.

Fig A4-5 (opposite) shows the curve as it appears on a *Mercator Projection* chart when the *Declination* is zero, and Fig A4-6 (opposite) shows three typical curves representing *Position Circle*s for three values of the *Altitude* when the *Geographic Position* is in *Latitude* 40˚N, *Longitude* 60˚W.

(3 continued)

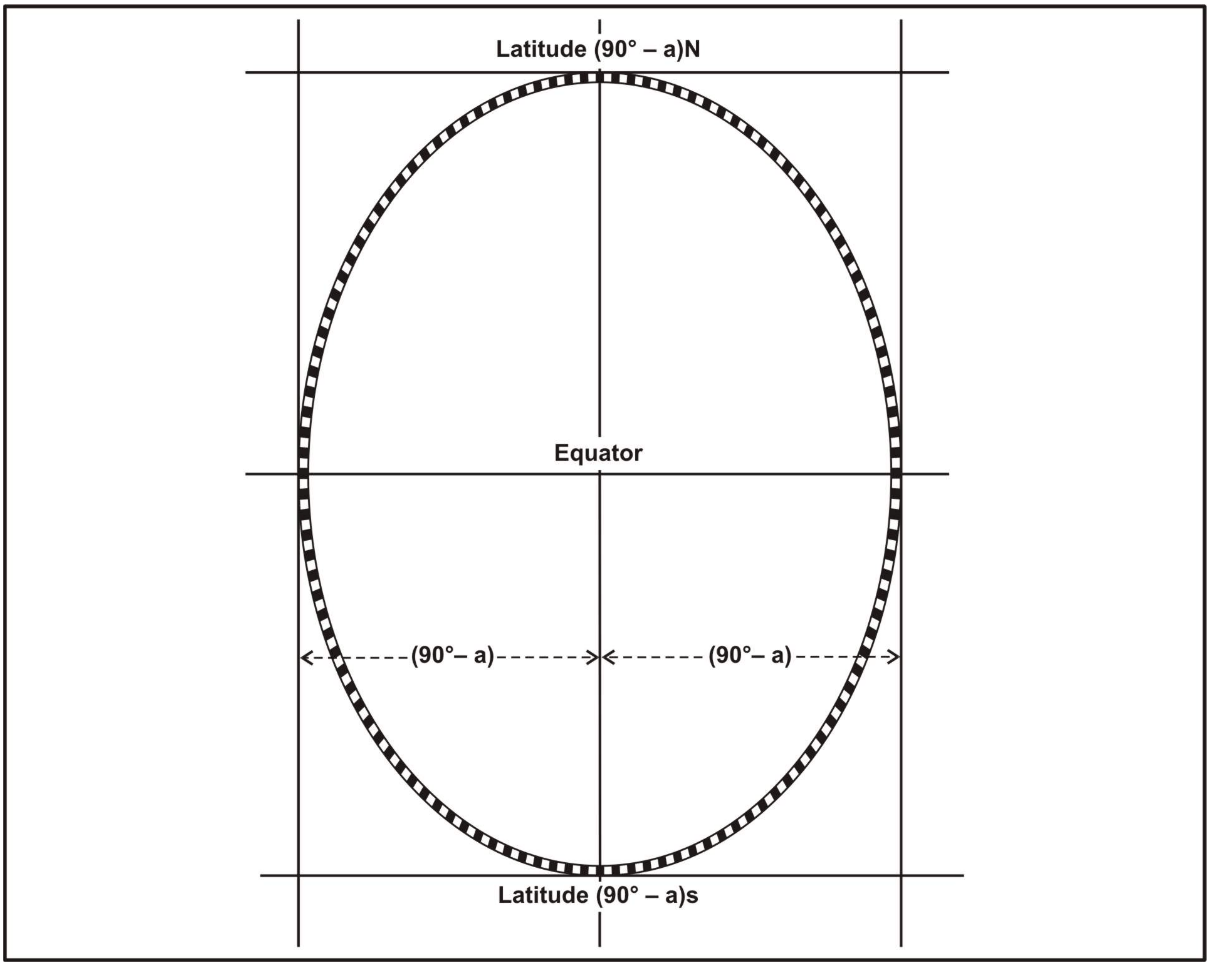


Fig A4-5. Position Circle Plotted on a Mercator Projection Chart with Declination Zero

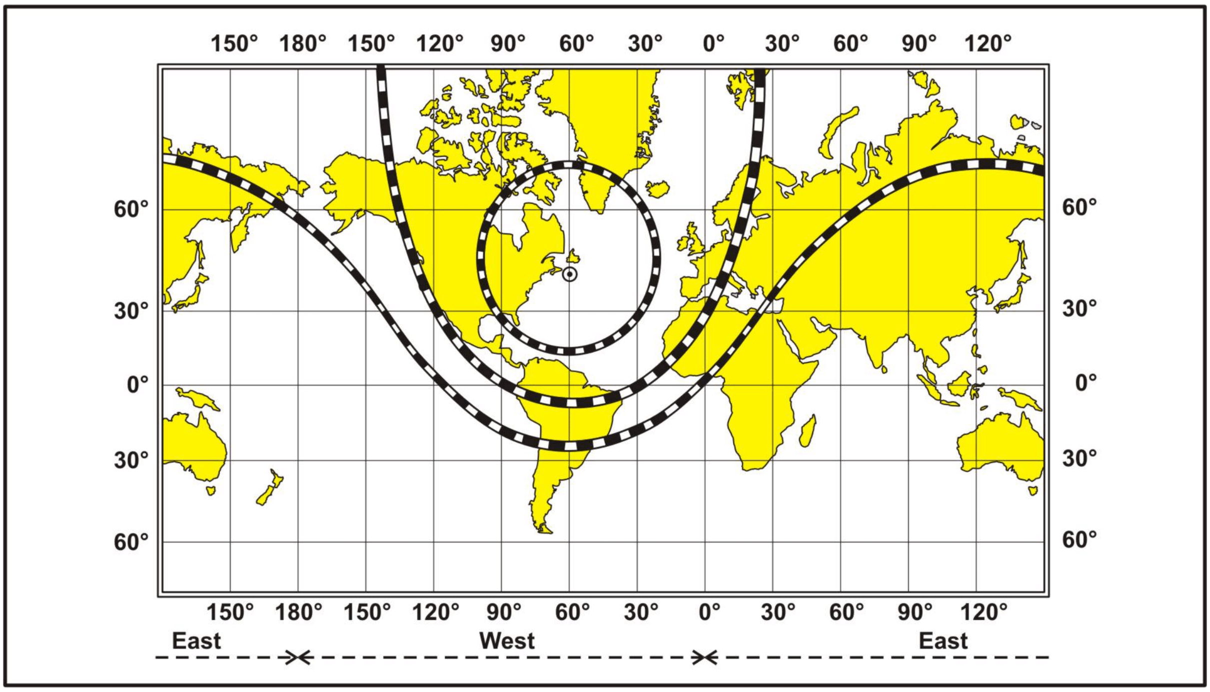


Fig A4-6. Typical Position Circles (for three values of the Altitude), when the Geographic Position is in Latitude 40˚N, Longitude 60˚W.

5. Modified Polyconic Projection

Although new charts on a *Scale* 1:50,000 or larger are now drawn on the *Transverse Mercator Projection*, there are 290 *UKHO* harbour plans / approaches in use (2008), traditionally described on the chart as being *Gnomonic,* while actually *Modified Polyconic Projections*.

a. **Polyconic Projection Properties.** In the *Polyconic Projection*, the *Central Meridian* alone is straight; distances between consecutive *Parallels* are made equal to the real distances along the surface of the *Spheroid,* to the *Scale* required for the chart.

b. **Polyconic Projection Construction.** Each *Parallel* is constructed as if it were the *Standard Parallel* of a simple *Conical Projection*. This means (see Chapter 4) that the circular arcs in which the *Parallels* are developed are not concentric, but their centres lie on the *Central Meridian*. The other *Meridians* are concave towards the *Central Meridian* and, except near the corners of maps or charts showing large areas, they intersect the *Parallels* at angles differing only slightly from right angles.

c. **Modified Polyconic Projection Construction.** In practice, all *Meridians* are drawn as straight lines on Admiralty charts and to this extent the *Polyconic Projection* has been modified, although the normal curvature of the limiting *Meridians* would be extremely small in any case, having regard to the *Scale* of the chart.

The coordinates (*x, y*) of any point *Q* on the *Modified Polyconic Projection* (Fig A4-7) may be found from the formulae:

$$x = v\Delta\lambda \cos\phi \qquad \text{. . . } \mathbf{A4.8}$$
$$y = \tfrac{1}{4}v(\Delta\lambda)^2 \sin 2\phi \qquad \text{. . . } \mathbf{A4.9}$$

where: ϕ is the *Latitude* of the *Parallel*,
$\Delta\lambda$ is the difference of *Longitude* from the *Central Meridian*,
v is the (transverse) radius of curvature at right angles to the *Meridian* at *Latitude* ϕ (see Fig A4-7 below); it should not be confused with the *Meridional* radius of curvature ρ (see Para 0314).

These formulae are accurate for *Projections* covering 2˚ of *Latitude* and 1˚ of *Longitude*. This may be extended to 2˚ of *Longitude* without appreciable inaccuracy by laying off coordinates from each of the extreme *Meridians*, to cover a further 30' of *Longitude*.

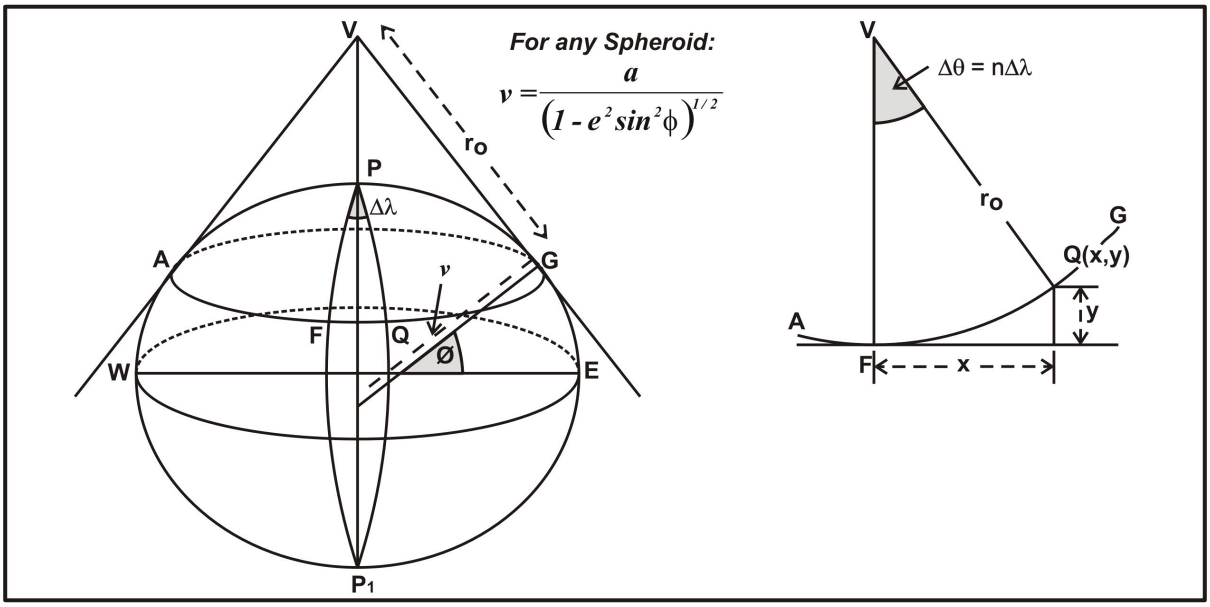


Fig A4-7. The Modified Polyconic Projection

6. Polar Stereographic Projection

The *Polar Stereographic Projection* (see Fig A4-8 below) is a *Perspective Conformal Projection* on a plane tangential to the *Sphere* at the *Pole*, obtained by projecting from the opposite *Pole*. Angles are correctly represented; *Parallels* of *Latitude* are represented by circles radiating outwards from and centred on the *Pole*. *Meridians* appear as straight lines originating from the *Pole*.

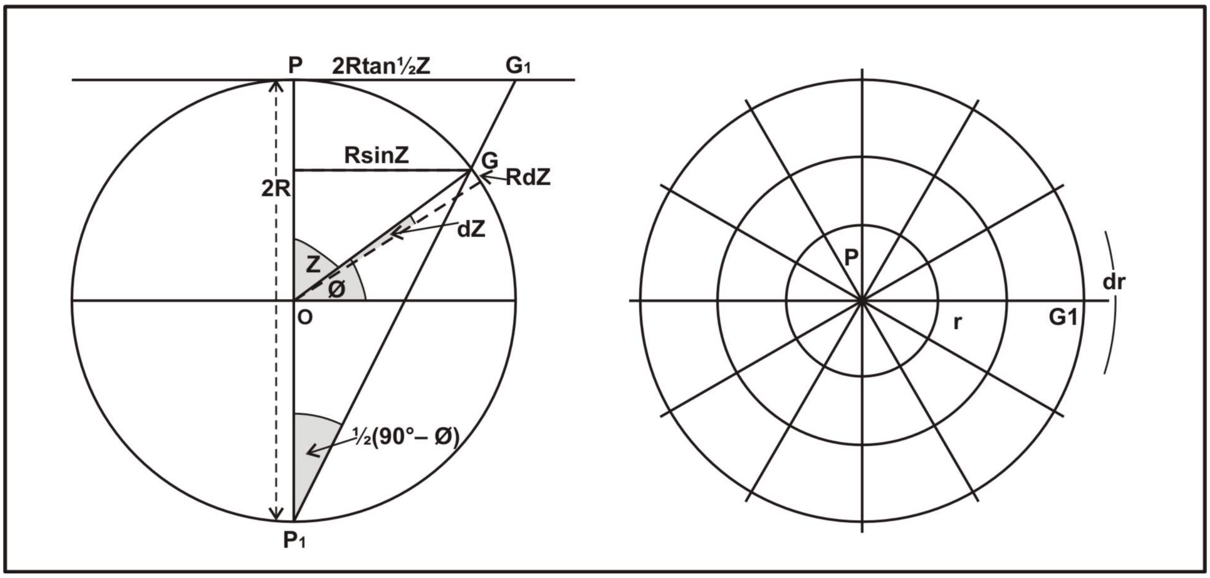


Fig A4-8. The Polar Stereographic Projection

If *R* is the Earth's radius and ϕ the *Latitude* of *G*, the angle PP_1G_1 is ½ (90° – ϕ °) and the radius PG_1 of the projected *Parallel* is 2*R* tan ½ (90° – ϕ °).

If the radius PG_1 is *r*, and the *Co-Latitude* of ϕ is Z, the *Scale* along the *Parallel* is G_1 is given by:

$$\frac{r}{R\ sin\ Z} = \frac{2R\ tan\ ½Z}{R\ sin\ Z} = sec^2 ½Z$$

The *Scale* along the *Meridian* at G_1 where *dr* is a small increase in *r* and *dz* a small increase in *Z* is:

$$\frac{dr}{Rdz} = \frac{2R\ ½\ sec^2 ½Z}{R} = sec^2 ½Z$$

The *Scale* is the same in each direction; thus the *Orthomorphic* property is established.

7. Gnomonic Projection

The *Gnomonic Projection* projects the Earth's surface from the Earth's centre onto the tangent plane. The *Gnomonic Projection* is only applied to a *Sphere* which represents the Earth. The *Gnomonic Projection* is NOT *Orthomorphic* and it does NOT have *Equal Area* properties. *Great Circles* are represented by straight lines on this *Projection* . (This summary is repeated from Para 04440d).

a. **Principal or Central Meridian.** The plane on which the *Parallels* and *Meridians* are projected is a tangent plane and, to avoid distortion, the tangent point *K* should be chosen in the centre of the area to be shown (see Fig A4-9 opposite).

Since the *Gnomonic Projection* is a *Perspective Projection*, the point on the tangent plane that corresponds to a point on the *Sphere* that represents the Earth is found by producing the radius at the point until it cuts the tangent plane. Thus (at Fig A4-9 opposite) *p* corresponds to *P* (the *Pole*), and all points on the *Meridian PK* project into the straight line *pK. PK* is known as the *Principal Meridian* or the *Central Meridian*.

If *B* is any point and *ABC* any *Great Circle* through it, the arc *AB* projects into the straight line *ab*. The *Meridian* through *B* is *PBL* and, since it is part of a *Great Circle*, *pbl* is also a straight line. The *Meridians* on the *Gnomonic Graticule* are thus straight lines radiating from *p*.

The straight line *Kl* corresponds to the *Great Circle* arc *KL*.

If ϕ_K and ϕ_A are the *Latitude*s of *K* and *A*, then:

$$KOP = arc\ KP = 90^\circ - \phi_K$$

And: $$AOP = arc\ AP = 90^\circ - \phi_A$$

Also: $$KOA = KOP - AOP$$

$$= (90^\circ - \phi_K) - (90^\circ - \phi_A)$$

$$= \phi_A - \phi_K$$

In triangle OKa:

$$Ka = OK \tan KOa = OK \tan KOA$$

$$Ka = R \tan\left(\phi_A - \phi_K\right) \qquad \ldots \mathbf{A4.10}$$

The chart distances of the *Pole* (*Kp*) and any point on the *Central Meridian* (*Ka*) from the tangent point are thus known.

It is thus clear from Fig A4-9 (opposite) that, if the *Latitude* of *A* is greater than that of *K*, *a* will lie on the line *K*p between *K* and *p*. If the *Latitude* of *A* is less, *a* will lie beyond *K* on *pK* produced.

(7a continued)

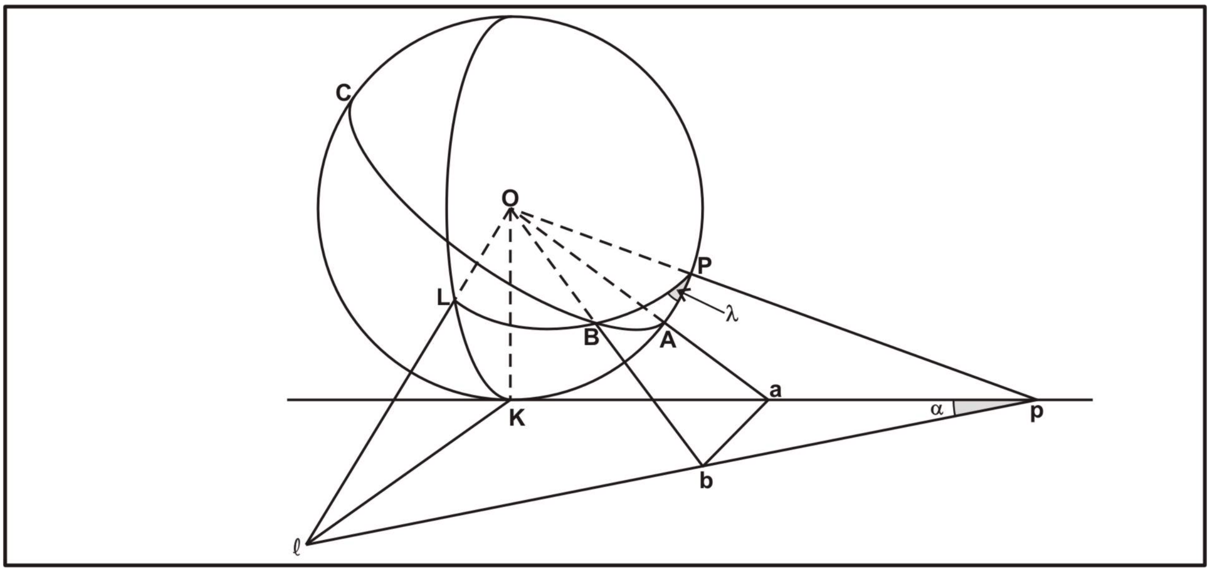


Fig A4-9. Gnomonic Projection - The Principal (or Central) Meridian

b. **Angle Between Two Meridians on the Chart.** The difference of *Longitude* between the *Meridians PBL* and *PAK* in Fig A4-9 (above) is the angle *LPK*, denoted by λ, and this angle is projected into the angle *lpK*, denoted by α.

Suppose the great circle *ABC* is chosen so that it cuts the *Meridian PK* at right angles. Its projection *ab* will then be at right angles to *Kp* and, from the plane right-angled triangle *pab*:

$$ab = ap \tan \alpha$$

Also, of the plane of the great circle *KLM* is made to cut the *Central Meridian* at right angles, the angle *pKl* is a right angle and, from the plane right-angled triangle *pKl*:

$$Kl = Kp \tan \alpha$$

From the plane right-angled triangles *lKO* and *pKO*:

$$Kl = OK \tan KOL$$

And: $$Kp = OK \tan KOP = R \cot \phi_K$$

By Napier's rules applied to the *Spherical* triangle *LKP*, right-angled at *K*:

$$\tan KL = \sin KP \tan \lambda$$

Hence, by combining these relations:

$$\tan \alpha = \sin \phi_K \tan \lambda \qquad \ldots \textbf{A4.11}$$

From this relation it is apparent that when ϕ_K is 90˚, (ie when the *Pole* is the tangent point), α is equal to λ and there is no distortion in the chart angles between the *Meridians*: they are equal to exact differences of *Longitude*. When the tangent point is not at the *Pole*, there is distortion and the angles between the *Meridians* are not represented correctly on the chart.

If the distance *ab* is required, it can be found by substitution. Thus:

$$\begin{aligned} ab &= ap \tan \alpha \\ &= (Kp - Ka) \sin \phi_K \tan \lambda \\ &= R\,[\cot \phi_K - \tan (\phi_A - \phi_K)] \sin \phi_K \tan \lambda \\ &= R \tan \lambda \cos \phi_A \sec (\phi_A - \phi_K) \end{aligned} \qquad \ldots \textbf{A4.12}$$

(7) c. **Parallels of Latitude.** As *Parallels* of *Latitude* are not *Great Circles*, they form a series of curves on the *Gnomonic Graticule*. In Fig A4-10 (below) *ABC* is a *Parallel* in *Latitude* ϕ, and *b* is the projection of *B*. As *B* moves along the *Parallel*, *b* describes a path which is not a straight line. The problem is to find formula (A4.13) to establish the path, and this can be done by referring *b* to the rectangular axes *KX* and *Kp*.

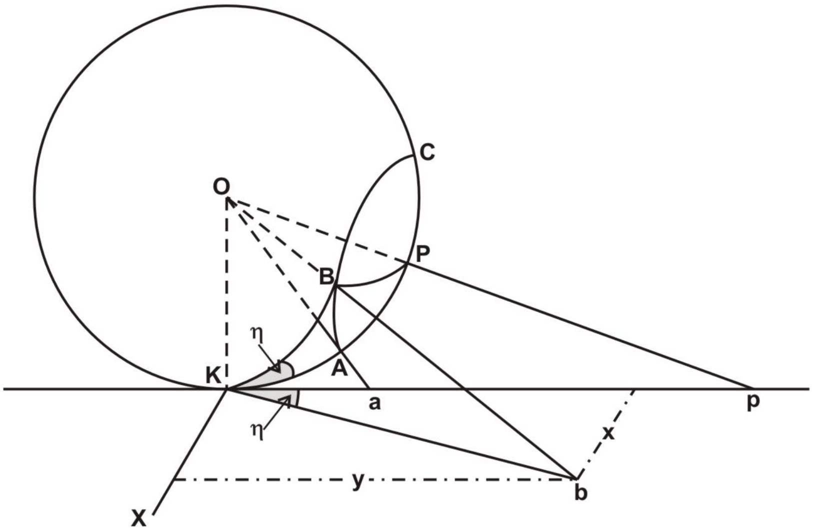


Fig A4-10. Gnomonic Projection - The Parallel of Latitude

If the angle *AKB* is denoted by η, the angle *bKp* will also be η because the *Great Circles KB* and *KP* can be regarded as '*Meridians*' radiating from '*Pole*' *K* which is a tangent point. There is thus no distortion when this angle is projected. Hence, if *x* and *y* are the coordinates of *b*:

$$x = Kb \sin \eta$$

And: $$y = Kb \cos \eta$$

And: $$x^2 + y^2 = Kb^2$$

From the right-angled *Plane Triangle KOb*:

$$Kb = OK \tan KOb$$

From the *Spherical Triangle PBK,* by the *Cosine Formula*:

$$\cos PB = \cos KB \cos KP + \sin KB \sin KP \cos \eta$$

ie: $$\sin\phi \sec KB = \sin \phi_K + \tan KB \cos \phi_K \cos \eta$$

For convenience take the radius of the *Sphere* as unity. Then:

$$\sin\phi \sec KB = \sin \phi_K + y \cos \phi_K$$

And: $$\tan^2 KB = x^2 + y^2$$

ie $$\sec^2 KB = 1 + x^2 + y^2$$

$$\therefore \sin^2 \phi (1 + x^2 + y^2) = \sin^2 \phi_K + 2y \sin \phi_K \cos \phi_K + y^2 \cos^2 \phi_K$$

Thus: $$x^2 \sin^2 \phi + y^2 (\sin^2 \phi - \cos^2 \phi_K) - 2y \sin \phi_K \cos \phi_K = \sin^2 \phi_K - \sin^2 \phi \quad \ldots \textbf{A4.13}$$

For all points on the *Parallel ABC*, ϕ is constant and ϕ_K is also constant. Thus the formula (A4.13) is therefore the equation of the curve that represents the *Parallel ABC* on the chart.

(7) d. **To Construct a Gnomonic Graticule.** When the tangent point is on the *Equator* or at the *Pole*, the *Graticule* admits a simple geometrical construction. When the tangent point is elsewhere, formula (A4.13) must be employed.

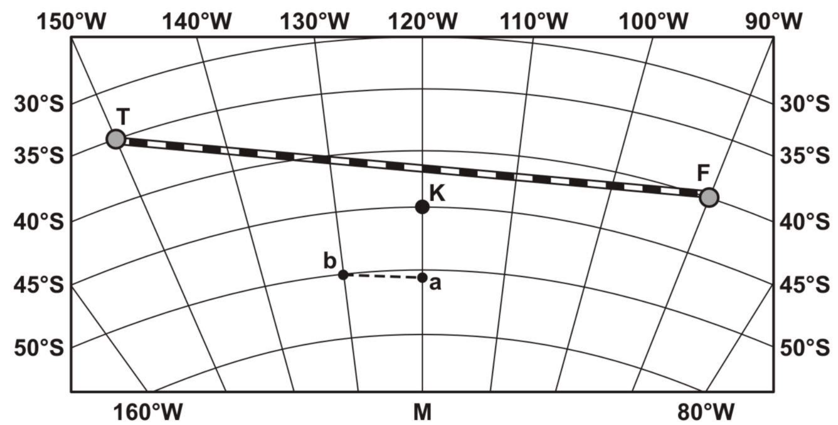


Fig A4-11. The Gnomonic Graticule

Fig A4-11 (above) shows the *Graticule* when the tangent point is in *Latitude* 45˚S, *Longitude* 120˚W. *MK* is the *Central Meridian*, and the other *Meridians* are inclined to it at angles given by:

$$\tan \alpha = \sin \phi_K \tan \lambda$$

where ϕ_K is 45˚ and λ has successive values 10˚, 20˚, 30˚, etc.

The position of the *Pole* (not shown in Fig A4-11) is given by:

$$Kp = OK \cot \phi_K$$

Kp can therefore be marked according to the chosen *Scale*, and the *Meridians* drawn as lines radiating from *p* at the angles discovered.

If *b* is the point corresponding to *Latitude* 50˚S, *Longitude* 130˚W, and *ba* is the perpendicular from *b* to *MK*, the length of *Ka* in the chosen *Scale* is given by:

$$Ka = \tan (\phi_A - \phi_K)$$

where ϕ_A is the *Latitude* of *A*, the point that *a* represents on the chart (see Fig A4-10 opposite).

If ϕ_B is the *Latitude* of *B*, the point that *b* represents on the chart, *Napier's Rules* applied to the triangle *PBA* give:

$$\tan \phi_A = \tan \phi_B \sec \lambda$$

where λ is the difference of *Longitude* between *A* and *B*.

This formula gives ϕ_A since ϕ_B is 50˚ and λ is 10˚. Hence *Ka* can be found. Also, in the chosen units:

$$ab = \tan \lambda \cos \phi_A \sec (\phi_A - \phi_K)$$

Thus: $$ab = \tan 10^\circ \cos \phi_A \sec (\phi_A - 45^\circ)$$

The point *b*, corresponding to *Latitude* 50˚S, *Longitude* 130˚W, can therefore be plotted with other points where this *Parallel* cuts the *Meridians*.

In this way all the *Parallels* can be inserted.

(7) e. **Equatorial Gnomonic Graticule.** When the tangent point is on the *Equator*, ϕ_K is zero, and the general formulae are simplified considerably. The *Graticule*, however, lends itself to a geometrical construction.

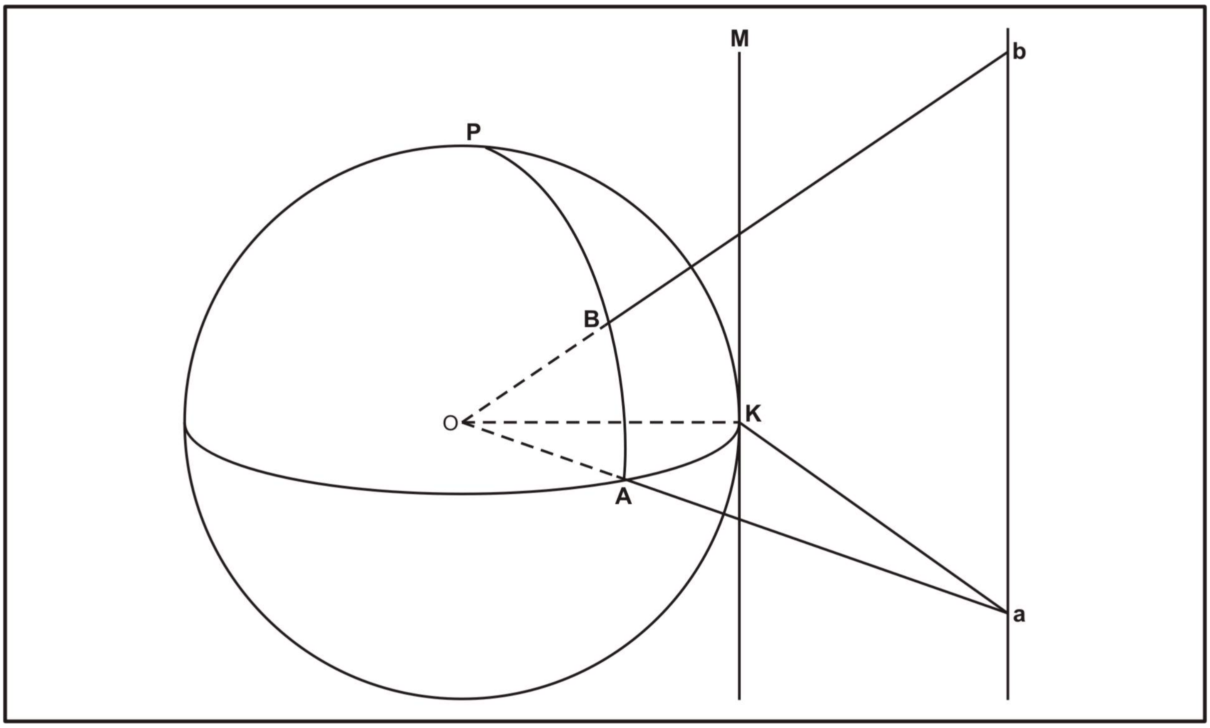


Fig A4-12. Gnomonic Projection - The Equatorial Graticule (1)

In Fig A4-12 (above) the *Central Meridian* is *KP*, and this is represented on the chart by *KM* which is at right angles to *OK*. The *Equator KA* projects into the straight line *Ka* at right angles to *KM*, and any other *Meridian*, *AP*, projects into a line at right angles to *Ka* and therefore *Parallel* to *KM*.

The distance between the projected *Meridian ab* and the *Central Meridian* is given by:

$$Ka = OK \tan KOA$$
$$= R \tan (\text{d.long between } K \text{ and } A)$$

Thus the positions of the *Meridians* can be decided.

If *B* is any point on the *Meridian AP* in *Latitude* ϕ , B projects into *b*, and *ab* represents this *Latitude* on the chart. Fig A4-13 (opposite) shows the geometrical construction for finding the position of *b*.

The plane of *Projection* is represented by *MKa* in the plane of the paper, and *Ka* is a tangent to the *Equatorial* circle at *K*. *A* is fixed on this circle by its exact difference of *Longitude* from *K*, and it projects into *a*. If ab_1 is now drawn at right angles to *Oa*, so that the angle aOb_1 is equal to the *Latitude* of *B*, the triangle aOb_1 is equal in all respects to the triangle *aOb* in Fig A4-12 (above). The position of *b* can thus be marked merely by making *ab* equal to ab_1.

Other points on the *Projection* of the *Parallel* through *B* can be found in the same way. Since, however, a *Graticule* is usually drawn for equal angular intervals of *Latitude* and *Longitude*, the work can be shortened by drawing radials at the required interval and using them for both *d.long* and *Latitude* as shown in Fig A4-14 (opposite).

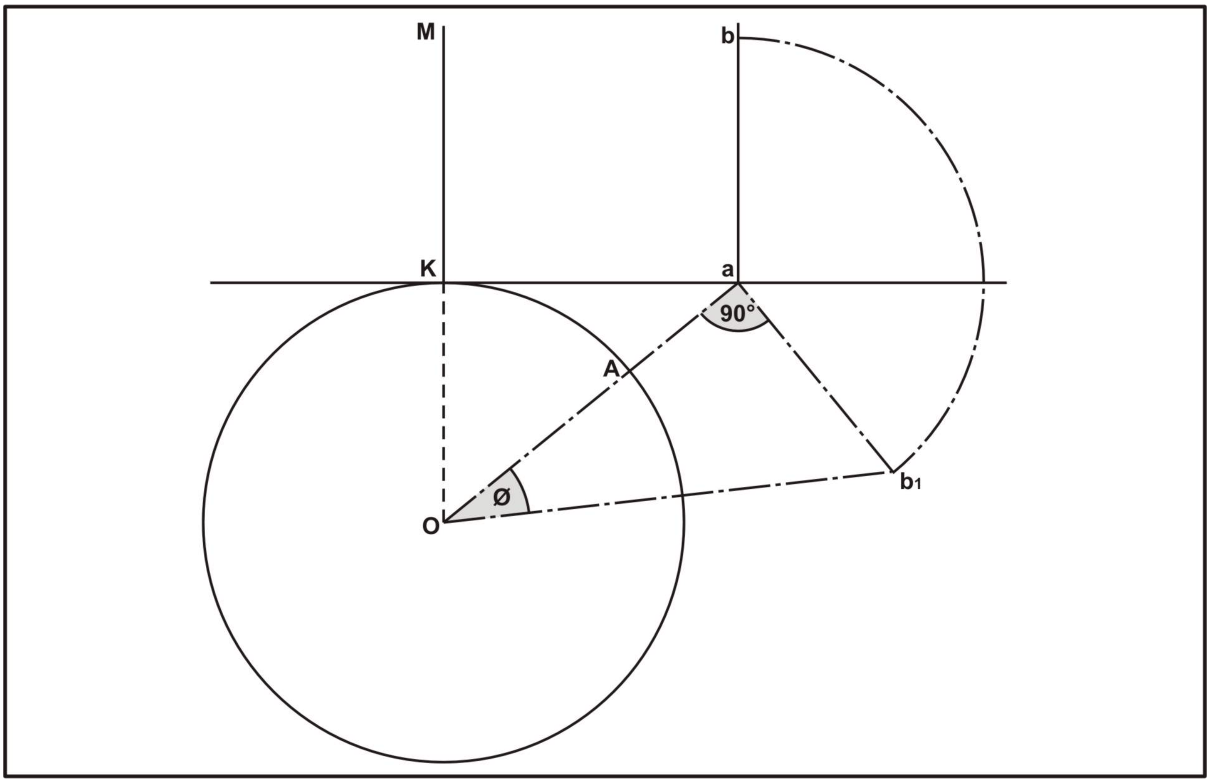


Fig A4-13. Gnomonic Projection - The Equatorial Graticule (2)

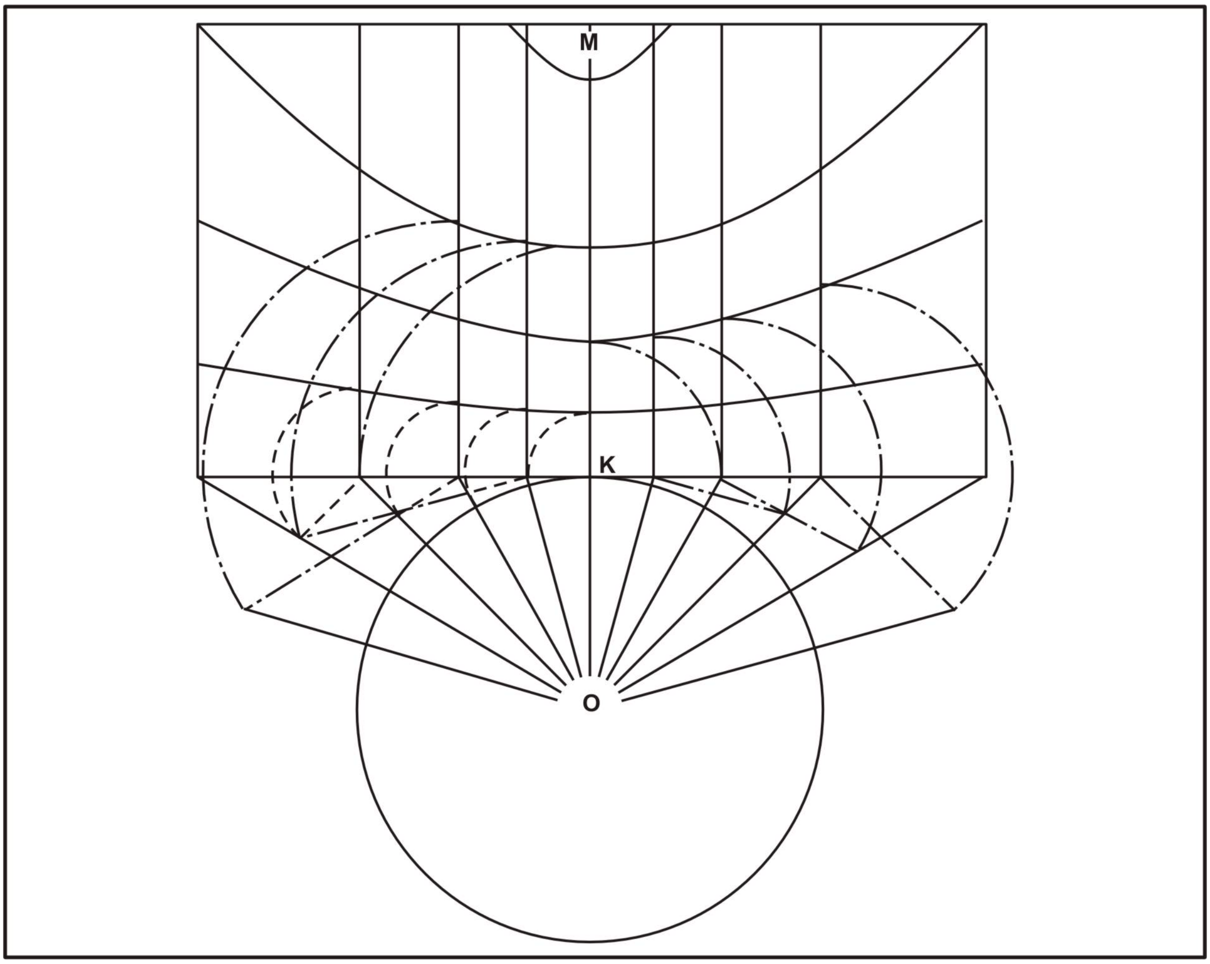


Fig A4-14. Gnomonic Projection - The Equatorial Graticule (3)

(7e cont)

This same construction can be used for finding the position of the *Vertex* and the *Latitude* of any point on a *Great Circle*, the *Longitude* of which is known.

Any *Great Circle* projects into a straight line. Also, a *Great Circle* cuts the *Equator* in two points 180˚ apart. In Fig A4-15 (below), *Q* is one of these points, and *q* its projection. Then, since the *Vertex Longitude* is 90˚ from the *Longitude* of *Q*, the position of the *Vertex v* is found merely by making the angle *QOU* a right angle. The angle uOv_1 measures the *Vertex Latitude*.

If the *Latitude* of any point *x* is required, it can be found in the same way, that is, by drawing *xy* at right angles to *uq* and yx_1 at right angles to *Oy*, and making yx_1 equal to *xy*. The angle yOx_1 then measures the *Latitude* of the point *X* on the Earth to which *x* corresponds on the chart.

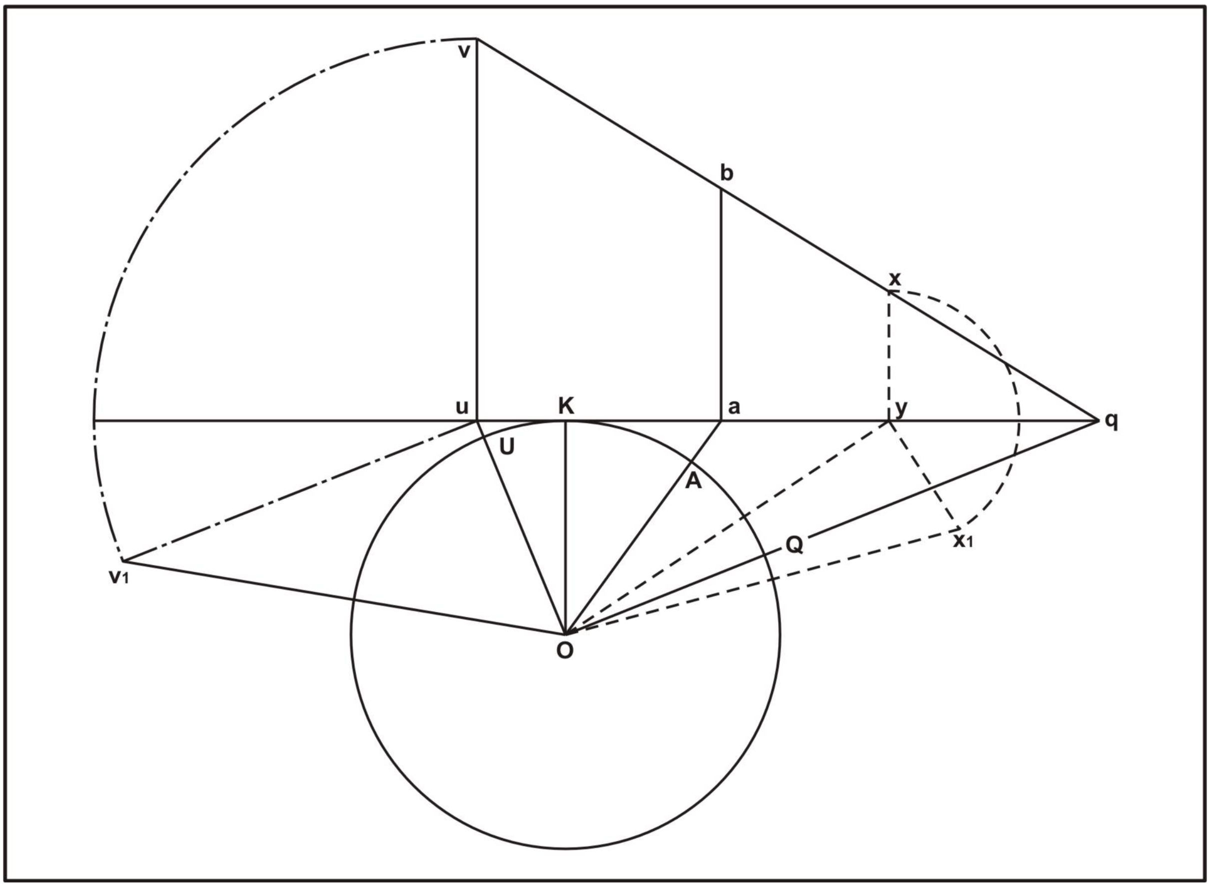


Fig A4-15. Gnomonic Projection - The Equatorial Graticule (4)

8. Transverse Mercator Projection - Conversion of Geographical / Grid Coordinates

The symbols and formulae to be used in the appropriate computer program for the conversion of *Geographic Position* to *Grid* coordinates and vice versa on the *Transverse Mercator Projection* are set out below and overleaf.

a. **Symbols.** The symbols used in these formulae, which correspond to those in use in *UKHO*, are as follows:

a = semi-major axis of *Spheroid* (metres)

b = semi-minor axis of *Spheroid* (metres)

e = *Eccentricity* of *Spheroid*

$$n = \frac{a-b}{a+b}$$

ϕ = *Latitude* (radians)

λ = *Longitude* (radians)

λ_o = *Longitude* of *Central Meridian* of *Grid* (radians)

$\Delta\lambda = \lambda - \lambda_o$

$t = \tan\phi$

ρ = radius of curvature of *Meridian* (metres)

$$\rho = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}}$$

ν = radius of curvature at right angles to *Meridian* (metres)

$$\nu = \frac{a}{\left(1-e^2\sin^2\phi\right)^{1/2}}$$

$$\eta^2 = \frac{\nu}{\rho} - 1 = \frac{e^2\cos^2\phi}{\left(1-e^2\right)}$$

S_ϕ = length of *Meridian* arc from *Equator* to *Latitude* ϕ (metres)

$$\theta = \frac{S_\phi}{b(1+n)\left(1+\frac{5n^2}{4}+\frac{81n^4}{4}\right)}$$

ϕ_1 = 'footpoint' *Latitude*

t_1 = variable, defined above, corresponding to ϕ_1

ρ_1 = variable, defined above, corresponding to ϕ_1

ν_1 = variable, defined above, corresponding to ϕ_1

η_1 = variable, defined above, corresponding to ϕ_1

E = *Grid Easting* (metres)

N = *Grid Northing* (metres)

FE = *False Easting* of True *Grid Origin*

FN = *False Northing* of True *Grid Origin*

E′ = *True Easting*

and: E′ = E − FE (points East of *Central Meridian*)

or: E′ = FE − E (points West of *Central Meridian*)

N′ = *True Northing*

N′ = N − FN

k_o = *Scale Factor* on *Central Meridian* (= 0.9996 for *UTM*)

(8) b. **To Find the Length of the Meridional Arc Given the Latitude.** The length of the *Meridional Arc* is set out in formula (A5.8d) at Appendix 5 Para 6, but is repeated here for convenience, in a slightly different form as formula (A4.14a).

$$S_\phi = a(1-e^2)\left[-\frac{35}{3072}e^6 \ \sin\ 6\phi + \left(\frac{15e^4}{256}+\frac{105e^6}{1024}\right)\ \sin\ 4\phi - \left(\frac{3e^2}{8}+\frac{15e^4}{32}+\frac{525e^6}{1024}\right)\ \sin\ 2\phi + \left(1+\frac{3e^2}{4}+\frac{45e^4}{64}+\frac{175e^6}{256}\right)\phi\right] \quad \text{. . . A4.14a}$$

c. **To Find the 'Footpoint' Latitude, Given the True Grid Coordinates.** If S_o is the length of the *Meridian* arc from the *Equator* to the true *Grid Origin*, then:

$$S_{\phi 1} = S_o \pm \frac{N'}{k_o}(+\ \text{in N hemisphere},\ -\ \text{in S hemisphere})$$

And where ϕ_1 is the *Latitude* of the foot of the perpendicular drawn from a point on the *Projection* to the *Central Meridian*, ϕ_1 can be found from $S_{\phi 1}$ using the formula:

$$\phi_1 = \frac{8011}{2560}n^5\ \sin\ 10\theta_1 + \frac{1097n^4}{512}\ \sin\ 8\theta_1 + \left(\frac{151n^3}{96}-\frac{417n^5}{128}\right)\ \sin\ 6\theta_1 + \left(\frac{21n^2}{16}-\frac{55n^4}{32}\right)\ \sin\ 4\theta_1 + \left(\frac{3n}{2}-\frac{27n^3}{32}+\frac{269n^5}{512}\right)\ \sin\ 2\theta_1 + \theta_1 \quad \text{. . . A4.14}$$

d. **To Convert From Geographical to Grid Coordinates.** If an accuracy of ± 0.01 metre is acceptable, terms containing $\Delta\lambda^6$ and higher powers of $\Delta\lambda$ may be ignored.

$$\frac{E'}{k_o\nu} = \Delta\lambda\ \cos\ \phi + \frac{\Delta\lambda^3\cos^3\phi}{6}(1-t^2+\eta^2) + \frac{\Delta\lambda^5\ \cos^5\phi}{120}(5-18t^2+t^4+14\eta^2-58t^2\eta^2) + \frac{\Delta\lambda^7\ \cos^7\phi}{5040}(61-479t^2+179t^4-t^6) \quad \text{. . . A4.15}$$

And:

$$\frac{N'}{k_o n} = \frac{S_\phi}{n} + \frac{\Delta\lambda^2}{2}\ \sin\ \phi\ \cos\ \phi + \frac{\Delta\lambda^4}{24}\ \sin\ \phi\ \cos^3\phi(5-t^2+9\eta^2+4\eta^4) + \frac{\Delta\lambda^6}{720}\ \sin\ \phi\ \cos^5\ \phi(61-58t^2+t^4+270\eta^2-330t^2\eta^2) + \frac{\Delta\lambda^8}{40320}\sin\phi\ \cos^7\phi(1385-3111t^2+543t^4-t^6) \quad \text{. . . A4.16}$$

e. **To Convert From Grid to Geographical Coordinates.** If an accuracy of ± 0.001" is acceptable, terms containing $(E')^7$ and higher powers of E' may be ignored.

$$\frac{\phi}{t_1} = \frac{\phi_1}{t_1} - \frac{(E')^2}{2k_o^2\rho_1\nu_1} + \frac{(E')^4}{24k_o^4\rho_1\nu_1^3}(5+3t_1^2+\eta_1^2-9t_1^2\eta_1^2-4\eta_1^4) - \frac{(E')^6}{720k_o^6\rho_1\nu_1^5}(61+90t_1^2+45t_1^4+46\eta_1^2-252t_1^2\eta_1^2-90t_1^4\eta_1^2) + \frac{(E')^8}{40320k_0^8\rho_1\nu_1^7}(1385+3633t_1^2+4095t_1^4+1575t_1^6) \quad \text{. . . A4.17}$$

And:

$$\Delta\lambda\ \cos\phi_1 = \frac{E'}{k_0\nu_1} - \frac{(E')^3}{6k_0^3\nu_1^3}(1+2t_1^2+\eta_1^2) + \frac{(E')^5}{120k_0^5\nu^5}(5+28t_1^2+24t_1^4+6\eta_1^2+8t_1^2\eta_1^2) - \frac{(E')^7}{5040k_0^7\nu_1^7}(61+662t_1^2+1320t_1^4+720t_1^6) \quad \text{. . . A4.18}$$

APPENDIX 5

THE SPHEROIDAL EARTH

1. Scope of Appendix

Appendix 5 contains the following information for the *Spheroidal* Earth:

- **Para 2: The Equation of the Ellipse.**
- **Para 3: Geodetic Latitude, Geocentric Latitude & Parametric Latitude.**
- **Para 4: The length of the Earth's radius in various Latitudes.**
- **Para 5: The length of one minute of Latitude - The Sea Mile.**
- **Para 6: The Length of the Spheroidal Meridional Arc.**
- **Para 7: Meridional Parts for the Spheroidal Earth.**

2. The Equation of the Ellipse

When a point *M* (see Fig A5-1 below) moves so that its distance from a fixed point *S* (the focus) is always in a constant ratio *e* (less than unity) to its perpendicular distance from a fixed straight line *AB* (the directrix), the locus of *M* is called an ellipse of *Eccentricity (e)*. The *Equation of the Ellipse* takes its simplest form when the co-ordinates of *S* are ($-ae$, 0) and the directrix *AB* is the line:

$$x = -\frac{a}{e}$$

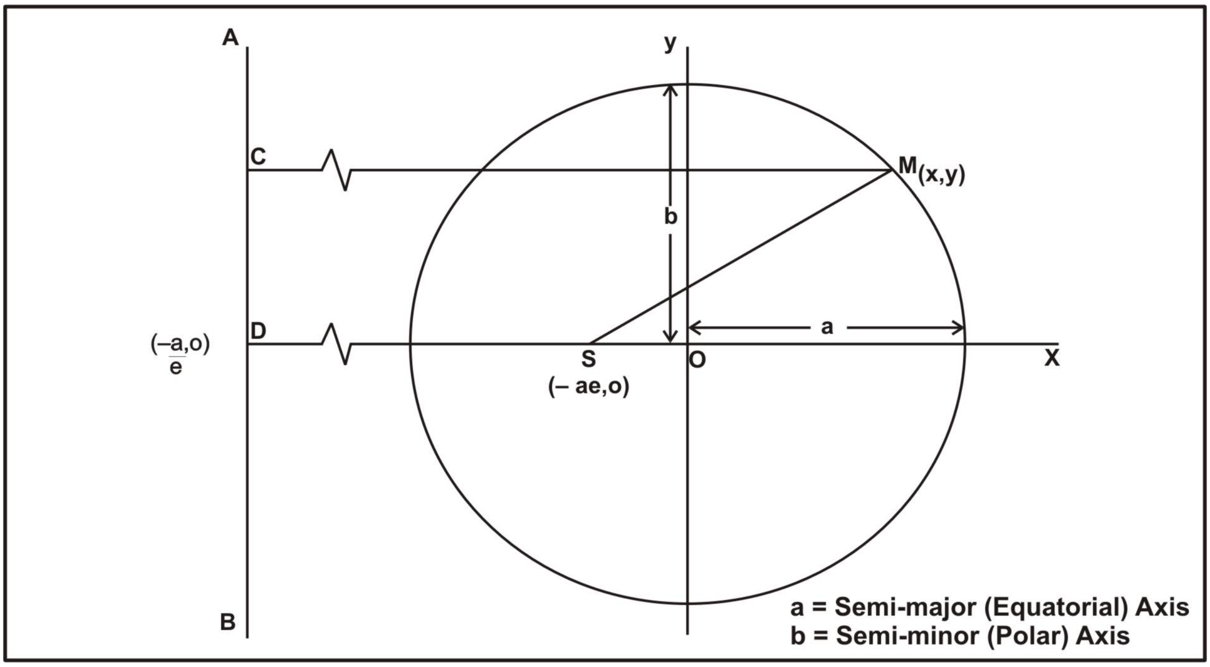


Fig A5-1. The Ellipse

From Fig A5.1: $MS = eMC$ and $MC = x + \frac{a}{e}$

$$(MS)^2 = y^2 + (x + ae)^2$$

$$\therefore e^2\left(x + \frac{a}{e}\right)^2 = (x + ae)^2 + y^2$$

ie $(1 - e^2)x^2 + y^2 = a^2(1 - e^2)$

This may be written in the form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{. . . A5.1}$$

(2 continued)

where: $$b^2 = a^2\left(1 - e^2\right)$$ . . . **A5.2**

ie $$e = \left(\frac{a^2 - b^2}{a^2}\right)^{1/2}$$ . . . (formula 3.2)

The ellipse corresponds to a cross-section of the Earth, where *a* is the *Equatorial* and *b* the *Polar* radius. As *b* is less than *a*, the Earth is 'flattened' in the *Polar* regions.

The *Flattening* or ellipticity of the Earth may be defined by a quantity *f* where:

$$f = \frac{a - b}{a}$$ . . .(formula 3.1)

From formulae (3.1) and (3.2):

$$e = \left(2f - f^2\right)^{1/2}$$ . . . (formula 3.3)

The quantities *a, e* and *f* are used regularly in the solution of *Rhumb Line* and *Great Circle* sailing problems on the *Spheroid.*

3. Geodetic Latitude, Geocentric Latitude and Parametric Latitude

a. **Geodetic Latitude and Geocentric Latitude.** As stated at Para 0312 and shown in Fig A5-2 (below), ϕ is the *Geodetic Latitude* and θ is the *Geocentric Latitude* of *M*.

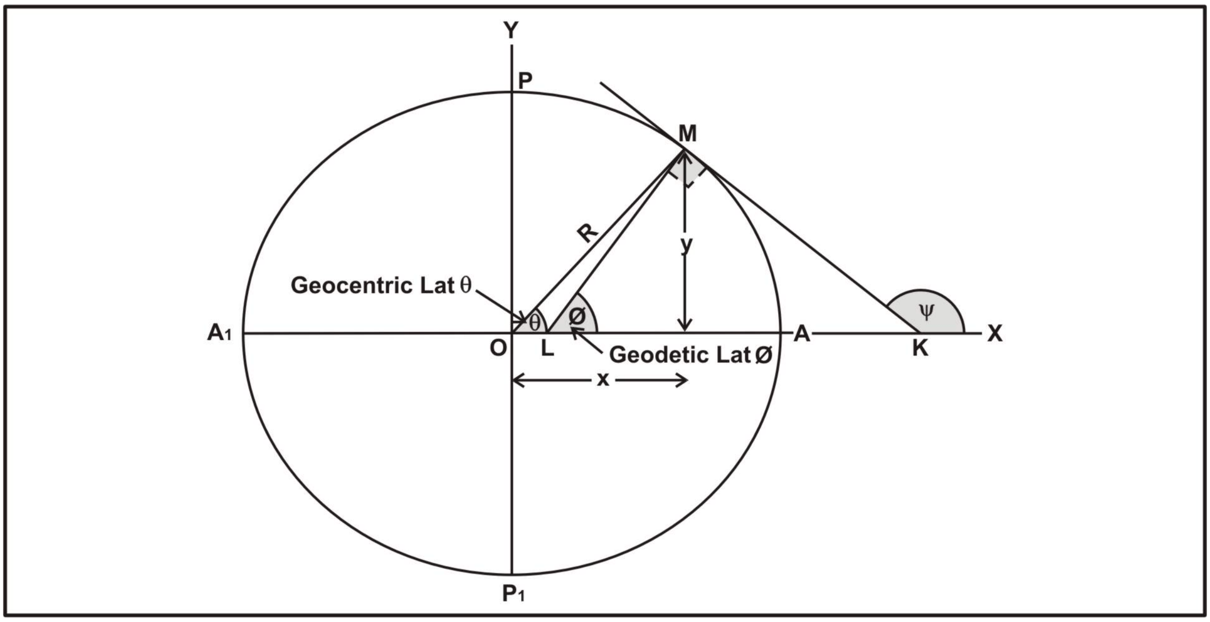


Fig A5-2. Geodetic Latitude and Geocentric Latitude

In Fig A5-2 (above), if the distance of the point *M* from the *Polar* axis *OP* is *x*, and its distance from the major axis *OA* is *y*, these distances or co-ordinates are connected by the *Equation of the Ellipse* on which *M* lies, ie:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ . . . (formula A5.1)

(3 continued)

Thus: $$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 - \frac{x^2 b^2}{a^2}$$

By differentiation:

$$2y\frac{dy}{dx} = -2x\frac{b^2}{a^2}$$

$$\frac{dy}{dx} = -\frac{x}{y}\frac{b^2}{a^2}$$

If ψ is the angle which the tangent *MK* makes with the X-axis, then, since the slope of the tangent is measured by the differential coefficient:

$$\tan\psi = \frac{dy}{dx} = -\frac{b^2}{a^2}\frac{x}{y}$$

But ψ is equal to (ϕ + 90˚) since *ML* is perpendicular to *MK*:

Hence: $$\tan\psi = -\cot\phi$$

$$\therefore\ \cot\phi = \frac{b^2}{a^2}\frac{x}{y} \qquad \text{. . . \textbf{A5.3}}$$

$$= \frac{b^2}{a^2}\cot\theta$$

But ϕ and θ are connected by formulae (3.4), (3.5) and (3.6):

$$\tan\ \theta = \frac{b^2}{a^2}\ \tan\ \phi \qquad \text{. . . (formula 3.4)}$$

$$= \left(1 - f\right)^2\ \tan\ \phi \qquad \text{. . . (formula 3.5)}$$

$$= \left(1 - e^2\right)\ \tan\ \phi \qquad \text{. . . (formula 3.6)}$$

The difference between the *Geodetic Latitude* and *Geocentric Latitude* is zero at the *Equator* and the *Poles* and has a greatest value when ϕ = 45˚.

For the *WGS 84 Spheroid*, where *f* = 1/298.257223563, the greatest value of the angle *OML* $\left(\phi - \theta\right)$ is approximately 11.54 minutes of arc (see Para 0312d).

(3) b. **The Parametric Latitude.** As stated at Para 0313 and shown in Fig A5-3 (below), β is the *Parametric Latitude* of *M*.

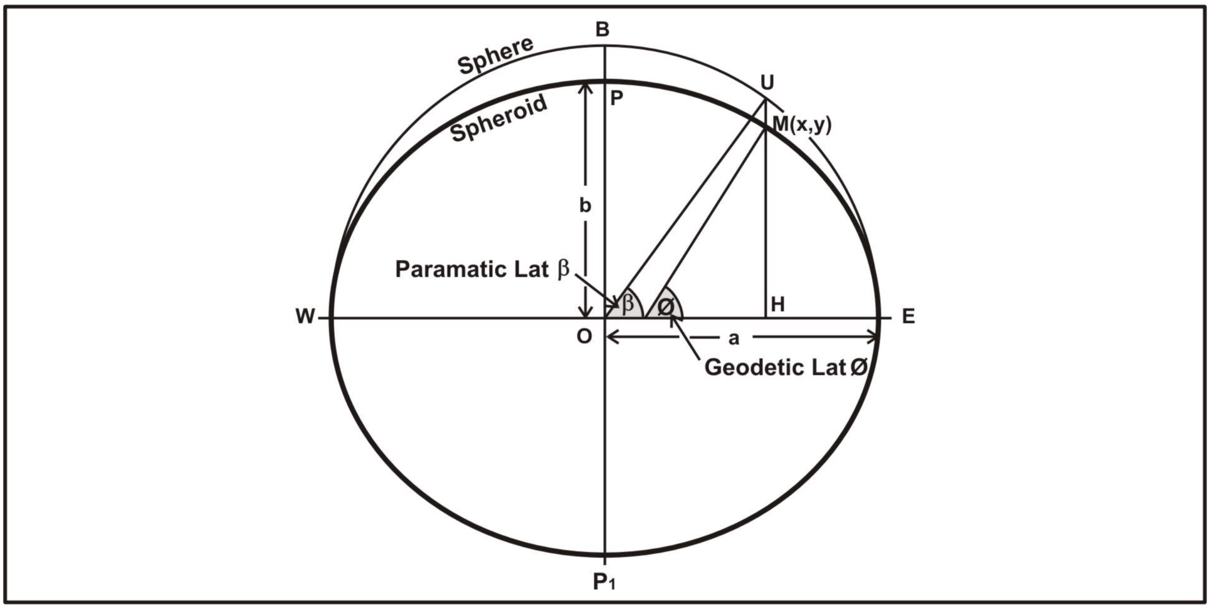


Fig A5-3. Parametric Latitude

If the co-ordinates of *M* are (*x, y*) and *WBE* is a semi-circle of radius *a*, centre *O*.

$$OH = OU \cos\beta$$

ie $$x = a \cos\beta$$

But: $$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = 1 - \cos^2\beta$$ from . . . (formula A5.1)

$$y^2 = b^2\left(1 - \cos^2\beta\right) = b^2 \sin^2\beta$$

$$y = b \sin\beta$$

$$\therefore \frac{y}{x} = \frac{b}{a}\tan\beta$$

From Fig A5-2: $$\frac{y}{x} = \tan\theta$$

And, $$\frac{x}{y} = \frac{b^2}{a^2}\tan\phi$$ from . . . (formula 3.4)

$$\therefore \tan\beta = \frac{b}{a}\tan\phi$$ . . . (formula 3.7)

$$= \left(1 - f\right)\tan\phi$$ . . . **A5.5**

The difference between the *Geodetic Latitude* and *Parametric Latitude* is zero at the *Equator* and at the *Poles* and has a greatest value when $\phi = 45^\circ$.

For the *WGS 84 Spheroid*, where f = 1/298.257223563, the greatest value is approximately 5.85 minutes of arc (see Para 0313b).

4. The Length of the Earth's Radius in Various Latitudes

In Fig A5-2 (see Para 3a), the required *Geocentric* radius is *OM* and, if this length is denoted by *R*, it follows that $x = R\cos\theta$ and $y = R\sin\theta$. Hence, by substituting for *x* and *y* in the equation of the ellipse:

$$\frac{R^2\cos^2\theta}{a^2} + \frac{R^2\sin^2\theta}{b^2} = 1$$

But, as: $b^2 = a^2(1-f)^2$ then:

$$R^2\left[(1-f)^2\cos^2\theta + \sin^2\theta\right] = a^2(1-f)^2$$

When terms in f^2 (10^{-5} x 1.1) are neglected, this equation becomes:

$$R^2(1-2f\cos^2\theta) = a^2(1-2f)$$

$$R = a\left(\frac{1-2f}{1-2f\cos^2\theta}\right)^{1/2}$$

When the right-hand side is expanded by the binomial theorem, terms in f^2 and higher powers again being omitted, the equation becomes:

$$R = a(1-f)(1+f\cos^2\theta)$$

$$= a(1-f\sin^2\theta)$$

Since θ varies from ϕ by a small quantity, *R* may be expressed in terms of the *Geodetic Latitude* (Para 3a) without appreciable error by direct substitution:

$$R = a(1-f\sin^2\phi) \qquad \ldots \textbf{A5.13}$$

5. The Length of One Minute of Latitude - The Sea Mile

The length of the *Sea Mile* (one minute of *Latitude* on the *Spheroid*) may be found from the general formula $\rho d\phi$ (see Fig A5-4 below) where ρ is the radius of curvature in the *Meridian* and $d\phi$ a small increase (in radians) in the *Geodetic Latitude* ϕ .

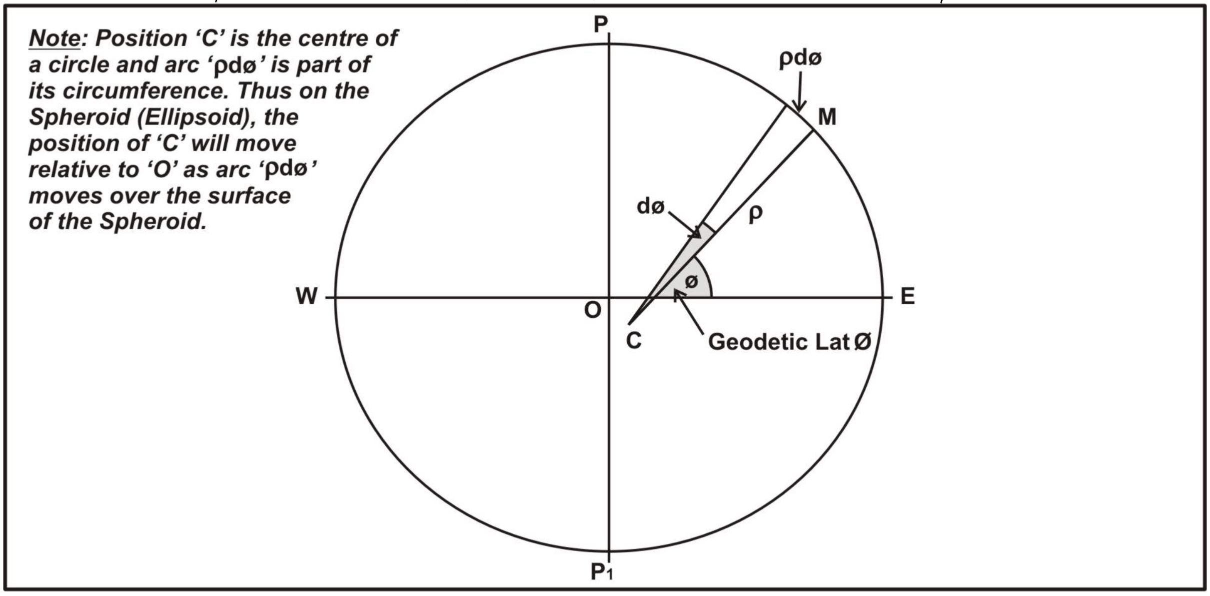


Fig A5-4. The Length of One Minute of Latitude (1)

Fig A5-5 (below) shows an expanded version of Fig A5-4, where $d\phi$ is a very small increase in ϕ . The co-ordinates of *M* are (*x, y*); those of M_1, representing this small increase, are (x − dx), (y + dy).

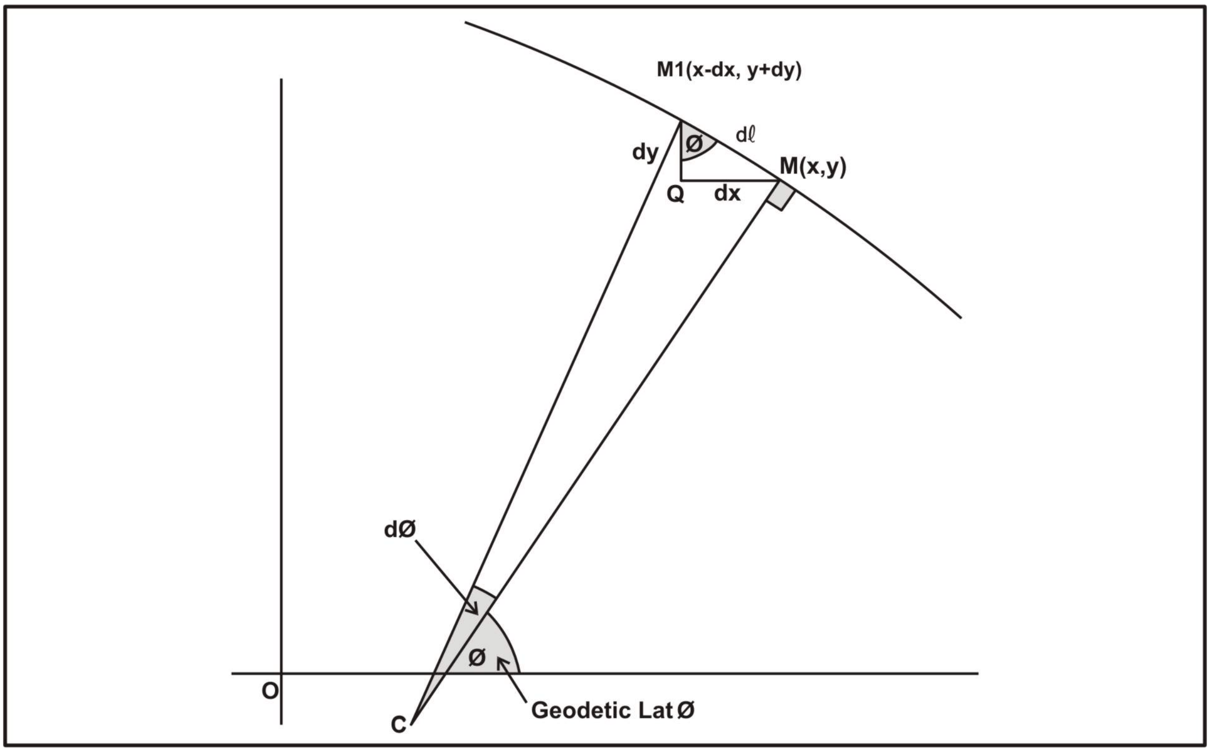


Fig A5-5. The Length of One Minute of Latitude (2)

(5 continued)

The triangle MQM_1 (see Fig A5-5 opposite) may be considered plane and, if the length of MM_1 is denoted by dℓ then:

$$\frac{d\ell}{dx} = -\frac{1}{\sin\phi}$$

But, as: $$d\ell = \rho d\phi$$

$$\frac{d\ell}{d\phi} = \rho \quad = \frac{dx}{d\phi} \text{ x } \frac{d\ell}{dx}$$

$$\therefore \ \rho = -\frac{1}{\sin\phi} \text{ x } \frac{dx}{d\phi} \qquad \textbf{. . . A5.6}$$

$\frac{dx}{d\phi}$ may be found as follows:

$$y = \frac{xb^2}{a^2}\tan\phi \qquad \text{from . . . (formula A5.3)}$$

Thus: $$= x\left(1 - e^2\right)\tan\phi \qquad \text{from . . . (formula A5.2)}$$

If this value of *y* is substituted in the general *Equation of the Ellipse* formula (A5.1) and the value of *b* from formula (A5.2) also substituted, then:

$$\frac{x^2}{a^2} + \frac{x^2\left(1 - e^2\right)^2 \ \tan^2\phi}{a^2\left(1 - e^2\right)} = 1$$

$$x^2 + \left(1 - e^2\right)x^2\tan^2\phi \ = a^2 \ = x^2\left(1 + \tan^2\phi - e^2\tan^2\phi\right) = x^2\left(\sec^2\phi - e^2\tan^2\phi\right)$$

$$x^2\left[\frac{1}{\cos^2\phi} - \frac{e^2\sin^2\phi}{\cos^2\phi}\right] = a^2$$

$$x = \frac{a \ \cos \ \phi}{\left(1 - e^2\sin^2\phi\right)^{1/2}}$$

$$= a \ \cos \ \phi\left(1 - e^2\sin^2\phi\right)^{-1/2} \qquad \textbf{. . . A5.7}$$

Differentiating:

$$\frac{dx}{df} = \frac{-a\left(1 - e^2\right)\sin\phi}{\left(1 - e^2\sin^2\phi\right)^{3/2}}$$

Substituting in formula (A5.6):

$$\rho = \frac{a\left(1 - e^2\right)}{\left(1 - e^2\sin^2\phi\right)^{3/2}} \qquad \text{. . . (formula 3.8)}$$

Thus, when $d\phi$ equals 1' of arc:

$$1' \ of \ Latitude = \frac{a\left(1 - e^2\right)}{\left(1 - e^2\sin^2\phi\right)^{3/2}}\sin \ 1' \qquad \text{. . . (formula 3.9)}$$

(5 continued)

Formula (3.9) is the theoretical expression for the *Sea Mile*. The expression may be expanded as follows:

$$1' \ of \ Latitude = a \sin 1' (1 - e^2) \left(1 + \frac{3e^2}{2} \sin^2 \phi + \frac{15e^4}{8} \sin^4 \phi + \ldots \right)$$

Approximating by disregarding terms of e^4 (10^{-5} x 4.5) and higher powers:

$$1' \ of \ Latitude = a \ \sin \ 1' \left(1 + \frac{3e^2}{2} \sin^2 \phi - e^2 \right)$$

$$= a \ \sin \ 1' \left[1 - e^2 + \frac{3e^2}{4} \left(1 - \cos \ 2\phi \right) \right]$$

$$= a \ \sin \ 1' \left(1 - \frac{e^2}{4} - \frac{3e^2}{4} \cos \ 2\phi \right)$$

$$= a \ \sin \ 1' \left[1 - \frac{e^2}{4} \left(1 + 3 \ \cos \ 2\phi \right) \right]$$

When figures for *a* and *e* for the *WGS 84 Spheroid* are given:

$1' \ of \, Latitude = 1852.22 - 9.315 \cos \ 2\phi$ *(metres)* . . . **A5.8**

Or: $1' \ of \, Latitude = 1.00012 \ - \ 0.00503 \ \cos \ 2\phi$ *(n. miles)* . . . **A5.8**

$\therefore$ $1' \ of \, Latitude = 1852.22 \ (metres) \ or \ 1.00012 \ (n.miles)$ *(at Latitude 45˚)*

Compared to the precise formula (3.9), this approximation gives a solution for the *Sea Mile* which is correct (for *WGS 84 Spheroid*) at the *Equator*, is 0.00127% in error at *Latitude* 45˚ and is 0.00169% in error at Latitude 90˚.

6. The Length of the Spheroidal Meridional Arc

In Fig A5-6 (below), where ϕ is the *Geodetic Latitude* and ρ the radius of curvature in the *Meridian*, the length R of the *Meridional Arc EM* may be found from formula (A5.8a):

$$\ell = \int_0^{\phi} \rho d\phi \qquad \text{. . . A5.8a (1987 Ed . . . 5.17)}$$

The value of ρ is given in formula (3.8) at Para 0314.

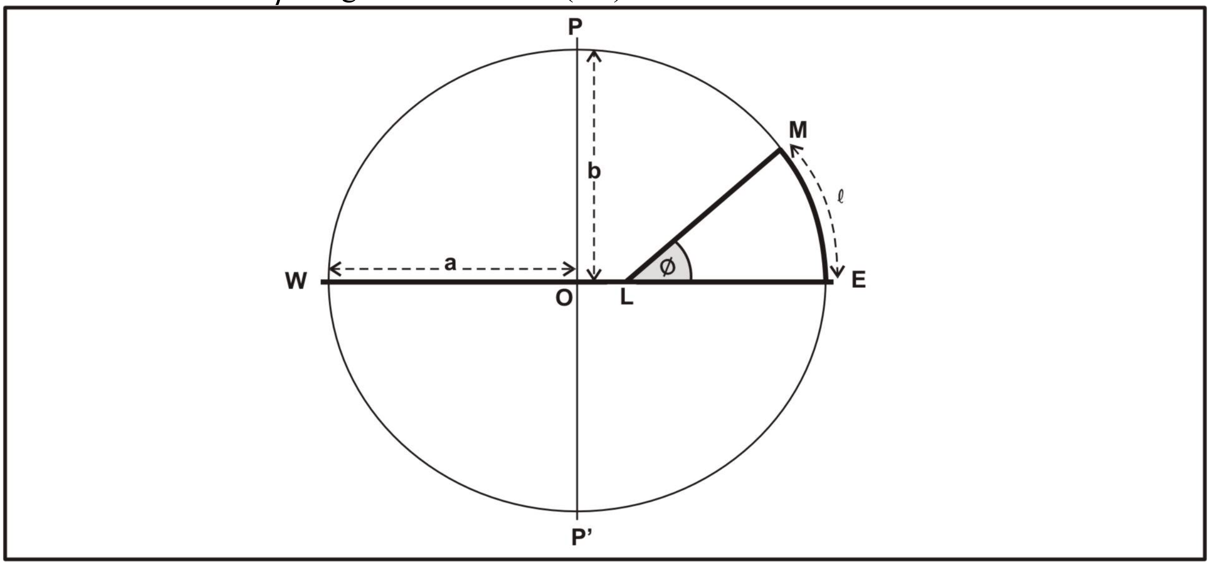


Fig A5-6. The Length of the Spheroidal Meridional Arc (Copy of Fig 5-6)

(6 continued)

Following from Fig A5.6 (opposite) and formula (A5.8a), the *Meridional Arc* length ℓ along a *Meridian* between two *Geodetic Latitudes* ϕ_1 and ϕ_2 may be found from formula (A5.8b):

$$\ell = \int_{\phi 1}^{\phi 2} \rho d\phi \qquad \textbf{. . . A5.8b}$$

$$= a\left(1 - e^2\right)\int_{\phi 1}^{\phi 2} \frac{1}{\left(1 - e^2 \sin^2\phi\right)^{3/2}} d\phi \qquad \textbf{. . . A5.8c (1987 Ed . . . 5.18)}$$

Expanding by the binomial theorem:

$$= a\left(1 - e^2\right)\int_{\phi 1}^{\phi 2}\left(1 + \frac{3e^2}{2}\sin^2\phi + \frac{15e^4}{8}\sin^4\phi + \frac{35e^6}{16}\sin^6\phi + \ldots\right)d\phi$$

Each term in the integral may now be integrated separately where:

$$\int \sin^2\phi \, d\phi = \int\left(½ - ½ \cos 2\phi\right) d\phi = \frac{\phi}{2} - \frac{\sin 2\phi}{4} + c$$

$$\int \sin^4 \phi \, d\phi = \int\left(\frac{3}{8} - \frac{\cos 2\phi}{2} + \frac{\cos 4\phi}{8}\right)d\phi = \frac{3\phi}{8} - \frac{\sin 2\phi}{4} + \frac{\sin 4\phi}{32} + c$$

$$\int \sin^6\phi \, d\phi = \left(\frac{10}{32} - \frac{15 \cos 2\phi}{32} + \frac{3 \cos 4\phi}{16} - \frac{\cos 6\phi}{32}\right)d\phi$$

$$= \frac{10\phi}{32} - \frac{15 \sin 2\phi}{64} + \frac{3 \sin 4\phi}{64} - \frac{\sin 6\phi}{192} + c \qquad \text{etc, etc.}$$

Thus:

$$\ell = a\left(1 - e^2\right)\left[\phi + \frac{3e^2}{2}\left(\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right) + \frac{15e^4}{8}\left(\frac{3\phi}{8} - \frac{\sin 2\phi}{4} + \frac{\sin 4\phi}{32}\right)\right.$$
$$\left. + \frac{35e^6}{16}\left(\frac{10\phi}{32} - \frac{15\sin2\phi}{64} + \frac{3\sin4\phi}{64} - \frac{\sin6\phi}{192}\right) + \ldots \right]_{\phi 1}^{\phi 2} \qquad \textbf{. . . A5.8d}$$

The *Meridional Arc* length ℓ may be determined from formula (A5.8e) for any *Spheroid* of known *Equatorial* semi-major axis a and *Eccentricity* e (see Para 0322, Table 3-1), and expressed, dependent on what unit is used for a (metres, *International Nautical Miles* etc).

$$\ell = \int_O^\phi \rho df = a\left(A_O\phi - A_2\sin2\phi + A_4\sin4\phi - A_6\sin6\phi + \ldots\right)$$

. . . A5.8e (1987 Ed . . . 5.19)

This may be expanded in the form:

$$\ell = a\left[A_o\phi - A_2\sin2\phi + A_4\sin4\phi - A_6\sin6\phi + \ldots\right]_{\phi 1}^{\phi 2} \qquad \textbf{. . . A5.8f}$$

Thus:

$$\ell = a\Big[A_o\left(\phi_2 - \phi_1\right) - A_2\left(\sin2\phi_2 - \sin2\phi_1\right)$$
$$+ A_4\left(\sin4\phi_2 - \sin4\phi_1\right) - A_6\left(\sin6\phi_2 - \sin6\phi_1\right) + \ldots \Big] \qquad \textbf{. . . A5.9}$$

Where, ϕ is measured in radians and A_0, A_2, A_4, A_6 are given by:

$$A_o = 1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 - \ldots$$

(6 continued)

$$A_2 = \frac{3}{8}\left(e^2 + \frac{1}{4}e^4 + \frac{15}{128}e^6 + \ldots\right)$$

$$A_4 = \frac{15}{256}\left(e^4 + \frac{3}{4}e^6 + \ldots\right)$$

$$A_6 = \frac{35}{3072}e^6 + \ldots$$

Formula (A5.9) [previous page] is large and complex, and ideally a computer is needed to calculate the *Meridional Arc* distance ℓ. However, the *Meridional Arc* distance ℓ may be calculated to a reasonably high degree of accuracy by disregarding terms of $e^6(10^{-7}$ x 3.1) and higher powers. With this approximation, the *Meridional Arc* distance ℓ from the *Equator* to *Latitude* ϕ may be found from formula (5.24):

$$\ell = a\left[\phi - \frac{e^2\phi}{4} - \frac{3e^2}{8}sin2\phi - \frac{3e^4}{64}\phi - \frac{3e^4}{32}\ sin\ 2\phi + \frac{15e^4}{256}\ sin\ 4\phi\right]$$

. . . (formula 5.24)

Tables giving the length of the *Meridional Arc* for any *Latitude* (eg at minute of arc intervals) may be constructed by computer from the general formula (A5.8e). Tables may be computed for any *Spheroid* and may be expressed in the same units of distance used for *a*. The length of the *Meridional Arc* between the two different *Latitudes* can then be obtained, and the course and distance calculated between two positions using formulae (5.22) and (5.23). Conversely, if the course, distance and initial positions are known, the final *Latitude* may be computed from the length of the *Meridional Arc* and the final *Longitude* from the difference of *Meridional Parts*.

An error can arise from the assumption that a distance in *International Nautical Miles* (n miles) can be said to equate to a *d.lat* measured in minutes of arc or *Sea Miles*. The maximum error in this assumption is of the order of 0.5%. For most purposes, little account need be taken of this difference between the n mile and the *Sea Mile* except when precise distances are required, particularly near the *Equator* or the *Poles*.

7. Meridional Parts for the Spheroidal Earth

Tables of *Meridional Parts* used by cartographers to compute the *Graticules* for *Mercator Projection* charts must be for *Spheroidal Meridional Parts*. Astronomical observations at sea are made with reference to a horizon which is part of the *Spheroidal* surface of the Earth; thus, tables of *Spheroidal Meridional Parts* are consistent with the co-ordinates of positions found from astronomical observations.

In Fig A5-7 (below), where $x = a \cos \beta$ and $y = b \sin \beta$, the elliptic *Meridional* section of the Earth may be expressed by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

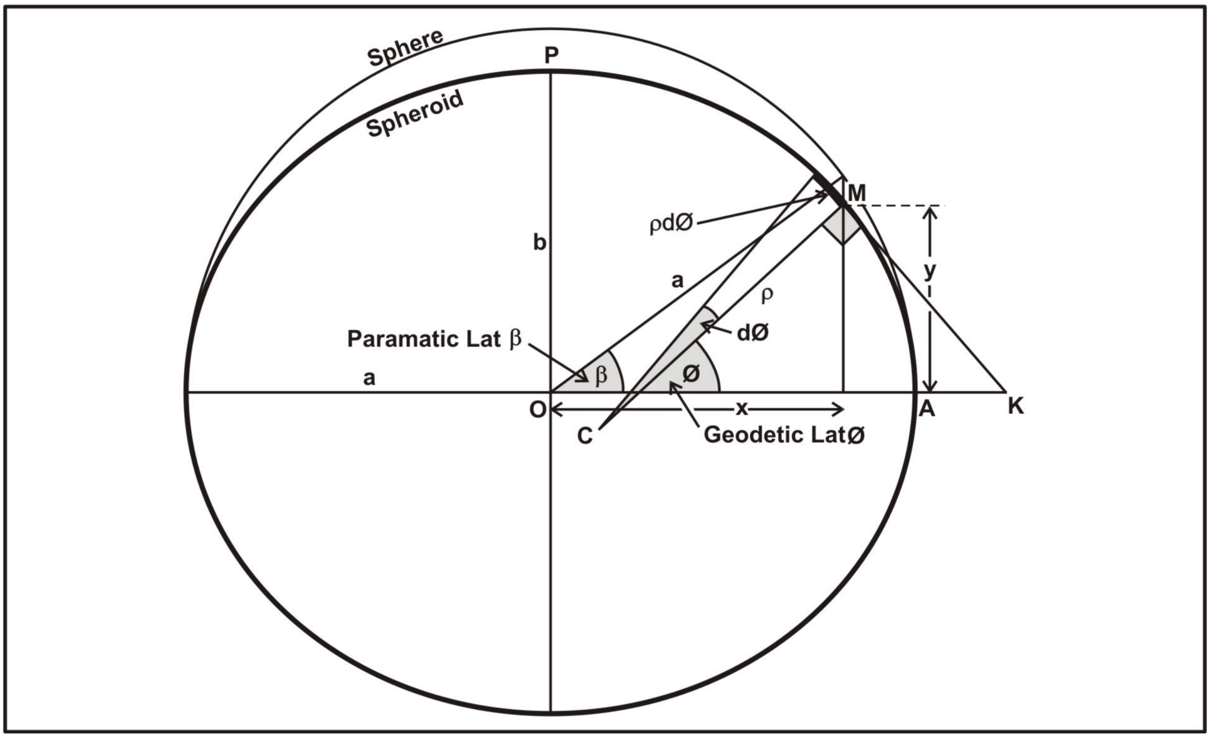


Fig A5-7. Meridional Section of the Spheroidal Earth

At a point *M* which has co-ordinates (*x, y*) with reference to *O*, the centre of the ellipse, let the *Geographic Latitude* be ϕ. If the radius of curvature at *M* is ρ, the length of an element of the *Meridian* is $\rho d\phi$.

In order to measure the *Meridional Parts* of ϕ, the *Meridional Element* $\rho d\phi$ must be expressed in terms of the length of 1 minute of *Longitude* at *Latitude* ϕ. The *Longitude Scale* for this *Latitude* is *x/a* times the *Longitude Scale* at the *Equator*, and the unit of *Longitude* at the *Equator* is the length of that *Equatorial Element* which subtends an angle of 1 minute of arc at the centre of the Earth. The length of this element is *a* divided by the number of minutes in 1 radian, that is: $\frac{a\pi}{10800}$

The length of a minute of *Longitude* at *Latitude* ϕ is thus:

$$\frac{x}{a} \times \frac{a\pi}{10800} \quad \text{or} \quad \frac{x\pi}{10800}$$

(7 continued)

And, the number of *Longitude* units in the *Meridional Element* $\rho d\phi$ is:

$$\rho d\phi \div \frac{x\pi}{10800} \quad \text{or} \quad \frac{10800}{\pi} \text{ x } \frac{\rho}{x} d\phi$$

The *Meridional Parts* at *Latitude L* are given by the equation:

$$Mer\ Parts\ L = \frac{10800}{\pi} \int_{O}^{L} \frac{\rho}{x} d\phi$$

Which, from formulae (3.8) and (A5.7):

$$Mer\ \ Parts\ \ L = \frac{10800}{\pi} \int_{O}^{L} \frac{a\left(1 - e^2\right)}{\left(1 - e^2 sin^2 \phi\right)^{3/2}} \text{ x } \frac{1}{a\ \ cos\ \ \phi\left(1 - e^2 sin^2 \phi\right)^{-1/2}} d\phi$$

$$= \frac{10800}{\pi} \int_{O}^{L} sec\phi \left(\frac{1 - e^2}{1 - e^2 sin^2 \phi} \right) d\phi \qquad \text{. . . } \mathbf{A5.11}$$

$$Mer\ \ Parts\ \ L = \frac{10800}{\pi} \int_{O}^{L} sec\phi \left[1 - e^2 cos^2 \phi \left(1 + e^2 sin^2 \phi + e^4 sin^4 \phi + e^6 sin^6 \phi + \dots \right) \right] d\phi$$

$$Mer\ \ Parts\ \ L = \frac{10800}{\pi} \int_{O}^{L} \left(sec\phi - e^2 cos\phi - e^4 sin^2 \phi\ cos\phi\ - e^6 sin^4 \phi\ cos\phi - \dots \right) d\phi$$

$$Mer\ \ Parts\ \ L = \frac{10800}{\pi} \left[log_e\ \ tan\left(45° + \frac{L°}{2}\right) - e^2 sinL - \frac{1}{3} e^4 sin^3 L - \frac{1}{5} e^6 sin^5 L - \dots \right]$$

. . . (formula 5.21a)

From formula (5.21a), a simplified numerical formula (ignoring e^6 and higher powers) for the *WGS 84 Spheroid*, giving the *Meridional Parts* '*m*' correct to three decimal places is:

$$Mer\ Pats\ L\ = 7915.7045 log_{10} tan\left(45° + \frac{L°}{2}\right) - 23.01358\ \ sin\ \ L - 0.05135\ \ sin^3 L$$

. . . (formula 5.21b)

APPENDIX 6

VERTICAL AND HORIZONTAL SEXTANT ANGLES

1. Scope of Appendix

Appendix 6 contains the following information:

- **Para 2: Vertical Sextant Angles (VSAs).**
- **Para 3: Horizontal Sextant Angles (HSAs).**

2. Vertical Sextant Angles (VSAs)

a. **Base of the Object Visible to the Observer.** As stated at Para 0803h, a *Position Line* may be obtained from the observation of the *Vertical Sextant Angle* (*VSA*) of an object (eg 10 foot pole, lighthouse etc) whose base is visible to the observer, by multiplying the known height of the object by the 'cot' of the observed angle.

Distance = Visible Height of Object x *Cot (Observed Angle)* . . . (formula 8.1)

A table of 'Distance by Vertical Angle' for ranges up to 7 *n. miles* is provided in Norie's Nautical Tables, for objects whose base is visible to the observer. At such short ranges, the effect of *Atmospheric Refraction* is ignored.

b. **Base of the Object Beyond the Observer's Horizon.** A *Position Line* may also be obtained from the observation of the *VSA* of an object (eg distant mountain peak) where the base is out of sight beyond the observer's horizon, but the calculation is more complex, and some approximations are necessary.

Parameters. At Fig A6-1 (overleaf), the following parameters are established:

- *C* is the *Centre of Curvature* of the Earth, with a local *Radius of Curvature R*.
- *AD* is the height of eye *h*.
- *B* is a mountain summit whose height *BE* is *H* above the prevailing sea level.
- *DE* is the required distance *d*, while the angle measured between the mountain top and the observer's horizon is represented by the angle *JAF*.

Terrestrial Atmospheric Refraction. The angle *JAF* takes account of the terrestrial *Atmospheric Refraction r*, which 'bends' the ray of light as it proceeds through the atmosphere between object and observer. Thus, the top of the mountain *B* is seen in the direction *AJ*, while the horizon *G* is seen in the direction *AF*. These two lines *AJ* and *AF* are tangential to their respective curved rays of light (shown as pecked lines in Fig A6-1). At long range with high objects, terrestrial *Atmospheric Refraction* amounts to approximately 8% of the distance in *n.miles* of the object, expressed in minutes of arc; thus an initial approximate estimate of distance is required.

Dip. AK is the horizontal at the observer's position and the angle *KAF* is known as the *Angle of Dip* (*Dip*), defined as the angle between the horizontal plane through the eye of the observer and the apparent visible horizon. It is always present when the observer's eye is above sea level. *Dip* is tabulated in the Nautical Almanac and in Norie's Tables. *Dip* and *Atmospheric Refraction* are explained fully in BR 45 Volume 2 Chapter 8; both must be subtracted from the *Observed Altitude* of the object to obtain its *True Altitude*.

(2b continued)

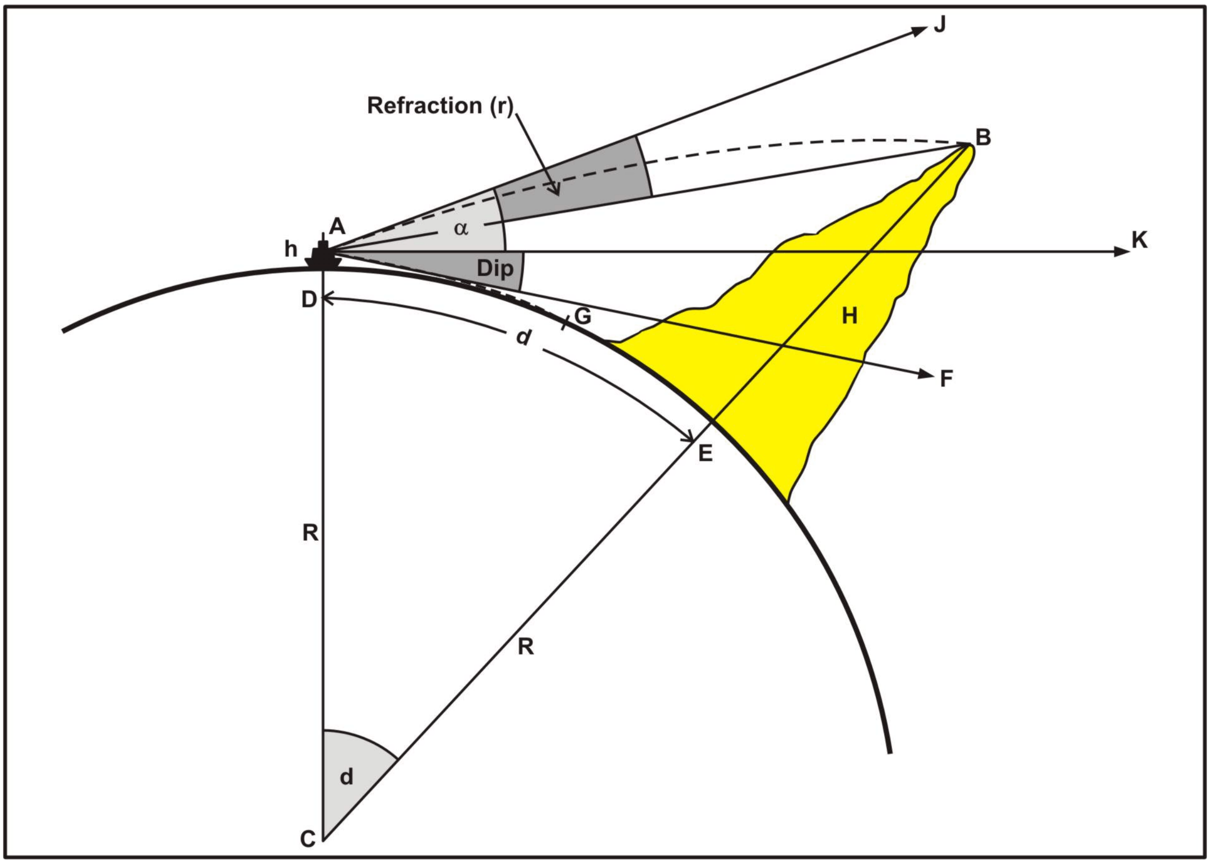


Fig A6-1. Position Line by Vertical Sextant Angle − Base of the Object Beyond Horizon(1)

Explanation. The apparent altitude of *B* as measured from the sea horizon, when reduced by *Dip*, is the angle *JAK*, α. The *True Altitude* of *B*, the angle *BAK*, is (α − *r*), where *r* is the amount of *Atmospheric Refraction JAB*.

Earth's Radius. The *Radius of Curvature* of the Earth varies slightly with both *Latitude* and the *Azimuth* of the cross-section concerned. However, for most practical purposes it is sufficiently accurate to use a fixed *Radius of Curvature* of 3437.75 *n. miles;* this may result in a distance error which should not exceed 0.32%. The calculated distance *d* would then be expressed directly in *n. miles*, regardless of *Latitude* or *Azimuth*.

Calculation. At Fig A6-1 (above) and at Fig A6-2 (opposite) in the triangle *CAB*:

$$CA = R + h$$

$$CB = R + H$$

$$CAB = 90° + \alpha - r$$

$$CBA = 180° - (d + 90° + \alpha - r) = 90° - d - \alpha + r$$

$$\frac{\sin\ CBA}{\sin\ CAB} = \frac{R+h}{R+H} = \frac{\sin\ 90° - (d+\alpha - r)}{\sin\ 90° + (\alpha - r)} = \frac{\cos\ (d+\alpha - r)}{\cos\ (\alpha - r)}$$

$$\cos\ (d + \alpha - r) = \frac{R+h}{R+H}\ \cos\ (\alpha - r) \qquad \textbf{. . A6.1}$$

Distance *d* may be found from formula (A6.1): see Example A6-1 (opposite). If the estimated distance is too much in error, a second approximation will be necessary.

(2b continued)

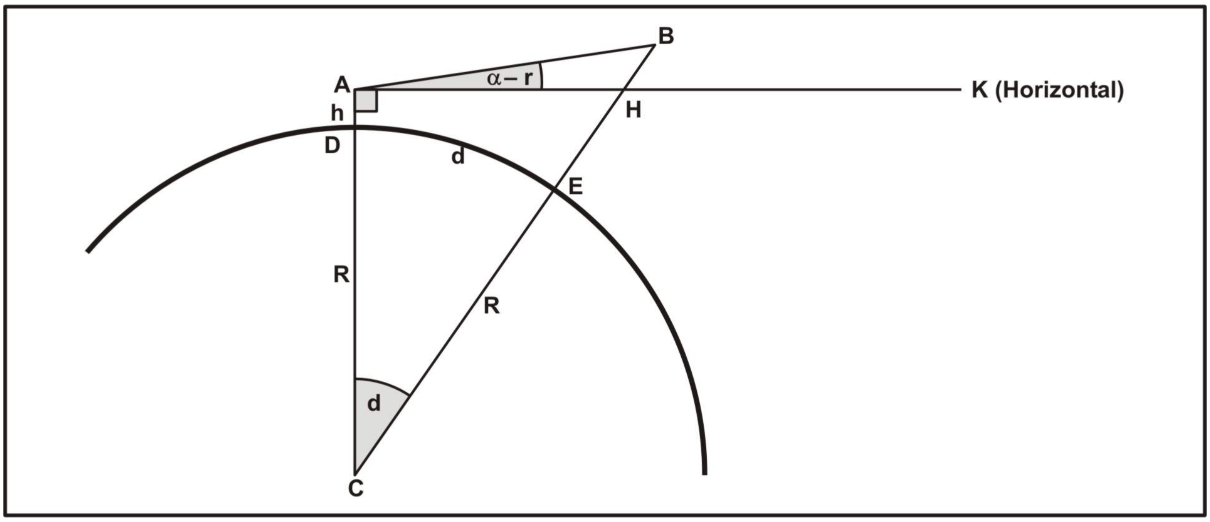


Fig A6-2. Position Line by Vertical Sextant Angle − Base of the Object Beyond Horizon(2)

Example A6-1. A mountain 1646 m (5400 ft) high is observed at a range of about 25 *n.miles*. The observer's height of eye is 7.6 m (25 ft). The *VSA* of the summit is 1˚59'.3. *Index Error* of the sextant is -1'.3. What is the range of the mountain?

Observed angle	1˚59.3	
Index error	-1'.3	
	1˚58'.0	
Dip	-4'.9	
Apparent Altitude (α)	1˚53'.1	
Refraction correction (r) (8% of 25 = 2.0)	-2'.0	
True altitude (α - r)	1˚51'.1	(1.8517˚)

R = 3437.75 *n. miles*
H = 0.8888 *n. miles*
h = 0.0041 *n. miles*

$$cos\left(d + 1.8517°\right) = \frac{3437.7541}{3438.6388}\ cos\ \ 1.8517° \qquad \text{. . . (formula A6.1)}$$

$$d = 2.2621° - 1.8517° \quad = 0.4104° \ = 24.6\ \ n.\ miles$$

Effect of Abnormal Refraction on Accuracy of Calculation. Long-range *Position Line*s obtained in this way are of little value if *Abnormal Refraction* is suspected (see BR 45 Volume 2 Chapter 8). *Abnormal Refraction* is likely to be present when the temperature of the water and that of air differ considerably.

CAUTION

PRACTICAL USE. This method of obtaining a *Position Line* has a limited application and while useful in giving a reasonably satisfactory long-range *Position Line* on a single isolated peak (eg Mount Teide at Tenerife in the Canaries), <u>it should ALWAYS be used with PARTICULAR CAUTION, and should NOT be used with a mountain peak which forms part of a mountain chain unless it has been positively identified</u>.

3. Horizontal Sextant Angles (HSAs)

a. **Rapid Plotting Without Instruments - HSA Lattice.** To enable *Fixes* obtained by *Horizontal Sextant Angle*s (*HSAs*) to be plotted rapidly without instruments, a lattice of *HSA* curves (*HSA Lattice*) may be constructed on a chart. Each curve gives the constant angle between a pair of suitably placed *Fixing* marks, and is an arc of a circle. If a set of curves is plotted for each of two pairs of marks, then, having observed the angle between each pair simultaneously, the observer can plot the resultant *Fix* immediately at the intersection of the two curves corresponding to the two angles. A sufficient number of curves must be drawn to enable the observed angles to be plotted conveniently by interpolation between the *HSA Lattice* lines.

b. **Preparation of an HSA Lattice.** The preparation of an *HSA Lattice* is illustrated at Fig A6-3 (opposite). At Fig A6-3, consider the pattern of arcs which may be generated from one pair of objects *A* and *B*. Three arcs are shown: *AEB, ADB, ACB*. Their centres *O*, *P, Q* respectively, all lie along the perpendicular bisector *FQ* of the base line *AB*. Then, consider one arc *AEB*. Let *QO* (the distance of the centre of the arc from the base line) be *x*, where *d* is the length of the base line and θ is the angle subtended by the chord *AB* on the circumference of the circle through *AEB*.

Thus: $x = ½d \cot \theta$ . . . **A6.2**

Formula (A6.2) may now be used to construct the lattice for all required angles.

c. **Fixing Objects Within the Boundaries of the Chart - Chart D6472.** Chart D6472 (Diagram for Facilitating the Construction of Curves of Equal Subtended Angles) is issued by *UKHO* and enables the observer to plot any lattice of *HSA* curves on any chart or plotting sheet, provided that all the *Fixing* objects lie within the boundaries of the chart. Full instructions as to how to use Chart D6472 are printed on it.

d. **Fixing Objects Outside the Boundaries of the Chart.** If the *Fixing* objects do not lie within the area of the chart, the following procedure will enable the observer to plot a customised *HSA Lattice*.

- Lay out on an appropriate space, such as the floor or deck, the chart or plotting sheet on which the *HSA Lattice* is required. Represent the *HSA* marks with pins placed in their correct relative positions.
- From the largest *Scale* navigational chart which shows the *Fixing* marks, measure, as accurately as possible, the distance between them. Convert these distances to the desired *Scale* of the *HSA Lattice* to obtain the distances between the pins on the floor. The simplest method for this *Scaling* up is to find a multiplication factor (eg if the navigational chart has a natural *Scale* of 1:50,000 and the *HSA Lattice* is to have a *Scale* of 1:10,000, then all chart lengths taken off the former must be multiplied by 5 [ie 50,000 / 10,000]. If two objects, *A* and *B*, are found to be 150 mm apart of the navigational chart, the pins should be placed 750 mm apart (ie 150 x 5) on the *HSA Lattice*).
- Measure the angle between the base lines (α in Fig A6-4 opposite) and lay this off on the floor. Measure the appropriate floor lengths and mark the position of the third object *C*. If the grid co-ordinates of the *Fixing* marks are known, the accuracy of all these measurements should be checked by calculation.

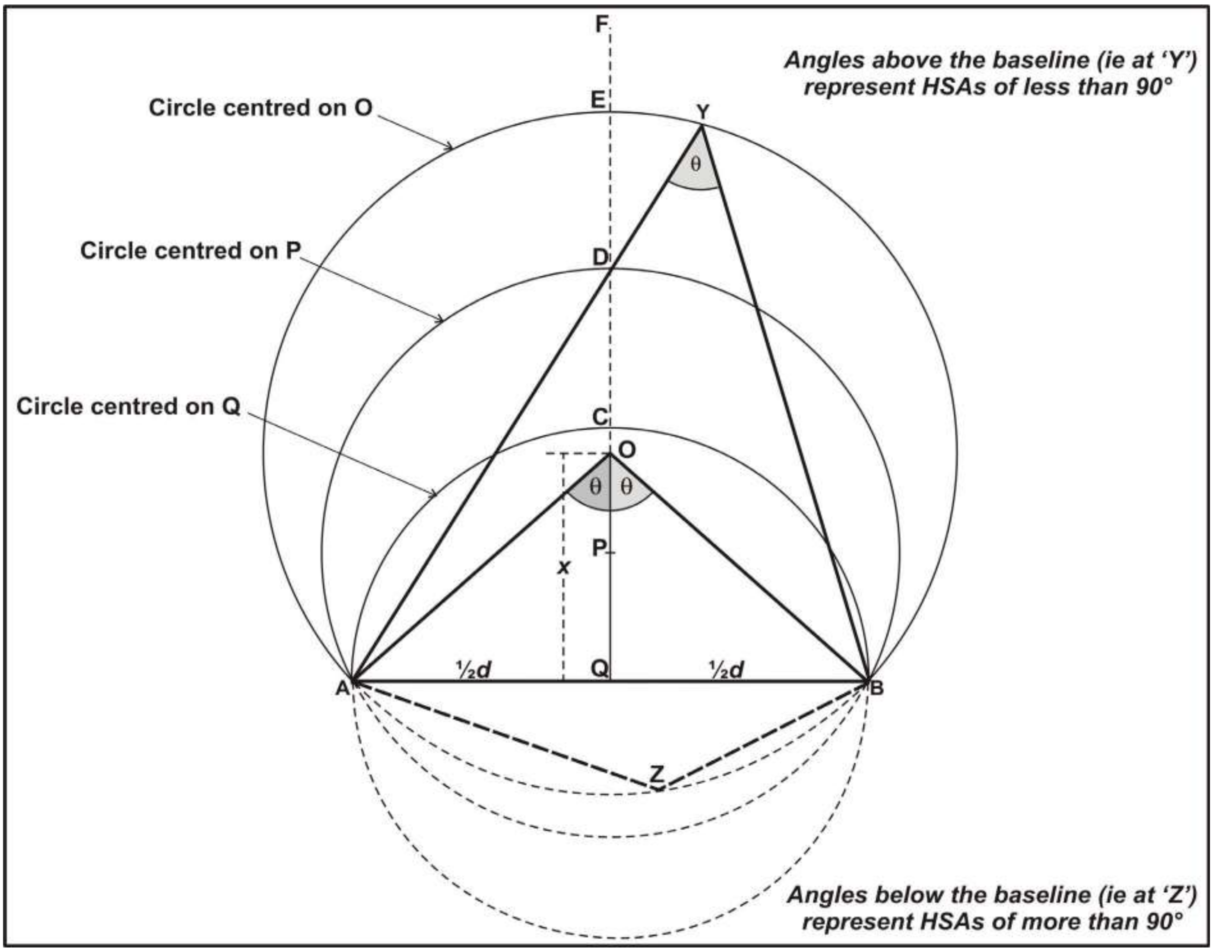


Fig A6-3. Pattern of Arcs Generated From One Pair of Objects

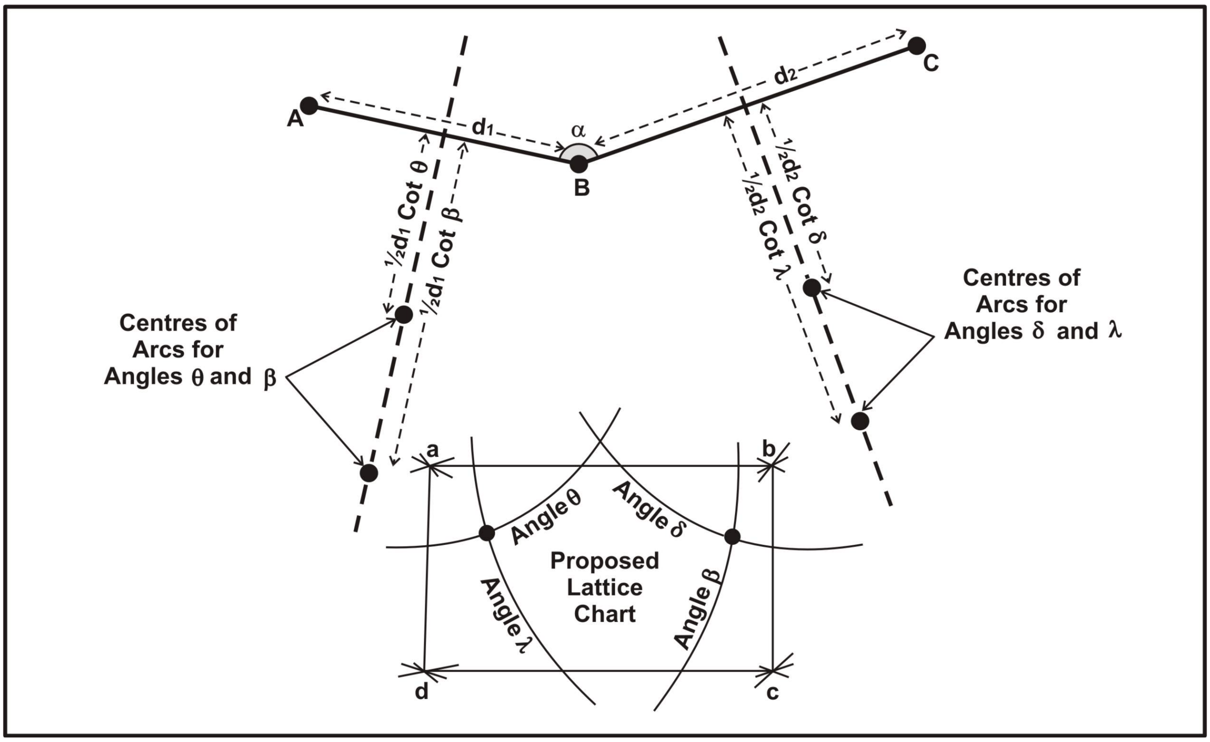


Fig A6-4. Construction of an HSA Lattice

(3d continued)

- Next, the exact position on the floor for the *HSA Lattice* chart must be found. On the largest *Scale* navigational chart which shows both the *HSA Lattice* area and the *Fixing* marks, draw in the limits of the *HSA Lattice* chart. Measure the distances from each of the *Fixing* marks to all four corners of the *HSA Lattice* (*Aa, Ab, Ad, Ac, Ba, Bb,* etc). Scale up these distances by the appropriate multiplication factor, and then, by striking off arcs on the floor, *Fix* the positions of the corners of the *HSA Lattice*. Pin down the outline *HSA Lattice* chart in this position.

- On the floor, draw the base lines and their perpendicular bisectors. Where the floor surface is unsuitable for drawing, tightly stretched thread can be used.

- On the perpendicular bisectors of the base lines mark the centres of the arcs to be drawn ($\frac{1}{2}d \cot \theta$ from the base line). Strike off two arcs from each pair of objects giving an intersection at each end of the *HSA Lattice* area. As a check, compare for accuracy the geographical positions of the intersections thus obtained with *Fixes* plotted by *Station Pointer* using the same angles on a navigational chart which shows the objects and *HSA Lattice* area. This will reveal any inaccuracy in the construction of the *HSA Lattice*.

- Finally, complete the *HSA Lattice*, using red ink for the curves generated from the left-hand angles as viewed from seaward, and green for the right-hand angles. On large-scale *HSA Lattices*, an alteration of firm and pecked lines in each pattern may improve the clarity of the *HSA Lattice*. If the curves do not cut at a satisfactory angle or are too widely spaced in any part of the chart, other objects can be taken and the curves generated from them drawn in the appropriate area, with colours other than red or green being used. The general form of the completed *HSA Lattice* is shown in Fig A6-5 (below).

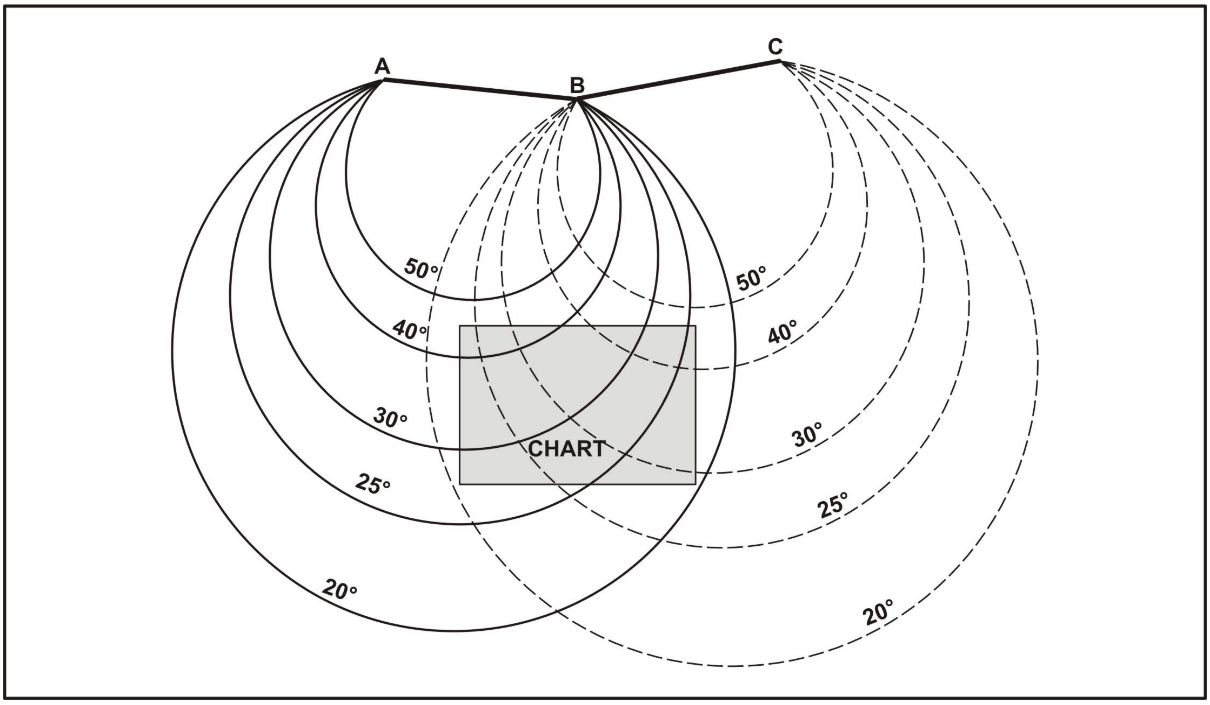


Fig A6-5. Lattice of HSA Curves

APPENDIX 7

DOUBLING THE ANGLE ON THE BOW

1. Scope of Appendix

Appendix 7 contains *'Doubling the Angle on the Bow'* and the effect of *Current* and/or *Tidal Stream* on this technique.

2. Doubling the Angle on the Bow and Effect of Current / Tidal Stream

a. **Doubling the Angle on the Bow - Concept.** If a ship holds a steady course until the bearing of an object on its bow is doubled, the position at which this occurs forms an isosceles triangle with the first position and the object, and its distance from the object is equal to the run between the observations. If the ship experiences a *Current* or *Tidal Stream* in the meantime, an allowance must be made to avoid an error in the final position calculation. In practice, it will usually be more convenient to solve a problem of this type by plotting it on the chart and transferring the position lines as necessary. However, the following theory may be regarded as general.

b. **Doubling the Angle on the Bow in a Current / Tidal Stream.** In Fig A7-1 (below), *AB* is the course of the ship, and *BC* is the *Tidal Stream*. *AB* and *BC* combine to give the *Ground Track*, *AC*.

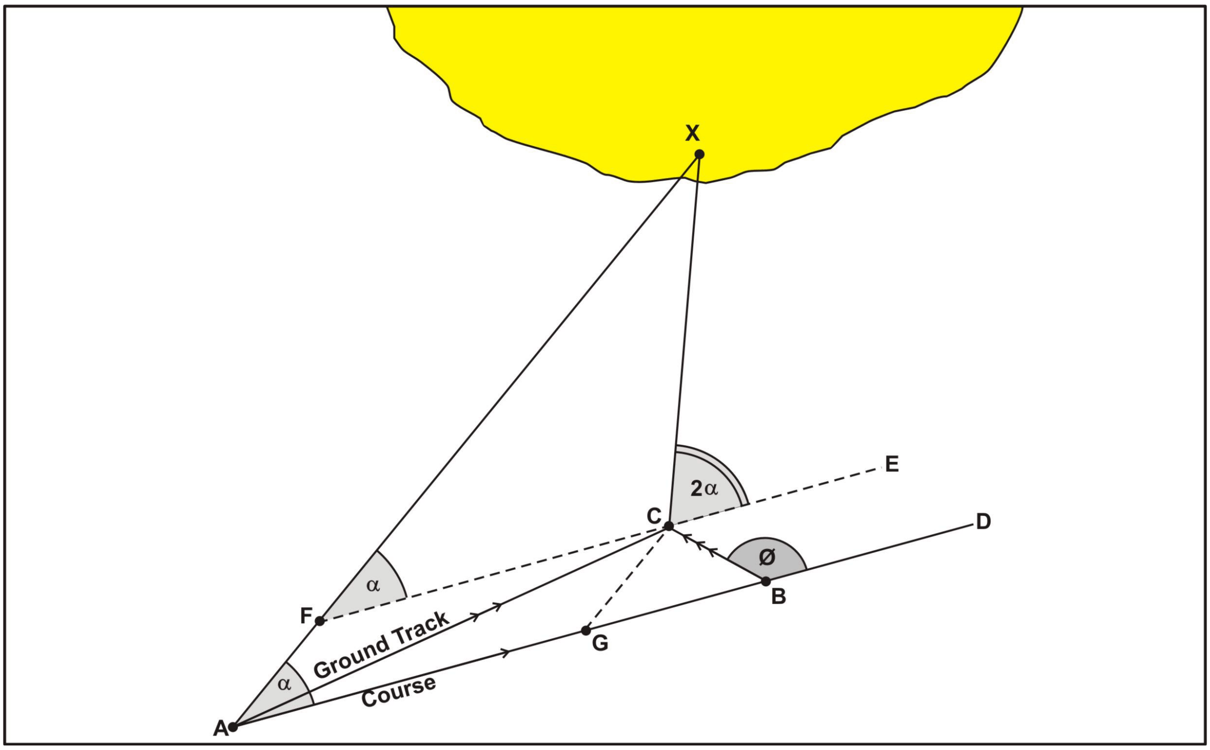


Fig A7-1. Doubling the Angle on the Bow in a Current / Tidal Stream

If *X* is some object observed from the ship, when the ship is at *A*, the angle on the bow is *XAB*, denoted by α. When the ship is at *C*, it is assumed for the purpose of this problem that the angle on the bow has been doubled. At this point the fore-and-aft line is in the direction *CE*, parallel to *AD*, and the angle *XCE* is thus 2α.

(2b continued)

EC produced meets *AX* in *F*. The angle *CFX* is therefore equal to the angle *CXF*, and *FC* is equal to *CX*. *CG* is drawn parallel to *XA*. The angle *CGB* is therefore equal to α, and, since *FAGC* is a parallelogram:

$$CX = AG = AB - GB$$

AB, the distance resulting from the ship's known speed and the duration of the run, can be found at once, but *GB* must be calculated from the triangle *GCB*.

Thus: $\dfrac{GB}{BC} = \dfrac{\sin\ GCB}{\sin\ BGC}$ And: $GB = BC\dfrac{\sin\ GCB}{\sin\ \alpha}$

If *BC* is denoted by *d* (the amount of *Drift* during the run) and the angle *CBD* by ϕ, the angle *GCB* is $(\phi - \alpha)$ and:

$$CX = AB - \frac{d\ \sin(\phi - \alpha)}{\sin\ \alpha} \ldots (\phi > \alpha)$$ **. . . A7.1a (1987 Ed . . . A7.8)**

If ϕ is less than α, *CX* is given by:

$$CX = AB + \frac{d\ \sin(\alpha - \phi)}{\sin\ \alpha}$$ **. . . A7.2a (1987 Ed . . . A7.9)**

These formulae are correct when the *Current* or *Tidal Stream* carries the ship to the <u>same</u> side as the object. However, when the *Current* or *Tidal Stream* carries the ship to the <u>opposite</u> side to the object, it can easily be shown that *CX* is given by:

$$CX = AB + \frac{d\ \sin(\phi + \alpha)}{\sin\ \alpha}$$ **. . . A7.3a (1987 Ed. . . A7.10)**

The distance of the ship from the object at the instant of the second observation can therefore be found in both cases.

Example A7-1. At 1000 an object is seen to bear 040˚ to an observer on board a ship steering 075˚ at 16 knots in a *Tidal Stream* setting 300˚ at 3 knots. At 1030 the same object bears 005˚. How far is the ship from the object at 1030?

At 1000 the angle on the port bow is (075˚−040˚) or 35˚. At 1030 the angle is (075˚−005˚) or 70˚. Also, the angle ϕ is (75˚−300˚ + 360˚) or 135˚. The ship's run in 30 minutes is 8', and d is 1'.5. Both the *Set* of the *Tidal Stream* and the object are to port. The distance of the ship from the object at 1030 is therefore:

$$Distance = 8' - \frac{1'.5\ \sin\ (135^\circ - 35^\circ)}{\sin\ 35^\circ}$$ . . . (formula A7.1a)

$$= 8' - \frac{1'.5\ \sin\ 100^\circ}{\sin\ 35^\circ} = 8' - 2.6' = 5.4'$$ . . . (formula A7.3a)

The position of the ship at 1030 is thus *Fixed* by a bearing and distance of 005˚ and 5'.4, and it is necessary to plot only the true bearing. If the *Set* had been in the opposite direction, 120˚, ϕ would have been equal to (120˚ − 75˚) or 45˚, and the distance would have been:

$$Distance = 8' + \frac{1'.5\ \sin\ (45^\circ + 35^\circ)}{\sin\ 35^\circ} = 8' + 2.6' = 10.4'$$ . . . (formula A7.3a)

(2) c. **Effect of the Tidal Stream when ϕ has Particular Values.** The general formula is simplified considerably when ϕ has certain values. These values and adjustments are:

- **When ϕ is Equal to Zero.** This means that the direction of the *Current* or *Tidal Stream* is the same as the course steered. Then, by substitution:
  $CX = AB + d$

- **When ϕ is Equal to 180˚.** The *Current* or *Tidal Stream* is now in a direction opposite to the course steered, and:
  $CX = AB - d$

- **When ϕ is Equal to α.** This means that the direction of the *Current* or *Tidal Stream* is that of the first true bearing, and:
  $CX = AB$

- **When ϕ is Equal to (180˚ − α).** The *Current* or *Tidal Stream* is now in a direction opposite to the first true bearing, and again:
  $CX = AB$

- **When ϕ is Equal to 2α.** This means that the direction of the *Current* or *Tidal Stream* is that of the second true bearing, and:
  $CX = AB - d$

- **When ϕ is Equal to (180˚ − 2α).** The *Current* or *Tidal Stream* is now in a direction opposite to the second true bearing, and:
  $CX = AB + d$

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(APPENDICES 8 & 9 ARE SPARE AND APPENDIX 10 IS IN PART 2)

INDEX-GLOSSARY

1. Purpose - 'Technical Terms'

The purpose of this 'Index-Glossary' is to provide a quick method of locating the primary definition and/or explanation of each of the 'technical terms' used in BR 45 Volume 1, together with the location of other occurrences of their use.

2. Italicised Terms.

'Technical terms' (as described above) are indicated in the text of BR 45 Volume 1 by being italicised. Exceptionally,'Notes', 'Examples' and 'lettered notations' referring to diagrams or included in formulae are also italicised to enhance clarity, but are NOT included in the Index-Glossary.

3. Primary and Secondary References.

Within this Index-Glossary, the primary definition and/or explanation of an 'italicised technical term' is given in **bold type**, with secondary occurrences given in ordinary type.

4. Explanatory Notes in the Index Glossary

Some words have more than one meaning (eg 'compass Bearing ....' and 'bearing in mind ....', or 'Sphere [Earth's shape]' and 'Sphere [soft-iron, for magnetic compass]'). Where appropriate, to avoid the risk of confusion between similar terms, a brief explanatory note to indicate the context is included in the Index-Glossary, in addition to the paragraph locations.

5. Scope

This Index-Glossary gives the paragraph location of instances of the italicised technical terms used in BR 45 Volume 1. Instances of words used in a sense which is NOT a technical term (eg 'bearing in mind ....' see Para 4 above) are NOT included in the Index-Glossary.

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
2nd and/or 3rd Trace Echoes (Radar)	**1515, 1518, 1530.**
ABC Table Method	**0210.**
Abnormal Refraction	**0803,** 0933, 1603, App 6.
Abnormal Waves	**1125,** 1812.
Absolute Position (Naval Command Systems)	**0111.**
Absolute Position (Navigational Errors)	**1602.**
Acceleration Distance (Manoeuvring Data)	**0714,** 1314
Accuracy (Navigational Errors)	**1602-1603, 1611,** 1620, **Anx 16A.**
Acquisition (Radar / ARPA)	**1703.**
Additional Military Layers	See AML.
Admiralty Digital Publications	See ADP.
Admiralty List of Lights and Fog Signals	See ALLFS.
Admiralty List of Radio Signals	See ALRS.
Admiralty Raster Chart Service	See ARCS.
Admiralty Sailing Directions (Pilots)	See Sailing Directions.
ADPs (Admiralty Digital Publications)	**0640.**
Advance (Manoeuvring Data)	**0714,** 1310-1311, **1314,** 1315.
Advising (Management)	**1910.**
Aerial Rotation (Radar)	**1510.**
Aero Lights (Light characteristics)	**0931.**
Aeromarine Lights (Light characteristics)	**0931.**
AGC (Automatic Gain Control - Radar)	**1514.**

Primary / Secondary References (Primary References in BOLD)

Agulhas Current	**1124-1125.**
AHO (Australian Hydrographic Office)	**0632.**
Airy Spheroid	**0322,** 0331.
AIS (Automatic Identification System)	0331, **0950-0954, 1230,** 1240-1241, 1317, 1319-1320, 1520, 1921, 1923.
Aleutian Current	**1124 -1125.**
ALLFS (Adm. List of Lights and Fog Sigs)	0612, **0640,** 0803, 0930-0931, **0932, 0934,** 0940, 1311.
ALRS (Admiralty List of Radio Signals)	0322, 0323, **0640,** 0910, 0912-0913, 1111, 1240-1241, 1311, 1518.
Alternating (Light characteristics)	**0930.**
Altitude (Sextant)	**0803,** App 4. See also Observed Altitude, True Altitude.
AMLs (ENC - Additional Military Layers)	**0632.**
Amphidromic Points (Tides)	**1052.**
Amplitude (Tidal Wave Height)	**1021.**
Amplitudes (Tidal Harmonic Constituents)	**1030-1031.**
Anchor Bearings	**1415.**
Anchoring	1401, **1410-1419,** 1501.
Angle of Dip	See Dip.
Angle on the Bow	**1527, 1703, 1722.**
Annual Notices to Mariners (UKHO)	**1221.**
Annual Summary of Notices to Mariners	1111, **1211.**
Antipodal / Antipodal Point (Tides)	**1012-1013,1015.**
Antipode (Tides)	See Antipodal / Antipodal Point.
Aphelion (Tides)	**1016.**
Apogean Tide (Tides)	**1015.**
Apogee (Tides)	**1015.**
Apparent Angle (Hydrographic Survey)	See Cocked-up Angle.
ARCS (Admiralty Raster Chart Service)	**0614,** 0930.
Arcs (S African Regional Datum)	**0322.**
Area (Projection property)	0410, **0411,** 0413-0414, 0451.
Areas to be Avoided (Traffic routing system)	1111, **1221.**
ARPA (Automatic Radar Plotting Aids)	0950, 1317, 1324, **1525, 1526,** 1701, 1702, 1921, 1923.
Arrival Gates	1112, **1214, 1312, 1330.**
Arrival Point	**1112.**
Aspect	See Angle on the Bow.
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Atmospheric Refraction (Radar)	**1515.**
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Attenuation (Radar)	**1510, 1516.**
Australian Hydrographic Office	See AHO.
Automatic Gain Control (Radar)	See AGC.
Automatic Identification System	See AIS and W-AIS.
Automatic Radar Plotting Aids	See ARPA.
Auxiliary Light (Light characteristics)	See Subsidiary (Auxiliary) Light.
Axis (Earth)	**0111,** App 2.
Azimuth	0324, 0541, **1326,** App 6.
Azimuth Circle	**0802.**
Ballistic Deflection (Gyro)	**0920.**
Ballistic Tilt (Gyro)	**0920.**
Bands (Light Structure Descriptions)	**0930.**
Bands of Latitude (UTM)	**0451.**
Bandwidth (Radar)	**1513,** 1523, **1529.**
Bank Effect (Canal Effect / Interaction)	**1220, 1332.**
Barycentre (Tides)	**1011-1012, 1016.**
Base Extension (Hydrographic Survey)	**1820, 1825.**
Base Extension Triangulation (Hyd. Survey)	**1820.**
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Beam Mark / Marks (Anchoring)	**1410, 1413.**
Beam Width (Radar)	**1510, 1512,** 1519, 1522, 1523.
Bearing (Projection property)	**0411,** 0414.
Bearing Error (Navigational Errors)	**1603-1623.**
Bearing Lattices	**0808-0809,** 1313.
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Benguela Current	**1124 -1125.**
Bergy Bits (Ice)	**1531.**
Bernoulli Phenomenon (Interaction)	**1220.**
Bias (Navigational Errors)	**1611,** 1612, Anx 16A.
Blind Pilotage	0902, 0923, 0612, 0715, 0720, 1111, 1233-1234, 1301, **1310,** 1311-1315, **1316,** 1317-1320, **1321,** 1323-1325, 1327-1328,1401, 1413, 1415, 1501, 1520, 1521, 1911-1912, 1921-1925, 1931-1933.
Block Coefficient (Cb)	**1220.**
Blockage Factor (Interaction)	**1220.**
Blunders (Navigational Errors)	1603, **1610,** 1621.
Bore (Tides)	**1021.**
Boundary Layer (Water flow)	**0925.**
Bow Dome (Warship sonar)	1332, 1412, **1414, 1418-1422, 1423, 1425.**
Bracketed Corrections (Obsolete from 1986)	**0624.**
Brazil Counter-Current	**1125.**
Brazil Current	**1124 -1125.**
Breast (Berthing)	**1422, 1425.**
Bridge Swinging Circle	**1415,** 1418.
British National Grid	0331, 0431, **0452.**
British Standard Nautical Mile	**0113.** This term is discontinued - see International Nautical Mile.
BSB (NGA RNC format)	**0632.**
Bubble Times	**0715, 1214,** 1234, **1312, 1330, 1415.**
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Cable (Distance)	**0113.**
CADET (Compass to True Add East)	**0124,** 0125, 0811.
California Current	**1124 -1125.**
Canal Effect	**1220,** 1234, 1322, **1332.**
Canal Speed (Canal Effect / Interaction)	**1220.**
Canary Current	**1125.**
Cardinal Marks (Buoyage System)	**0941.**
Carrier Sense TDMA (AIS)	See CSTDMA.
Cartesian Coordinates	**0206, 0324,** 0333, 0412, 0414, 0450.
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CATZOC (Categories: Zones of Confidence)	**0625-0626,** 0805.
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CDMVT (Cadbury's Dairy Milk Very Tasty)	**0124,** 0811.
Celestial Pole	**0920.**
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Central Meridian (Transverse Mercator)	**0414,** 0421, **0431,** 0450, **0451, 0452,** App 4.
Centre of Curvature (Earth)	**App 6.**
Centre of Gravity (Ship)	**1220.**
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CEP (Circular Error Probable - Nav. Errors)	**1602, 1615-1616, Anx 16B.**
Character / Characteristic (Lights)	**0930,** 0933, 0942.
Charge (Navigational Charge of a vessel)	**1312,** 1323, 1326, **1910, 1912, 1923-1924, 1931-1932.**
Chart Correction Log	**1311.**
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Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
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Circular Error Probable (Navigational Errors)	See CEP.
Circular Normal Distribution (Nav. Errors)	**Anx 16B.**
Clarke Spheroid (1880)	0424, **0531.**
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Closest Point of Approach	See CPA.
Clutter (Radar)	**1510, 1513-1514,** 1515, **1523,** 1526.
Coastal Navigation	0714, 0716, **0721,** 0902, 0923, 1101, 1110-1111, 1201, **1210-1214, 1222, 1230-1238,** 1301, 1311, 1316, 1320, 1327, **1527, 1911, 1920, 1930.**
Coastal Warnings (Radio)	**0615.**
Coastlining Board (Hydrographic Survey)	**1829.**
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COG (Course Over G'd / Ground Track AIS)	**0951**
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Commercial Service (Galileo)	See CS.
Compass North	**0122, 0123.**
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Compass Rose Correction (Rose Correction)	**0124.**
Composite Errors (Navigational Errors)	**1610.**
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Composite Group Occulting (Lights	**0930.**
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Conduct (Navigational Conduct of Ship)	**1238, 1910, 1912.**
Cone (Projections)	**App 4.**
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Conical Projections	**0413,** 0414, App 4.
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Conning (Systems - IBS)	**1236.**
Constant of the Cone (Mercator Projection)	**0421,** App 4.
Contiguous Zones	**1111.**
Continuous Uniform Distribution (Nav Error)	See Rectangular Distribution.
CONTRARY Name (Hemispheres)	**0208, 0211.**
Control (Hydrographic Survey)	**1820.**
Control / Controlling (Manoeuvre of ship)	**1910, 1912, 1924, 1932.**
Control Points (Hydrographic Survey)	**1820.**
Controllable Pitch Propellers	See CPP.
Conventional Direction of Buoyage	**0941.**
Convergence (Meridians)	**0421.**
Coral	0803, **1222, 1237,** 1328.
Co-Range (Charts)	1050, **1052.**
Coriolis Force	**1122.**

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Corpos Mortos (Stern-To Berthing)	**1420.**
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Corrected Mean Latitude Sailing	0201, **0205,** 0207, 0501.
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Course (Naval Command Systems)	**0120.**
Course (Rhumb Line - Spherical)	**0511,** 0520, **0522.**
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Course Over Ground (Ground Track - AIS)	See COG.
Courses to Steer	1310, **1312, 1314,** 1316, 1319, 1323, 1413, 1415.
Course-Up (Radar / ARPA)	**1526.**
CPA (Closest Point of Approach)	0713, 1316, 1526, **1527,** 1703, 1733.
CPP(Controllable Pitch Propellers)	1332, **1334.**
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Cross Track Error (Hydrographic Survey)	**1827.**
Cross Track Error (Nav Errors - Bias)	**1611.**
CS (Commercial Service - Galileo)	**0914.**
CSTDMA (Carrier Sense TDMA - AIS)	**0951.**
Current / Currents	**0712,** 0713, 0715-0716, 0804, **0805,** 0806, 0916, 0925, **1002, 1040-1042,** 1110-1112, 1120, **1121-1125,** 1210, 1212, 1214, 1222, 1231, 1234, 1236, 1238, 1312, 1314, 1316, 1320, 1330-1331, 1333, 1411-1412, 1527, 1603, 1621, App 7.
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Cylindrical Projections	**0413, 0420,** App 4.
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d.long	**0112,** 0203-0204, **0205-0206**, 0207, **0208,** 0511, 0522, 0541, Apps 3-4.
d[rms] (Navigational Errors)	**1602 1612.**
1d[rms] (Navigational Errors)	**1615, Anx 16B.**
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Datum / Datums (Geodetic)	0101, 0110, **0311, 0320-0321,** 0322, **0323,** 0324, 0331, 0452, 0550, 0551, 0711, 0807, 0910, 0913, 1222, 1321, 1330, 1602, **1820, 1829,** 1921-1922.
Datum Shift	**0323, 0324,** 0624, **0805,** 0910.
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Daymarks	**0940**
Daytime Lights (Light characteristics)	**0931**
DChayka (Russian *LORAN* -C equivalent)	**0912**
Dead Range (Blind Pilotage)	**1316**
Dead Reckoning	See DR.
Declination (Projections)	**App 4.**
Declination (Tides)	**1014-1018, 1020,** 1062.
Deep Water Routes (Traffic routing systems)	**1221.**
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Departure (Sailings)	**0202-0206,** 0421, **0511,** App 3.
Detection (Radar / ARPA)	**1703.**
Deviation (Magnetic)	**0122-0125,** 0807, 0811, **1113, 1230,** 1610, App 10
DG (Degausing)	0922, **1230**
DG (Directional Gyro)	**0920**
DGLONASS (Satellite navigation)	**0912.**

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
DGPS / GPS (Satellite navigation)	0323, 0331, 0423, 0451, 0714, 0716, 0803, **0805,** 0808, **0910 0911-0912, 0916,** 0918, 1110, 1111, 1210, **1211**, 1212, 1222, 1231, 1234-1236, 1313, 1316, 1321, **1521,** 1527, 1803, **1806-1807, 1810,** 1811, 1813-1814, 1820-1821, 1826, **1827, 1829,** 1922-1933.
Diagonal Stripes (Structure Descriptions)	**0930.**
Diamond of Error (Navigational Errors)	**1614,** 1620, **Anx 16B.**
Diaphone (Fog signal)	**0934.**
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Difference of Meriodinal Parts (DMP)	See DMP.
Differential Chayka	See DChayka.
Differential GLONASS	See DGLONASS
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Differentiation (Radar)	**1513, 1514,** 1520.
Digital Navigation	0901- 0902, **0919,** 1903, 1910, **1911, 1923-1924.**
Digital Selective Calling (VHF Comms)	See DSC.
Dilution of Precision (GPS)	See DOP.
Dip (Magnetic)	**1610.**
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DIPCLEAR (Diplomatic Clearance)	**1110-1111, 1210 1211, 1802.**
Diplomatic Clearance	See DIPLCLEAR.
Dipping Range (Lights)	**0933.**
Direction (Naval Command Systems)	**0120.**
Direction Light (Light characteristics)	**0930.**
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Distance (Geodesic - Spheroidal)	**0541.**
Distance (Great Circle)	**App 2.**
Distance (Rhumb Line - Spherical)	**0511, 0522, 0541.**
Distance (Rhumb Line - Spheroidal)	**0530, 0531.**
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Distance Error (Navigational Errors)	**1603, 1623.**
Distance Meters	**0803.**
Distance to New Course (Manoeuvring)	See DNC.
Diurnal (Tidal Streams)	**1040.**
Diurnal (Tides)	**1015, 1017, 1018, 1020,** 1062.
Diurnal Inequality (Tidal Streams)	**1040 1042, 1045,** 1816.
Diurnal Inequality (Tides)	**1015, 1017, 1020, 1030,** 1062.
Dived Navigation	**1911.**
DMP (Difference of Meriodinal Parts)	0207, **0422,** 0424, 0510, **0511,** App 3.
DNC (Distance to New Course - Manoeuvre)	**0714.**
DNC (Manoeuvring Data)	**1314.**
DNC (NGA [non-ENC] Vector chart format)	**0632,** 0902.
Doldrums	**1122,** 1125.
DOP (GPS - Dilution Of Precision)	0910, **1806, 1829.**
Double Normal MSR (Anchoring)	**1412.**
Double Reduced MSR (Anchoring)	**1412.**
Doubling the Angle on the Bow	**0806,** App 7.
Douglas Protractor	**0808, 0811.**
DR (Dead Reckoning)	**0710, 0712,** 0713-0716, 0805, 0806-0807, 0810, 0812, 0916, 0925, 0942, **1231,** 1328, 1603, 1610, 1623, 1921.
DR Stations (Running Survey)	**1814.**
Drift / Drift Rate	**0713, 0713,** 0720, 0806, 1322, 1527, 1603.
Drift Current	**1041, 1121, 1125.**
Dropping Anchorage	**1414,** 1417, **1421.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
DSC (Digital Selective Calling-VHF Comms)	0950, **1240.**
Duration of Turn (Manoeuvring Data)	**0714**, 1314.
Dutchman's (Speed) Log	**0925.**
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Earth Curvature Graph (Nomograph)	**1524.**
East Greenland Current	**1124.**
East Australian Coast Current	**1124-1125.**
Ebb (Tidal Streams)	**1040.**
EBL (Electronic Bearing Line)	**1317,** 1720, 1933.
Eccentricity (Spheroid)	**0311, 0322,** 0422, Apps 4-5
ECDIS (Electronic Chart Display Information System)	See also WECDIS. 0210, **0311**, 0451, 0550, **0551, 0631, 0632,** 0701, 0712, 0714, 0715- 0716, **0720-0721,** 0805, 0808, 0811, 0910, 0919, 0950, 1051, 1102, 1110, 1113, 1202, 1210, 1214, 1222, 1230, **1231,** 1232, 1236, **1238,** 1302, **1311,** 1312-1316, **1317,** 1319-1320, **1321,** 1323-1325, 1327-1328, 1402, 1413, 1415, 1502, **1528,** 1701, 1702, **1810, 1814-1815,** 1903, **1911,** 1920, **1921-1922, 1925.**
Echo Sounders	0613, 0626, 0803, 0805, **0923-0924,** 0942, **1211,** 1222, 1235, 1310, **1312, 1315,** 1319-1320, **1323,** 1324-1325, 1328, 1416, **1807, 1810,** 1813-1814**, 1821, 1827, 1831, 1923-1924, 1931-1933.**
Eclipse (Light characteristics)	**0930.**
Ecliptic Plane (Tides)	**1017.**
ECS (Electronic Chart System)	**0311, 0631,** 0701, 0720, 0910.
ED 50 (European Datum 1950)	**0322.**
Eddy / Eddies (Tides)	**1044.**
EGNOS (European Geostationary Navigation Overlay Service)	**0915, 1806.**
El Niño (variable) Current	**1124-1125.**
Electro Optic Surveillance Systems	See EOSS.
Electronic Bearing Line	See EBL.
Electronic Chart Display Information System	See ECDIS and WECDIS.
Electronic Chart System	See ECS.
Electronic Charts (Generic term)	See ENCs, RNCs and DNCs.
Electronic Navigation Charts (Database)	See ENC..
Electronic Plotting Aid	See EPA.
Electronic Tracking Aid	See ETA.
Elevation	**0624, 0803, 0930,** 0931, 0932-0933, 0940, 1233, **1820,** 1823, 1830.
Ellipsoid (Earth - Oblate Spheroid)	**0311.**
eLORAN	0716, **0805,** 0912, **0918,** 1111, 1231.
Emergency Position Indicating Radio Beacon	See EPIRB.
Emergency Wreck Marking Buoys (Buoyage)	**0941.**
e-Nav / e-Navigation	0901, **0919.**
ENC (Electronic Navigation Charts)	0331, 0614, 0615-0616, 0625, 0631, **0632-0633**, 0712, 0720, 0902, 0919, 0930, 0940, **1042,** 1110, 1210, 1214, 1317, 1321, 1911.
Enhanced LORAN	See eLORAN.
EOSS (Electro Optic Surveillance Systems)	**0926,** 1312, 1316, 1320, 1323, 1325.
EP (Estimated Position)	**0710, 0712,** 0713, 0715-0716, 0805-0807, 0810, 0812, 0916, 0942, **1231,** 1234, 1312- 1314, 1323-1325, 1328, 1603, **1610,** 1623,1814, 1921, 1931, 1932.
EPA (Electronic Plotting Aid)	**1525.**
EPE (GPS - Estimated Position Error)	0910, **1806, 1829.**
EPIRB (Emergency Position Indicating Radio Beacon)	**1518.**
Equal Area (Projection property)	**0414,** 0622, App 4.
Equation of the Ellipse	**App 5.**
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Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Equatorial (Radius / Axis)	**0110,** 0115, 0310, **0311**, 0541, App 5.
Equatorial Counter-Current	**1122, 1125.**
Equatorial Currents (North and South)	**1124-1125.**
Equatorial Element (Spheroid)	**App 5.**
Equatorial Trough	**1122, 1125.**
Equinoctial Tides	**1017.**
Equinox / Equinoxes (Tides)	**1017, 1020.**
Equipotential Surface (Geoid)	**0311.**
Equivalent Probability Circle (Nav Errors)	**1615,** 1620, **1623, Anx 16B.**
50% Equivalent Probability Circle	**Anx 16B.**
70% Equivalent Probability Circle	**1610.**
95% Equivalent Probability Circle	**1615-1616,** 1620, **1622-1623, Anx 16B.**
Error Circle (Navigational Errors)	**Anx 16B.**
Error Ellipse (Navigational Errors)	**1614-1616,** 1620, **Anx 16B.**
95% Error Ellipse	1614, 1620, **Anx 16B.**
Estimated Position	See EP.
Estimated Position Error (GPS)	See EPE.
ETA (Electronic Tracking Aid)	**1525.**
ETRS 89 (European Regional Datum)	**0321-0322.**
Eurofix	0911, **0912.**
European Datum (1950)	See ED 50.
European Geostationary Navigation Overlay Service	See EGNOS.
European Regional Geodetic Datum	See ETRS 89.
Exclusive Economic Zones	**1111.**
Execution / Executing (Navigation)	**1910-1912.**
Falkland Current	**1124-1125.**
False Easting (Transverse Mercator Grids)	**0451-0452,** App 4.
False Northing (Transverse Mercator Grids)	**0451-0452,** App 4.
False Origin (Transverse Mercator Grids	0431, **0450, 0452.**
FATDMA (,ed Access TDMA - AIS)	**0951.**
Faults (Navigational Errors)	1603, **1610,** 1620, **Anx 16B.**
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Fibre Optic Gyro	See 'FOG'.
Final Diameter (Manoeuvring Data)	**0714.**
First Quarter (Moon)	**1017-1018, 1020.**
Fisheries Zones	**1111.**
Fix / Fixed / Fixing (Position)	0120, **0710-0711,** 0712-0714, 0716, 0721, 0801, **0803, 0805-0812,** 0910-0911, 0916, 0918, 0924, 1113, 1211-1212, 1222, **1231-1232,** 1234, **1238,** 1310, 1312-1314, 1316, 1323, 1325, 1328, 1413, 1415, 1418, **1521,** 1527, 1602-1603, 1610-1612, 1620, 1623, Anx 16B, 1804, **1806, 1811,**1813-1814, 1820-1821, 1823-1824, **1827, 1829,** 1830, 1921, 1923-1924, 1931-1932, Apps 6-7, App 10.
Fixed (Light characteristics)	**0930,** 0942.
Fixed Access TDMA (AIS)	See FATDMA.
Fixed Pitch Propellers	**1334.**
Flash / Flashing (Light characteristics)	**0930.**
Flat Earth	**0330, 0333,** 0411, 0550.
Flattening (Spheroid)	**0311, 0322,** 0422, 0541, App 5
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Flood (Tidal Streams)	**1040.**
FLOOs (Fleet Operating Orders - BRd 9424)	**1111,** 1113.
Florida Current	**1125.**
Fluxgate (Magnetic) Compasses	**0922.**
FOG (Fibre Optic Gyro)	0121, **0920.**
Fog (Signals)	**0934, 1235,** 1310.
Fog (Visibility - Restricted)	0933, 1110 -1111, 1124, 1210, **1211,** 1212, 1232, **1235,** 1311, 1531.
Fog Detector Lights (Light characteristics)	**0931.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
Fog Lights (Light characteristics)	**0931.**
Foreign Charts	**0611,** 0624, **1110-1111, 1310-1311.**
Four-Part Formula (Trigonometry)	**App 2.**
Four-Point Bearing (Distance abeam)	**0806.**
FPP (Fixed Pitch Propellers)	**1334.**
Full Moon (Tides)	**1017, 1018, 1020.**
Gain (DGPS / GPS)	**1806.**
Gain (Radar)	**1514,** 1520.
Galileo (Satellite navigation)	**0914-0916.**
Gauss Conformal Projection	See Transverse Mercator Projection.
General Direction of Buoyage	**0941.**
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Geocentric Latitude	**0312,** 0331, App 5.
Geodesic (Great Circle equivalent)	**0110,** 0115, 0208, 0210, 0313, **0315, 0540,** 0551.
Geodesic Azimuth	**0541.**
Geodesic Course	See Course (Geodesic - Spheroidal).
Geodesic Distance	See Distance (Geodesic - Spheroidal).
Geodesy	**0301.**
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Geodetic Latitude	**0111, 0312-0314, 0321,** 0331, 0540, **0541,** App 5.
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Geographic Latitude	**0111,** App 5.
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Geographic Position	**0803, 1814, 1820,** 1826, App 4.
Geographical Range (Light characteristics)	0803, **0930, 0932** 0933.
Geoid (Earth - Oblate Spheroid)	**0311, 0321.**
Global Maritime Distress and Safety System	See GMDSS.
Global Navigation Satellite Systems	See GNSS.
Global Positioning System	See GPS.
GLONASS (Satellite navigation)	0913, **0915, 0916.**
GMDSS (Global Maritime Distress and Safety System)	**0917, 1230,** 1240.
Gnomonic / Gnomonic Projection (charts)	**0413-0414, 0440-0442,** 0521, 0551, 0611, **0622,** 0624, App 4.
Gnomonic Graticule / Projection Graticule	**0413, 0414 0440,** 0622, App 4.
GNSS (Global Navigation Satellite System)	0321, 0323, **0915,** 0954.
GNSS-1	See GNSS.
GNSS-2	See GNSS.
Gong (Fog signal)	**0934, 0940.**
GPS (Satellite navigation)	GPS-specific instances: **0910,** 0913-0914, **0915-0917,** 0954, 1610, **1806, 1829.** Otherwise, see DGPS, GPS 'Assisted' Accidents, GPS Denial, GPS Lattice.
GPS 'Assisted' Accidents	**0805,** 1231.
GPS Denial	**0805,** 0808, **0916, 1211,** 1222, 1231, 1234, 1313, 1921-1922.
GPS Lattice	**0805,** 1313.
Gradient Current	1041, **1121.**
Graticule (Projections)	**0412,** 0414, 0420, **0421,** 0423, **0424,** 043,0 0450, Apps 4-5.
Graticule Shape (Projections)	See Graticule (Projections).
Gravitational Force (Tides)	**1012-1013.**
Great Circle	**0110,** 0111, 0113, 0115, 0120, 0122, 0201, **0208-0211,** 0315, 0414, **0425,** 0440, **0441- 0442,** 0520, 0521, 0522, 0540, 0551, 0622, 0920, 1012-1013, 1110 -1111, App 2, Apps 4-5.
Greenwich Meridian	See Prime Meridian.
Grid / Grids (see also UTM Grid)	0320, 0331, 0401, **0412,** 0414, 0421, 0430-0431, **0450-0451,** 0621, 0805.
Grid (Coordinates)	**App 4.**
Grid Convergence	**0450, 0452.**

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Grid Eastings	0450, **0451,** App 4.
Grid Length (Transverse Mercator Grids)	**0450**.
Grid Navigation (Polar regions)	**0920.**
Grid Northings	**0421,** 0450, **0451,** 0454, App 4.
Grid Orientation	See Grid Convergence.
Grid Origin	**0333,** 0431, **0450, 0452,** App 4.
Grivation	**0454.**
Gross Tonnage	**1525.**
Ground Speed (Speed made good)	**0713, 0716,** 1214, 1231, **1703.**
Ground Stabilised (Radar / ARPA)	1316, **1526-1527, 1703,** 1722, **1933.**
Ground Track (Course made good)	0710, **0713,** 0925, 1236, 1314, 1322, 1415, 1527, 1610, **1611, 1703,** App 7.
Group Anchorages	**1401.**
Group Flashing (Light characteristics)	**0930.**
Group Occulting (Light characteristics)	**0930.**
Group Quick Flashing (Light characteristics)	**0930.**
Group Very Quick Flashing (Light characteristics)	**0930.**
Growlers (Ice)	**1531.**
Guard Zones (ARPA)	**1526.**
Guinea Current	**1125.**
Gulf Stream	**1124-1125.**
Gyro / Gyro Compass	0120, **0121,** 0122, 0125, 0712, 0811, **0920,** 0921, 1113, **1230,** 1522, 1526-1527, 1603, 1610-1611, 1622, Anx 16A, 1722.
Gyroscope	**0121, 0920,** 1811, 1814, 1816, 1820.
Gyroscopic Inertia	**0920.**
Half Beam Width Error (Radar)	1232, **1512, 1522-1523.**
Half Log Haversine Method (Trigonometry)	**0210, App 2.**
Handbrake Turn (Manoeuvring)	**1315.**
Harmonic Constants (Tides)	**1030-1032, 1042, 1050,** 1053.
HAT (Highest Astronomical Tide)	**0624, 1062,** 1820.
Haversine Formula / Method (Trigonometry)	**0210, App 2.**
HCRF (UKHO / AHO RNC format)	**0632.**
Head Rope (Berthing)	1333, **1425.**
Heading (General)	**0120,** 0122, 0124-0126, 0920.
Heading (Naval Command Systems)	**0120.**
Headmark / Headmarks	0942, 1310, 1312-1314, 1319, 1321, **1322,** 1324, 1410, 1413, 1923, 1931.
Head-Up (Radar / ARPA)	**1526.**
Heave-To / Heaving-To	**1211.**
Heavy Weather	**1211.**
Heights (Hydrographic Survey)	**1820.**
Heights (Light Structure Descriptions)	**0930.**
Highest Astronomical Tide	See HAT.
Homogeneous Geodetic Datums	**0324.** See also Non-Homogeneous Geodetic Datums.
Horizontal Control (Hydrographic Survey)	**1820.**
Horizontal Danger Angle	**0803, 1233.**
Horizontal Sextant Angle	See HSA.
Horn (Fog signal)	**0934, 0940.**
HOT (Height of Tide)	**0715,** 0803, 0805, 0932-0933, **1021-1022, 1030,** 1050-1051, 1060, **1210, 1213,** 1233-1234, 1310-1312, 1316, 1319, 1320, 1323, 1330, 1410, 1411, **1521, 1820, 1827.**
Hour Angle (Projections)	**App 4.**
HSA (Horizontal Sextant Angle)	**0803, 0805, 0808, 0810,** 0811, **1233, 1521,** 1811, **1827, 1829,** 1830, App 6, App 10.
HSA Lattice	**0808, 0810,** 1313, App 6, App 10.
Humbolt Current	See Peru Current.
Hurricanes	**1812.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
Hydrographic Chart Raster Format (HCRF)	See HCRF.
Hydrographic Note / Notes	**0616, 1804-1805, 1810, 1812,** 1832.
Hydrolapse	**1515.**
IALA (Fog signals - Policy / Use)	**1211,** 1235.
IALA Maritime Buoyage System	**0941.**
IBS (Integrated Bridge Systems)	**1236.**
Icebergs	**1531.**
IHO (International Hydrographic Organisation)	0614, **0630,** 0632.
IMO (International Maritime Organisation)	0331, **0630-0631,** 0714, 0901, 0919, **1211,** 1221, 1240, 1304, 1321, 1525, 1902.
Impede / Impeded (ColRegs - TSS)	**1221.**
Impeller Speed Logs	**0925.**
Impounded Water (Hydrographic Survey)	**1828.**
Index Error (Radar)	1232, 1310, 1316, 1520, **1521.**
Index Error (Sextant)	0803, 0810, App 6, **App 10.**
Inertial Navigation Systems	See INS.
Inland AIS	**0950,** 0951.
Innocent Passage	**1110-1111, 1210-1211.**
INS (Inertial Navigation Systems)	**0921, 1230,** 1611, 1921.
Inshore Traffic Zone	See ITZ.
Install / Installed (ENCs / RNCs)	**1110, 1210, 1214, 1311, 1320.**
Integrated Bridge Systems	See IBS.
Intensity (Light characteristics)	**0930,** 0932-0933.
Interaction	**1220,** 1234, 1310- 1312, 1319.
Intermediate Course (Manoeuvring Data)	**0714,** 1314.
Intermediate Distance (Manoeuvring Data)	**0714,** 1314.
International Date Line	0451, **1214.**
International Hydrographic Organisation	See IHO.
International Maritime Organisation	See IMO.
International Nautical Mile / Miles	Specific instances of importance (general usage not listed): 0110, **0113,** 0115, 0310, 0422, **0541, 0803,** App 5.
International Regulations for Prevention of Collisions	See ColRegs
International Spheroid (1924)	**0322,** 0331, 1111.
Interrupted Quick Flashing (Lights)	**0930.**
Intersection (Fixing)	**1811, 1824,** 1830.
Intertidal (Tides)	**1521.**
Inverse (Oblique) Mercator Projection	0411, **0414.**
Irish Grid (Transverse Mercator Projection)	**0453.**
Isogonal / Isogonic	**0122.**
Isolated Danger Marks (Buoyage System)	**0941.**
Isophase (Light characteristics)	**0930.**
ITZ (Inshore Traffic Zone - TSS)	**1221.**
Jack / Jackstaff	**1230.**
Japan (Kuro Shio) Current	**1124-1125.**
Kamchata (Oya Shio) Current	**1124 -1125.**
kn	See Knot.
Knot (kn)	**0113.**
Kuro Shio Current	See Japan Current.
La Niña (variable) Current	**1124 -1125.**
Labrador Current	**1124 -1125.**
Lag / Lagging (Tides)	**1017.**
Lamberts Conical Orthomorphic Projection	0411, **0414,** 0421, App 4.
Land Mile	See Statute Mile.
Land Survey Datums (Tides)	**1060.**
Large Corrections (Obsolete from 1972)	**0624.**
Laser Rangefinder	**0803.**
Last Quarter (Moon)	**1017-1018, 1020.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
LAT (Lowest Astronomical Tide)	**0624, 1060-1062.**
Lateral Marks (Buoyage System)	**0941.**
Latitude	Specific instances of importance (general usage not listed): **0111, 0314, 0321, 0711, 1820.** See also Bands of Latitude (UTM), Co-Latitude, Corrected Mean Latitude, Corrected Mean Latitude Sailing, Difference of Latitude, Geocentric Latitude, Geodetic Latitude, Geographic Latitude, Latitude / Longitude Grid, Latitude Rider (Gyro), Latitude Scale, Linear Latitude, Magnetic Latitude, Mean Latitude, Mean Latitude Sailing, Parallels of Latitude, Parametric Latitude, Safe Parallel of Latitude, True Latitude, Vertex Latitude (Great Circle).
Latitude / Longitude Grid	**0805.**
Latitude Rider (Gyro)	**0920.**
Latitude Scale	**0414, 0421, 0422, 0424, 0623,** 0711.
LDL (Limiting Danger Line)	**0715,** 0720, **0721,** 0923, 1110, **1111, 1210, 1214,** 1231, 1310, **1311,** 1312, **1315,** 1316, 1328, 1410, **1413,** 1418, **1911.**
Leading Lights (Light characteristics)	**0930.**
Leading Lines	1313, **1322.**
Leading Marks	1313, **1322.**
Least Squares (Navigational Errors)	**App 10.**
Least Squares Minimum Variance (N. Error)	**Anx 16B.**
Leeway	**0712,** 0713, 0716, 0806, 0925, 1231, 1234, 1236, 1310, **1312,** 1314, 1323, 1417, 1527, 1603, Anx 16A, 1722, 1814.
Leeway Angle	**0712,** 0713, 1722.
Leeway Vector	**0712,** 0713,1322.
LEP (Linear Error Probable)	**Anx 16A.**
Light Characteristics	**0930.**
Limiting Danger Line	See LDL.
Linear Error Probable (Navigational Errors)	See LEP.
Linear Latitude	**0114,** 0115.
Linear Longitude	**0114,** 0115.
Linear SD (Linear Standard Deviation - 1σ)	**1602, 1611, 1613,** 1614, **1615,** 1620, **Anx 16B.**
Linear Standard Deviation (Nav Errors)	See Linear SD.
Load / Loaded (ENCs / RNCs)	**1110, 1210, 1214, 1311, 1320.**
Load Line / Load Line Zones (Stability)	**1110-1111, 1210.**
Local Direction of Buoyage	**0941.**
Local Geodetic Datums (Spheroid)	**0311, 0321,** 0324.
Local Warnings (Radio)	**0615.**
Logline (Tidal Stream observation)	**1816.**
Long Flashing (Light characteristics)	**0930,** 0941.
Long Range Fixing (Radar)	1232, **1530.**
Longitude	Specific instances of importance (general usage not listed): **0111, 0314, 0321, 0711, 1820.** See also Difference of Longitude, Geodetic Longitude, Latitude / Longitude Grid, Linear Longitude, Longitude Scale, Longitude Units, Meridian, UTM Zones of Longitude, Vertex Longitude (Great Circle), Zones of Longitude (Transverse Mercator).
Longitude Scale	**0414, 0421-0424,** 0440, 0522, 0541, **0623,** 0711, App 3, App 5.
Longitude Units	**0422.**
Loom (Lights)	**0930.**
LORAN Denial	**1211, 1234.**
LORAN-A	**0918.**
LORAN-C	0716, **0805,** 0911, **0912, 0918,** 1111, 1231.
Lowest Astronomical Tide	See LAT.
Loxodromes	**0202.**
Lubbers Line	**0121, 0802, 1230,** 1319, 1610.
Luminous Range (Light characteristics)	**0930, 0932,** 0933.

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Lunar Day (Tides)	**1014-1015.**
Lunar Declinational Diurnal Harmonic Constituent	**1031.**
Lunar Equilibrium Tide (Tides)	**1013-1014.**
Lunar Month (Tides)	**1017.**
Lunar Semi-Diurnal Harmonic Constituent	**1031.**
Luni-Solar Declinational Diurnal Harmonic Constituent	**1031.**
Luni-Solar Tide Raising Force	**1017, 1020.**
Magnetic Anomaly	**1113, 1230.**
Magnetic Compass	0120, **0122-0125,** 0922, 1113, **1230,** 1238, 1313, 1322, 1610, 1820-1821, **1827, 1829,** App 10.
Magnetic Compass Rose	**0124.**
Magnetic Deviation	See Deviation.
Magnetic Dip	See Dip.
Magnetic Latitude	**0122,** 0125, **1113, 1230.**
Magnetic Meridian	**0122,** 0123-0124.
Magnetic North	**0122,** 0123-0124, 0454.
Magnetic Pole	**0122.**
Magnetic Signature	**1230.**
Magnetic Variation	See Variation.
Main Light (Light characteristics)	**0930.**
Man OverBoard	See MOB.
Manoeuvring (Plotting) Form	**1715.**
Manoeuvring Data	**0714,** 1314, 1319.
Marine Navigation	See Navigation.
Maritime and Coastguard Agency (UK)	See MCA.
Maritime Buoyage System	See IALA Maritime Buoyage System.
Maximum Unambiguous Range (Radar)	**1515, 1530.**
Maximum Likelihood (Navigational Errors)	See Least Squares Minimum Variance.
MCA (Maritime & Coastguard Agency - UK)	0950, 1525, 1902.
Mean Error Value (Navigational Errors)	**1611.**
Mean High Water Interval (Tides)	See MHWI.
Mean High Water Neaps (Tides)	See MHWN.
Mean High Water Springs (Tides)	See MHWS.
Mean Higher High Water (Tides)	See MHHW.
Mean Higher Low Water (Tides)	See MHLW.
Mean HW (Hydrographic Survey -Tides)	**1828.**
Mean Latitude	**0205-0207,** App 3.
Mean Latitude Sailing	0201, **0205,** 0501.
Mean Level (Hydrographic Survey -Tides)	**1828.**
Mean Low Water (Tides)	See MLW.
Mean Low Water Neaps (Tides)	See MLWN.
Mean Low Water Springs (Tides)	See MLWS.
Mean Lower High Water (Tides)	See MLHW.
Mean Lower Low Water (Tides)	See MLLW.
Mean LW (Hydrographic Survey -Tides)	**1828.**
Mean Neap Range (Tides)	See MNR.
Mean Sea Level (Tides)	See MSL.
Mean Spring Range (Tides)	See MSR.
Mean Square Deviation (Navigational Errors)	See Variance.
Mean Tide Level (Tides)	See MTL.
Mediterranean Moor (Anchoring)	See Stern-To Berthing.
Mer Parts	See Meridional Parts.
Mercator / Mercator Projection	0201, 0202, 0113, 0411, **0414,** 0420, **0421-0425,** 0430-0431, 0441-0442, 0450, 0511, 0521, 0551, 0611, 0620, 0624, 0805, Apps 3-4.
Mercator Graticule (Projections)	**0421-0424.**
Mercator Sailing	0201, 0205, **0207,** 0501, 0510, **0511.**
Meridian of Longitude	See Meridian.

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Meridian / Meridians / Meridional	**0110,** 0111-0115, 0120-0122, 0202, 0204, 0208-0209, **0312-0313,** 0314, 0411- 0412, 0414, 0421-0425, 0431, 0440, 0450, 0510-0511, 0520- 0522, 0920, 1014, 1017, 1052,1820, 1826, 1831, Apps 3- 5.
Meridional Arc	**0530, 0531,** Apps 4-5.
Meridional Element (Spheroid)	**App 5.**
Meridional Grid (Projections)	**0414.**
Meridional Parts (Projections)	0207, 0414, **0422-0423, 0510- 0511, 0530- 0531,** 0620, 0805, Apps 3- 5.
Messenger (Berthing)	**1420-1422**
MHHW (Mean Higher High Water - Tides)	0624, 0803, 0930, **1050-1062,** 1313.
MHLW (Mean Higher Low Water - Tides)	**1050-1062.**
MHWI (Mean High Water Interval - Tides)	**1052.**
MHWN (Mean High Water Neaps - Tides)	**1042, 1045-1046, 1050, 1062,** 1319.
MHWS (Mean High Water Springs - Tides)	0624, 0803, 0930, **1042, 1045-1046, 1050, 1062,** 1233, 1313, 1319 1813.
Middle Latitude	See Corrected Mean Latitude.
Military Command (Ships et al)	**1910, 1912.**
Minimum Keyboard Display (AIS)	See MKD.
Minimum Range (Radar)	**1511.**
Minimum Swing Radius / Radii (Anchoring)	See MSR.
Mini-SOCs (Mini-Standard Operator Checks)	**1230.**
Minor Surveys (Hydrographic)	**0923.**
Mixed Tides	**1020.**
MKD (Minimum Keyboard Display - AIS)	**0950.**
MLHW (Mean Lower High Water - Tides)	**1050, 1062.**
MLLW (Mean Lower Low Water - Tides)	0624, **1050, 1062.**
MLW (Mean Low Water - Tides)	**0624.**
MLWN (Mean Low Water Neaps - Tides)	0624, **1045, 1050, 1062,** 1319.
MLWS(Mean Low Water Springs - Tides)	**1045, 1050, 1062,** 1319.
MNR (Mean Neap Range - Tides)	**1045-1046, 1050,** 1062.
MOB (Man OverBoard)	**1111.**
Modified Polyconic Projections	**App 4.**
Moiré Direction Light (Light characteristics)	**0930.**
Monsoon	**1125.**
Monsoon Current / Monsoon Drift	**1125.**
Mooring (Anchoring)	**1412.**
Mooring Swivel (Anchoring)	**1412.**
Morse Code (Fog signal)	**0934.**
Morse Code (Light characteristics)	**0930.**
Most Probable Position	See MPP.
Mozambique Current	**1124-1125.**
MPP (Most Probable Position)	**0712,** 1610, 1622, **1623, Anx 16B.**
MRE (Multiple Regression Equations)	**0324.**
MSAS (MTSAT Satellite Augmentation System)	**0915, 1806.**
MSL (Mean Sea Level - Tides)	**0311, 0624,** 0803, 0930, **1050, 1060, 1062,** 1313, 1820.
MSL Seasonal Variations (Tides)	See Seasonal Variation (MSL - Tides).
MSR (Mean Spring Range - Tides)	**1045-1046, 1050, 1052,** 1062.
MSR (Minimum Swing Radius - Anchoring)	1410, **1412.**
MTL (Mean Tide Level - Tides)	**1062.**
MTSAT **S**atellite **A**ugmentation **S**ystem	See MSAS.
MTSAT(Multi-functional Transport Satellite)	**0915,** 1806.
Multi-functional Transport Satellite	See MTSAT.
Multiple Regression Equations (MRE)	See MRE.
n.mile	See International Nautical Mile. General usage not listed.
NAD 27 (North American Datum 1927)	**0322.**
NAD 83 (North American Datum 1983)	**0321, 0322.**
Napier's Rules	0520, **App 2,** App 4.

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
National Hydrographic Offices	See NHO.
Nautical Charts (IMO - SOLAS)	**0632.**
Nautical Mile	See International Nautical Mile.
Nautical Twilight	**1051.**
Naval Gunfire Support	See NGS.
NAVAREA Warnings	**0615.**
Navigation	0622, **0630,** 0631-0632, 0701, 0716, 0721, 1231, 1238, **1910, 1912.**
Navigation Plan	See NavPlan.
Navigational Publications (UKHO)	See NP.
Navigational Record Book	**0716,** 0807-0808, **1238,** 1312, 1327, 1415, 1923, 1931-1932.
NAVPAC	0116, **0210,** 0211, 0521, 0550, **0551,** 0811, 1311.
Navplan / Navplans (Navigation Plan / Plans)	**1110, 1210, 1310-1312,** 1316-1320, 1323, **1330,** 1903, 1923, 1931.
NAVSTAR (GPS)	**0910.** See also GPS and DGPS.
NAVTEX	0615, **1230.**
NC (New Chart)	**0615, 0624,** 0626.
NE (New Edition - chart)	0611 **0615, 0624,** 0626. See also UNE.
NE Trade Winds	**1122.**
Neaps / Neap Tides	**1017-1018, 1020, 1051.**
Negative Surge (Tides)	**1022.**
NELS (NW European LORAN-C System)	**0912.**
New Chart	See NC.
New Edition (chart)	See NE.
New Moon (Tides)	**1017-1018, 1020.**
Newton's Universal Law of Gravitation	**1010.**
NGA (US National Geospatial & Chart Agency)	**0632.**
NGS (Naval Gunfire Support)	0322, **0331.**
NHO (National Hydrographic Offices)	**1022,** 1221, **1311.**
Night Vision Aids	**1531.**
nm / n.m	See International Nautical Mile. General usage not listed.
NMs (Notices to Mariners)	0611, 0614, **0615,** 0616, **0624,** 0626, 0931, 0941 1110-1111, 1210-1211, 1311, **1805, 1812.**
No Go Line	See LDL.
No Headmark (procedure)	**1313,** 1322.
NO's Pilotage Notebook	1210, **1238,** 1310, 1312, 1316, **1319,** 1322, 1413.
NO's Workbook	1110, 1210, **1214, 1238, 1312,** 1316, 1413.
Nominal Range (Light characteristics)	**0930-0932.**
Non-Homogeneous Geodetic Datums	**0324.** See also Homogeneous Geodetic Datums.
Nord Algerie Grid (Transverse Mercator)	**0453.**
Nord Maroc Grid (Transverse Mercator)	**0453.**
Nord Tunisie Grid (Transverse Mercator)	**0453.**
Normal Distribution (Navigational Errors)	**1611,** 1620-1621, **1623, Anxs 16A-16B.**
Normal MSR (Anchoring)	**1412.**
Normal Probability Distribution (Nav Errors)	See Normal Distribution.
North American Datum 1983	See NAD 83.
North Atlantic (Drift) Current	See North Atlantic Current.
North Atlantic Counter-Current	**1125.**
North Atlantic Current	**1125.**
North Equatorial Current	See Equatorial Currents (North and South).
North Pacific Current	**1125.**
North-Up (Radar / ARPA)	**1526-1527.**
Northwest European LORAN-C System	See NELS.
Notices to Mariners (NMs)	See NMs.
NP (Navigational Publication - UKHO)	0612, **0640.**
Oblate Spheroid	**0110, 0115,** 0208, 0310, 0540.
Observed Altitude (Sextant)	App 6. See also Altitude, True Altitude.
Observed Position (Astronomical)	**0711,** 0805.

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Obstruction Lights (Light characteristics)	**0931.**
Occasional Lights (Light characteristics)	**0930.**
Occulting (Light characteristics)	**0930.**
Ocean Navigation	**0721,** 1101, **1110-1111,** 1201, 1210-1211, 1234, **1911,** 1920, **1930.**
Officer in Tactical Command	See OTC.
Offset EBL (Offset Electronic Bearing Line)	**1317.**
Offset VRM (Offset Variable Range Marker)	**1317.**
One-way Routes (Traffic routing systems)	**1221.**
Onset Depth (SWE - Interaction)	**1220, 1234,** 1319.
% Onset Depth	See Onset Depth.
... % Onset Depth	See Onset Depth.
Open Service (Galileo)	See OS.
Oppos ition (Tides)	**1017.**
Optical Rangefinder	**0803.**
Ordnance Datum (Tides)	**1061.**
Ordnance Survey Great Britain 1936	See OSGB 36.
Orientation (Hydrographic Survey - charts)	**1803, 1813-1814, 1820, 1827.**
Orientation Modes (Radar / ARPA)	**1526, 1527,** 1722.
Orthogonal Position Lines	**1613-1616,** 1620, **Anx 16B.**
Orthomorphic / Orthomorphic Projection	**0411, 0414,** 0420- 0421, 0430, 0622, App 4.
Orthomorphism	See Orthomorphic / Orthomorphic Projection.
OS (Open Service - Galileo)	**0914.**
OSGB 36 (Ordnance Survey Great Britain 36)	**0322,** 0323-0324, 0331.
OSSN 80 (Datum)	**0322.**
OTC (Officer in Tactical Command)	**1110-1111, 1210.**
Overfall / Overfalls (Tides)	**1044.**
Overhead Clearances	**1310-1312, 1320.**
Oya Shio Current	See Kamchata Current.
Parallel / Parallels	See Parallels of Latitude.
Parallel Index / Parallel Indices	See PI.
Parallel Sailing	0201, **0203,** 0204, 0501.
Parallels of Latitude	**0111-0112,** 0115, 0202-0205, **0313,** 0411-0412, 0414, 0420, **0421,** 0422-0423, **0424,** 0431, 0450, 0511, 0520, 0521, 0540, 0541, Apps 3-5. See also Latitude.
Passage Graph	**1112,** 1113, 1214, 1234.
Patch Antenna (DGPS / GPS)	**1806.**
PCS (Propulsion Control System)	**1236.**
PDF (Probability Density Function)	**Anx 16A.**
Pelorus (Bridge)	**0802,** 1230, 1319, 1413.
Percentage of the Day (Tides)	See Percentage Springs.
Percentage Onset Depth (SWE - Interaction)	See Onset Depth.
Percentage Springs (Tidal Streams)	**1045-1046,** 1319.
% Springs	See Percentage Springs.
... % Springs	See Percentage Springs.
0% Springs (MNR)	**1045.**
100% Springs (MSR)	**1045.**
Perigean Tide (Tides)	**1015.**
Perigee (Tides)	**1015.**
Perihelion (Tides)	**1016.**
Period (Light characteristics)	**0930.**
Perpendicularity (Sextant)	**App 10.**
Personal Error (Sextant)	**App 10.**
Persons on Board (AIS field)	See POB.
Perspective Conformal Projection	See Perspective Projection.
Perspective Projection	**App 4.**
Peru (Humbolt) Current	**1124-1125.**
Phase (Light characteristics)	**0930.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
Phase Lag (Tidal Harmonic Constituents)	**1031.**
Physical Surface (Earth - Oblate Spheroid)	**0311.**
Pilotage	0610, 0612, 0714, 0716, **0721,** 0807, 0808, 0923-0924, 0926, 0942, 1111, 1201, 1222, 1230, 1234, 1301, **1303-1305,1310-1319, 1320-1325, 1327-1328,** 1331, 1401, 1413, 1415, 1425, 1501, **1527, 1910-1912, 1921-1925, 1931-1933.**
Pilots (Admiralty Sailing Directions)	See Sailing Directions.
Pin-Mast	**1230.**
PIs (Parallel Index / Parallel Indices)	**1210, 1214,** 1231, **1232,** 1233-1234, 1310, **1316,** 1317, 1319, 1325, 1328, 1330, **1521,** 1522, 1527, **1528,** 1922, 1924, **1933.**
Pitometer Speed Logs	**0925.**
Pivot Point (Manoeuvring)	**0714,** 1314, 1332 **1334,** 1417.
Plane Sailing	0201, **0204,** 0206, 0501, 0511.
Plane Triangle	**Apps 1-2,** App 4.
Planning (Navigation)	**1910-1911.**
POB (Persons on Board - AIS field)	**0950.**
Point of No Return (Pilotage)	**1312.**
Points (Compass)	**0123,** 0126, 0806.
Polar (Axis / Radius)	**0110,** 0115, 0310, **0311, 0313,** 0541, App 5.
Polar (Charts)	See Polar Stereographic Projection.
Polar (Regions)	0910, 0913, **0920,** 1515.
Polar Coordinates	**0206.**
Polar Cosine Rule (Trigonometry)	**App 2.**
Polar Easterlies (Winds)	**1122.**
Polar Stereographic Projection	**0414,** 0450, 0621, 0920, App 4.
Polar Triangle (Trigonometry)	**App 2.**
Pole / Poles / Polar (North or South)	**0110,** 0113-0116, 0203, 0208-0209, 0211, 0311-0314, 0324, 0411, **0414,** 0421-0422, 0425, 0431, 0451, 0520, 1122, App 2, Apps 4-5.
Pole Logship (Tidal Stream observation)	**1816.**
Polyconic Projection	**0414, 0622,** App 4.
Portable Survey System (Hydrog. Survey)	See PSS.
Portugal Current	**1125.**
Position Circle / Circles	0620, **0808, 1603, 1623,** App 4, App 10.
Position Line / Lines	**0710, 0711,** 0712, 0803-0805, **0806,** 0808, 0933, 1212, 1312, 1415, **1530,** 1601-1603, 1610-1623, Anxs 16A-16B, App 6, App 10.
Position Probability Area	See PPA.
Positive Surge (Tides)	**1022.**
PPA (Position Probability Area)	**0710, 0712,** 1530, 1610, **1622-1623.**
PPS (GPS - Precise Positioning Service)	**0910.**
Precautionary Areas (Traffic routing system)	**1221.**
Precession (Gyroscopic)	**0920.**
Precise Navigation	**0324.**
Precise Positioning Service (GPS)	See PPS.
Precision (Navigational Errors)	**1602.**
Preliminary Notices to Mariners	See T&Ps.
Presentation Modes (Radar / ARPA)	**1526-1527,** 1722.
Pressure Zone (Interaction)	**1220,** 1332.
PRF (Pulse Repetition Frequency)	**1510.**
Prime / Priming (Tides)	**1017.**
Prime Meridian (Greenwich Meridian 0˚)	**0110,** 0111, 0114, **0423,** 0431, 0451-0452, 1214.
Principal Meridian	See Central Meridian.
Probability Density Function (Nav Errors)	See PDF.
Probability Heap (Navigational Errors)	**Anx 16B.**
Probable Error (Navigational Errors)	See LEP.
Projection / Projections / Projected (chart)	0401, **0410-0411,** 0412-0414, 0420, 0431, 0450, App 4.
Propulsion Control System	See PCS.
PRS (Public Regulated Service - Galileo)	**0914.**

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Pseudo AIS (contact)	See Virtual AIS.
Pseudo Ranging (GPS)	**0910, 0911.**
PSS (Portable Survey System)	**1827.**
Public Regulated Service (Galileo)	See PRS.
Pulse Length (Radar)	**1511, 1513,** 1519, **1521, 1523.**
Pulse Repetition Frequency (Radar)	See PRF.
PZ 90 (GLONASS Datum)	**0913.**
Quadrantal Spherical Triangle	**App 2.**
Quadrature (Tides)	**1017.**
Quadrifilar Helix Antenna (DGPS / GPS)	**1806.**
Quartering Bearings (Pilotage)	**1314,** 1322.
Quick Flashing (Light characteristics)	**0930,** 0941.
Race / Races (Tides)	**1044.**
Racon (Radar Transponder)	1518, **1529.**
Racon Flash (Radar Transponder)	**1529.**
Radar Image Overlay	See RIO.
Radar Reflectors	**1518.**
Radial Error (Nav Errors - RMS Error)	**1612, 1615, Anx 16B.**
Radial SD (Radial Standard Deviation)	**1612.**
Radial Standard Deviation (Nav Errors)	See Radial SD.
Radian Rule	**0127,** 0805-0806, 0811, 1312, 1319, **1322,** 1324, 1522, Anx 16B.
Radio Navigational Warnings	**0615,** 1311.
Radius of Curvature (Earth)	**0113, 0314, 0421,** App 6.
Ramark (Radar Beacon)	1518, **1529.**
Ramark Flash (Radar Beacon)	**1529.**
Random Errors (Navigational Errors)	0811, 1603, **1610-1611**, 1614, 1620, **1621, 1623 Anxs 16A-16B,** App 10.
95% Random Errors (Nav Errors)	**1623.**
Range (Tidal)	0624, 0942, **1014, 1020-1021, 1031, 1040, 1052,** 1062, 1313, 1820, 1828.
Range Discrimination (Radar)	**1511.**
Range Lattices (Fixing)	**0805.**
Range of the Day (Tidal Stream Predictions)	**1042, 1045.**
Rangefinders	See Optical Rangefinder and Laser Rangefinder.
RAS (Replenishment at Sea)	0811, 1110-1111, 1210, **1220,** 1238, 1327.
Raster / Raster Navigation Charts	See RNC.
Rate for the Day (Tidal Nurdle)	**1046.**
Recommended Tracks (Traffic routing)	**1221.**
Rectangular Distribution (Nav Errors)	**Anx 16A.**
Rectangular Errors (Navigational Errors)	1620, **Anx 16A.**
Rectilinear Tidal Streams	**1040.**
Reduced Latitude	See Parametric Latitude.
Reduced MSR (Anchoring)	**1412.**
Reed (Fog signal)	**0934.**
Region B (IALA Maritime Buoyage System)	**0941.**
Region A (IALA Maritime Buoyage System)	**0941.**
Regional Geodetic Datums (Spheroid)	**0311, 0321,** 0324.
Relative Bearing / Relative Bearings	**0126,** 0803,1230.
Relative Distance (Rel Vel)	**1735.**
Relative Motion (Radar / ARPA)	1316, **1526-1527, 1703,** 1712, 1720, 1721.
Relative Motion (Vector)	**1710,** 1712.
Relative Navigation	See RELNAV.
Relative Position (Navigational Errors)	**1602.**
Relative Speed (Rel Vel)	**1710, 1711,** 1712-1713, 1720, 1722, 1732-1736.
Relative Target Trails (Radar / ARPA)	**1526-1527.**
Relative Track (Radar)	0713, **1703, 1711,** 1712-1714, 1720, 1722, 1732-1736.
Relative Vectors (Radar / ARPA)	**1526-1527.**

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Relative Velocity	**1701, 1702, 1715, 1720, 1722, 1730.**
RELNAV (Relative Navigation)	**1321, 1922.**
Repeatability (Navigational Errors)	**1602.**
Replenishment at Sea	See RAS.
Resection (Fixing)	**1811.**
Restricted Visibility	See Fog.
Retroflector (Buoyage System)	**0941.**
Rhumb Line	0110, **0202,** 0204-0205, 0207, 0209-0210, 0414, 0421-0422, 0425, **0441- 0442,** 0511, 0521, 0530-0531, 0551, 1110, App 3, App 5. See also Course & Distance.
Rhythmic (Light characteristics)	**0930.**
Rigidity-in-Space (Gyroscope)	**0920.**
Ring Laser Gyros	See RLG.
RIO (Radar Image Overlay)	**0720,** 1231-1232, **1321,** 1323, 1415, 1528, 1921, 1923-1924.
Rising Range (Lights)	**0933.**
RLG (Ring Laser Gyros)	**0920,** 0921.
RMS Error (Root Mean Square Error)	**1611-1612, Anxs 16A-16B.**
RNC (Raster Navigation Charts)	0331, 0614- 0616, 0631, **0632-0633,** 0902, 0919, 0930, 0940, 1110, 1210, 1214, 1317, 1911.
Roaring Forties	**1122.**
Root Mean Square Distance (Nav Errors)	See d[rms].
Root Mean Square Error (Nav Errors)	See RMS Error.
Rose	See Compass Rose.
Rose Correction	See Compass Rose Correction.
Rotary Tidal Streams	**1040.**
Rounding-Off Error (Navigation Errors)	1611, **Anx 16A.**
Run-back (Transferred Position Line)	**0804,** 0805.
Running Anchorage	**1414,** 1418, **1420.**
Running Fix	0804, **0805, 0806.**
Running Survey	**1814.**
Run-on (Transferred Position Line)	**0804, 0805.**
Safe Latitude	See Safe Parallels of Latitude.
Safe Parallel	See Safe Parallels of Latitude.
Safe Parallels of Latitude	**0209, 0442, 0522,** 0622.
Safe Water Marks (Buoyage System)	**0941.**
Safety of Life Service (Galileo)	See SoL.
Safety Swinging Circle	**1410, 1412-1413, 1415.**
Sailing Directions (Admiralty Pilots)	0612, **0640,** 0712, 0930, 0940- 0941, 1042, 1110 -1111, 1221, 1234, 1311, 1323, **1812,** 1816, **1830,** 1831.
Sailings (Methods)	**0201,** 0207, **0208-0209,** 0501, **0511,** 0550.
SAME Name (Hemispheres)	**0208, 0211.**
Sandwaves	1213, **1234.**
SAR (Search and Rescue Service - Galileo)	**0914.**
SART (Search and Rescue Transponder)	**1518, 1529.**
Satellite Based Augmentation Systems	See SBAS.
SBAS (Satellite Based Augmentation Systems)	**0915.**
Scale (General)	0113, 0124, 0324, 0610-0611, 0620, **0622-0626,** 0631, 0713, 0716, 0805, 0810, 0930, 0940, 1046, 1060, 1110-1112, 1210, 1214, 1310-1312, 1321, 1410, **1521, 1803, 1813-1814, 1820,** 1822-1823, 1825-1828, 1830, 1924, App 3-4, App 6.
Scale (Projection property)	**0411,** 0413-0414, 0421, 0423-0424, 0431.
Scale Error (Hydrographic Survey)	**1820.**
Scale Factor (Tidal Nurdle)	**1046.**
Scale Factor (Transverse Mercator Grids)	**0450.**
Scend [or 'Send'] (Waves)	**1410.** Scend is the vertical movement of waves or *Swell* alongside a wharf, jetty, cliff, rocks etc.

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
SD (Standard Deviation)	**1611,** 1614, 1616, **Anxs 16A-16B.**
2SD (Twice Standard Deviation)	**1611.**
SE Trade Winds	**1122.**
Sea Command (Ships et al)	**1910.**
Sea Ice	**1531.**
Sea Mile	**0113,** 0114, 0202, 0314, 0421- 0422, 0424, 0623, **0803,** 0925, 0930, 0932, 1515, 1524, App 5.
Sea Orders / Sea Order Book (Command)	**1912.**
Sea Speed (Rel Vel)	**1703,** 1722.
Sea Stabilised (Radar / ARPA)	**1526, 1527, 1703,** 1722.
Seafarer (RNCs)	**0632.**
Search and Rescue Service (Galileo)	See SAR.
Search and Rescue Transponder (Radar)	See SART.
Seasonal Variation (MSL - Tides)	**1050-1060.**
Secant Projections	**0413.**
Secondary Ports (Tides)	**1030, 1050,** 1053, 1060, **1828.**
Sector Light (Light characteristics)	**0930.**
Seiche (Tides)	**1022.**
Seismic Waves (Tsunamis)	**1023.**
Self Organising TDMA (AIS)	See SOTDMA.
Semi-Diurnal / Semi-Diurnal Tidal Streams	**1040, 1042, 1045-1046,** 1816.
Semi-Diurnal / Semi-Diurnal Tides	**1014-1018, 1020, 1030, 1050,** 1062.
Semi-Systematic Errors (Navigational Errors)	**1610,** 1611, 1621.
Separation Line (TSS)	**1221.**
Separation Zone (TSS)	**1221.**
Set	**0713,** 0720, 0806, 1527, 1603, App 7.
Sextant	**App 10.**
SGS 90 (Soviet Geocentric Co-ord System)	**0913.**
Shadow Zones (Radar)	**1519, 1530.**
Shallow Water Effect (Interaction)	See SWE.
Shallow Water Effects / Corrections (Tides)	**1021, 1031-1032.**
Shape (Hydrographic Survey)	**1820.**
Shape (Projection property)	**0411,** 0414.
Ship Proximity Interaction	See Interaction.
Ship Reporting Systems (Routing Systems)	**1240.**
Ship's Head	See Heading.
Ship's Log	**0716,** 1230, **1238, 1415.**
SHM (Simplified Harmonic Method) - Tides	**1032 ,1050.**
SHM for Windows® (UKHO Tides software)	**1032 ,1042, 1050,** 1311.
Side Error (Sextant)	**App 10.**
Sight Reduction Method (Astro)	**0210.**
Sigma	See σ.
σ [Sigma] (Navigational Errors)	**1615-1616, 1621 Anx 16A, Anx 16B.**
1σ [1 Sigma] (Nav Errors - Linear SD)	**1602, 1611, 1613,** 1614, **1615, Anxs 16A-16B.**
2σ [2 Sigma] (Nav Errors)	**1602, 1611, 1615,** 1620, 1621, **Anx 16A.**
3σ [3 Sigma] (Nav Errors)	**1602,** 1621, **Anx 16A.**
σ_r [Sigma $_r$] (Nav Errors, Radial SD)	**1612, Anx 16B.**
2σ_r [2Sigma $_r$] (Nav Errors)	**1615.**
Simple Conical Orthomorphic Projection	See Conical Projections.
Simplified Harmonic Method (UKHO Tides)	See SHM and SHM for Windows®.
Simplified Symbols (ENC - IHO S.52)	**0632.**
Sine Formula / Method / Rule etc	0210, **0211,** 1322, 1820, **1825, Apps1- 2.**
Single Normal MSR (Anchoring)	**1412.**
Single Reduced MSR (Anchoring)	**1412.**
Siren (Fog signal)	**0934**
SIRGAS (South American International Geodetic Reference System)	**0321.**
Skew Orthomorphic Projection	0411, **0414.**

Italicised Technical Term	Primary / Secondary References (Primary References in BOLD)
Small Circle	**0110,** 0111, 0115, 0208, 0803, 1013, App 2, App 4.
Small Corrections (Obsolete from 2000)	**0624,** 0626.
Smelling the Ground (Interaction)	**1220, 1332.**
SMG (Speed Made Good)	See Ground Speed.
SOA (Speed of Advance)	**1110-1112, 1210-1211, 1214,** 1234.
SOCs (Standard Operator Checks)	**1230.**
SOG (Speed Over the Ground / Ground Speed - AIS field)	**0951.**
SoL (Safety of Life Service - Galileo)	**0914.**
Solar Day (Tides)	**1016.**
Solar Equilibrium Tide (Tides)	**1016.**
Solar Semi-Diurnal Harmonic Constituent	**1031.**
SOLAS (Safety Of Life At Sea)	**0631, 0632,** 1525.
Solstice / Solstitial Tides (Tides)	**1016-1017, 1020,** 1902.
Somali Current	**1125.**
Sonar Speed Logs	**0925.**
SOTDMA (Self Organising TDMA - AIS)	**0951.**
Sounding Board (Hydrographic Survey)	**1821, 1827.**
Sounding Book (Hydrographic Survey)	**1827.**
Sounding Datum (Hydrographic Survey)	**1820, 1827-1828, 1831.**
Sounding Marks (Hydrographic Survey)	**1820, 1824, 1826.**
Source Data (Diagram - charts)	**0624-0626,** 0805.
South Equatorial Current	See Equatorial Currents (North and South)
Southern Ocean Current	**1122, 1124-1125.**
Soviet Geocentric Co-ordinate System 1990	See SGS 90.
Spatial Correlation (Speed) Logs	**0925.**
Special Marks (Buoyage System)	**0941.**
Special Sea Dutymen	See SSD.
Speed Factor (Manoeuvring Data)	See Acceleration Distance.
Speed Log	0712, 0920, **0925,** 1526-1527, 1603, 1622, 1722, 1814.
Speed Made Good	See Ground Speed.
Speed of Advance	See SOA.
Speed Over the Ground (Ground Speed - AIS)	See SOG.
Sphere / Spherical (Earth)	**0110,** 0114, **0115-0116,** 0201, 0203-0205, 0208-0211, 0310-0311, 0313, 0315, 0330, **0332,** 0413, 0414, 0420, 0422, 0440, 0510, 0520, 0530, 0550, 0551, 1012, App 2, App 4.
Sphere / Spheres (Soft-iron, Magnetic Compass)	**0122,** 0125, **1230.**
Spherical Distance (Geodesic)	See Distance (Geodesic - Spherical).
Spherical Great Circle Composite Track	0501, **0522.**
Spherical Great Circle Sailing	0501, **0521,** 0551. See also Great Circle.
Spherical Mercator Sailing	**0511.**
Spherical Meridional Parts	**0511.**
Spherical Sailing	0201, 0207, **0208-0209.**
Spherical Triangle	0520, **App 2,** App 4.
Spheroid / Spheroidal (Earth)	0101, 0111, 0114, **0116,** 0201, 0205, 0207-0208, 0210, **0311,** 0312-0313, 0315, 0320, **0321,** 0322, 0330-0331, 0332, 0410, 0413-0414, 0420-0422, 0424, 0440, 0450, 0452, 0530, 0531, 0540, 0541, 0550, 0551, 0711, 0913, Apps 4-5. See also Datum, Oblate Spheroid and WGS 84.
Spheroidal Arc (Transverse Mercator Grids)	**0450.**
Spheroidal Corrections (Geodesic)	**0541.**
Spheroidal Great Circle Sailing	**0501.**
Spheroidal Meridional Parts	App 5.
Spheroidal Projection	**0414.**
Spheroidal Rhumb Line Sailing	**0501.**
Spin Axis (Gyroscope / Gyro Compass)	**0121, 0920,** 0921
Spring Tides / Springs	0712, **1017-1018, 1020, 1051,** 1330, 1816.
SPS (Standard Positioning Service - GPS)	**0910, 0917.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
Square Corner Reflectors (Radar)	**1518.**
Squat (Interaction)	1213, **1220,** 1234, 1311, **1332, 1807, 1827.**
Squint Error (Radar)	**1522.**
SR (SunRise)	**0715, 1110, 1210,** 1214, 1311.
SS (SunSet)	**0715, 1110, 1210,** 1214, 1311.
SSD (Special Sea Dutymen)	0924, **1230,** 1312, 1327, **1912, 1923-1924, 1931-1933.**
Stabilisation Modes (Radar / ARPA)	**1526, 1527,** 1722.
Stand (Tides)	**1021.**
Standard Deviation (Navigational Errors)	See SD.
Standard Distance (NATO - Manoeuvring)	**1412.**
Standard Operator Checks	See SOCs.
Standard Parallel / Parallels (Projections)	**0413,** 0414, 0421, App 4.
Standard Port (Tides)	**1021, 1030, 1040, 1042, 1046, 1050,** 1053, **1828.**
Standard Positioning Service (GPS)	See SPS.
Station Pointer	**0808, 0810-0811,** 1811, 1820-1821, 1826-1827, App 6, App 10.
Statute Mile	**0113.**
Steady Turning Diameter (Manoeuvring)	**1314.**
Steadying Point (Manoeuvring Data)	**1314.**
Stereographic Projection	0411, **0414.**
Stern Rope (Berthing)	**1424-1425.**
Stern Swinging Circle	**1415,** 1418, **1816.**
Sternboard (Manoeuvring)	1417, **1420, 1422,** 1425. A *Sternboard* is a prolonged movement of a vessel astern under its own power, usually for a distance of at least 1 cable or more. *Sternboards* are often carried out by warships when unberthing.
Sternmark / Sternmarks	0942, 1310, 1312-1313, 1319, 1321, **1322,** 1324, 1923, 1931.
Stern-to Berthing (Anchoring)	1412, **1420-1424.**
Storm Surges	**1022,** 1060, 1062, 1812.
Stripes (Light Structure Descriptions)	**0930.**
Sublunar / Sublunar Point (Tides)	**1012-1013, 1015.**
Sub-Refraction	**1515.**
Subsidiary Light (Light characteristics)	**0930.**
Subtense Method (Hydrographic Survey)	**1820.**
Suction Zone (Interaction)	**1220,** 1332.
Sud Algerie Grid (Transverse Mercator)	**0453.**
Sud Maroc Grid (Transverse Mercator)	**0453.**
Sud Tunisie Grid (Transverse Mercator)	**0453.**
SunRise	See SR.
SunSet	See SS.
Super-Refraction	**1515,** 1529, 1530.
Surface Current / Surface Drift	**0712-0713,** 0715, 0925, **1002, 1040 1120,** 1121, **1122**.
Surface Drift	1603.
Surface Drift Current	See Surface Current / Surface Drift.
SWE (Shallow Water Effect - Interaction)	**1220,** 1234, 1312, 1319, 1322, 1332-1333.
SWE Onset Depth (Interaction)	See Onset Depth.
Swell	1213, **1410.** *Swell* is the wave motion caused by a meteorlogical disturbance, which persists after the disturbance has died down or moved away.
Swept Gain (Radar)	**1513, 1514,** 1520.
Systematic Errors (Navigational Errors)	1603, **1610,** 1611, 1620, App 10, **Anx 16B.**
T&Ps (Temporary and Preliminary NMs)	**0615,** 1110- 1111, 1210, 1311.
Tactical Diameter (Manoeuvring Data)	**0714,** 1314.
Tactical Miles	**1520.**
Tactical Navigation	**1911.**
Tangent Projections	**0413.**
Tangential Plane	**0330,** 0332-0333.
TCPA (Time to CPA)	**1526.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
TDMA (Time Division Multiple Access - AIS)	**0951.**
Tectonic Plates (Seismic Waves)	**1023.**
Temperature Inversion	**1515.**
Temporal Correlation (Speed) Logs	**0925.**
Temporary and Preliminary NMs	See T&Ps.
Temporary NMs	See T&Ps.
Ten-Foot Pole	**1813, 1821, 1827, 1829.**
Territorial Sea	1110, **1111, 1210-1211,** 1221, **1802.**
Thames AIS	See Inland AIS.
Threshold Level (Radar)	**1510,** 1526.
Tidal Bore	See Bore.
Tidal Nurdle (Tidal Stream Predictions)	**1046.**
Tidal Current (American for Tidal Stream)	**1002, 1040.**
Tidal Curve (Hydrographic Survey)	**1827, 1829.**
Tidal Levels	1050, **1060-1061.**
Tidal Stream / Tidal Streams	0624, **0712-0713,** 0715-0716, 0804- 0806, 0916, 0925, 0942, 1001, **1002, 1032, 1040,** 1041, **1042-1043, 1045-1046,** 1051, 1053, 1112, 1120, 1210, 1212, 1214,1222, 1231, 1234, 1236, 1238 1310-1312, 1314, 1316, 1319-1320, 1322-1324, 1328, 1330-1331, 1333, 1410-1412, 1414-1415, 1417-1419,1527, 1603, 1621, Anx 16A, 1722, 1804, 1811, 1813, **1816,** App 7.
Tidal Stream Atlas / Atlases	0712, **1042,** 1044, **1045,** 1046
Tidal Waves (Tides - NOT Seismic Waves)	**1020 -1023.**
Tide / Tides	0712, 1001, **1002, 1010, 1012-1017, 1020-1022, 1040,** 1042-1044, **1050, 1052,** 1062, 1213, 1234, 1240, 1323, 1410, 1415, 1420, 1816, 1820, 1828, 1830.
Tide Pole (Hydrographic Survey)	**1821, 1828.**
Tide Raising Force / Effect	1002, **1010, 1013-1018, 1020, 1030-1031,** 1040-1041.
Tide-Rip / Tide-Rips	See Overfall.
Time Division Multiple Access (AIS)	See TDMA.
Time to CPA	See TCPA.
Time Zones	**1110,** 1111, 1210, **1214,** 1238, **1828.**
TotalTide® (UKHO tides software)	0712, 1001, **1042,** 1046, **1050-1051,** 1214, 1311.
Track Control (Manoeuvring)	1211, **1236.**
Track Control System	**1236.**
Tracking (Radar / ARPA)	**1703.**
Tractive Force	See Tide Raising Force.
Trade Winds	**1122,** 1515.
Traditional Symbols (ENC - IHO S.52)	**0632.**
Traffic Lanes (TSS)	**1221.**
Traffic Separation Scheme	See TSS.
Transfer (Manoeuvring Data)	**0714,** 1310-1311, **1314,** 1315.
Transferred Position Line	**0710, 0804, 0805, 0806.**
Transponder (Radar)	**1518,** 1520, **1529.**
Transverse Mercator Projection	0411, **0414, 0430, 0431, 0450,** 0451, 0611, 0621-0622, **0623,** 0624, App 4.
Transverse Mercator Projection Grid	**0450, 0451.**
Traverse Sailing	0201, **0206,** 0501.
Trial Manoeuvres (ARPA)	**1526.**
Triangulation (Hydrographic Survey)	**1820,** 1821, **1822, 1825, 1827, 1831.**
Triangulation Station (Hydrographic Survey)	**1820, 1824- 1827, 1829.**
Trim (Squat / Interaction)	**1220.**
Tropical Storms	**1812.**
True Altitude (Sextant)	App 4, **App 6.** See also Altitude, Observed Altitude.
True Easting (Transverse Mercator Grid)	**App 4.**
True Horizontal Angle (Hydrographic Survey)	**1824.**
True Latitude	**0111.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
True Motion (Radar / ARPA)	1316, **1526-1527, 1703,** 1712, 1721.
True North	**0120,** 0121-0122, 0450.
True Northing (Transverse Mercator Grid)	**App 4.**
True Speed (Rel Vel)	1711, **1712,** 1713, 1720, 1722, 1732, 1736.
True Target Trails (Radar / ARPA)	1316, **1526-1527, 1722.**
True Track (Rel Vel)	0713, 1711, **1712,** 1713, 1720, 1722, 1732, 1734-1736.
True Vectors (Radar / ARPA)	**1526-1527.**
TSS (Traffic Separation Scheme)	0611, 0941, 1110-1111, 1210- 1211, **1221,** 1240.
Tsunamis (Seismic Waves)	**1023,** 1812.
Turning at Rest (Manoeuvring)	0803, **1322.**
Turning Circle (Manoeuvring)	**0714,** 1314, 1322-1323.
Two-way Routes (Traffic routing systems)	**1221.**
Typhoons	**1812.**
UK Transverse Mercator Projection (UKTM)	**0452.**
UKHO (UK Hydrographic Office)	0111, 0207-0208, 0322, 0323, 0601, **0610,** 0611-0616, 0624, 0640, 0712, 0803, 0805, 0901, 1032, 1042, 1050,1060-1062, 1111, 1115, 1211, 1214, 1221, 1240, 1311, 1313, 1521, 1802-1803, 1805, 1812, 1814-1815, 1820, 1826-1829, 1831, App 6.
Ultra Quick Flashing (Light characteristics)	**0930.**
Underkeel Clearances	**1022,** 1051, **1052, 1210, 1212-1213, 1220, 1234,** 1310, **1311,** 1312, 1319, 1328, 1410.
UNE (chart - Urgent New Edition)	**0615, 0624.** See also NE.
Uninterruptible Power Supply	See UPS.
Union Jack	**1230.**
United Kingdom Hydrographic Office	See UKHO.
Universal Polar Stereographic Projection	**0414.**
Universal Transverse Mercator	See UTM .... (Grid, Projection, Zones etc).
UPS (Uninterruptible Power Supply)	**0631.**
Urgent New Edition (chart)	See UNE.
US National Geospatial & Chart Agency	See NGA.
UTM Grid (Universal Transverse Mercator)	**0431,** 0450, **0451,** 0452.
UTM Grid East / Eastings	**0451.**
UTM Grid North / Northings	**0421,** 0450, **0451.**
UTM Projection	**0414, 0451.**
UTM Zones of Longitude	**0451,** 0452. See also Zones of Longitude.
Variable Range Marker	See VRM.
Variance (Navigational Errors - Mean Square)	**Anx 16A.**
Variation (Magentic)	**0122-0124,** 0125, 0454, 0624, 0807, 0811, 1820.
Vector (chart)	0614, **0632.**
Velocity Of Sound (In water)	See VOS.
Velocity Triangle (Rel Vel)	**1713, 1720,** 1721-1722, 1730, 1734-1736.
Velocity Triangle Rules (Rel Vel)	**1714,** 1720, 1730.
Velocity Vector Modes (Radar / ARPA)	**1526-1527,** 1722.
Vertex / Vertices (Great Circle)	0201, 0207, **0209, 0442, 0520-0521,** 0522, App 2, App 4.
Vertex Latitude (Great Circle)	**0520,** App 4.
Vertex Longitude (Great Circle)	**0520,** App 4.
Vertex / Vertices (Great Circle)	0201, 0207, **0209, 0442, 0520-0521,** 0522, App 2, App 4.
Vertical Clearances	**0624,** 1051, **1062, 1820.**
Vertical Control (Hydrographic Survey)	**1820.**
Vertical Danger Angle	**0803, 1233.**
Vertical Datums (Tides)	**0320,** 1051, **1062.**
Vertical Lights (Light characteristics)	**0930.**
Vertical Sextant Angle	See VSA.
Very Quick Flashing (Light characteristics)	**0930,** 0941.
Vessel Traffic System	See VTS.
Vigias (Hydrographic Survey)	**1815.**
Virtual AIS (contact symbols)	**0952 0954.**

Italicised Technical Term	**Primary / Secondary References (Primary References in BOLD)**
VOS (Velocity Of Sound - in water)	**1807.**
VRM (Variable Range Marker)	**1317,** 1720, 1933.
VSA (Vertical Sextant Angle)	**0803,** 0805, **1813, 1829,** 1830, App 6.
VTS (Vessel Traffic System)	0952, **1221,** 1240-1241, 1320, 1323.
WAAS (Wide Area Augmentation System)	**0915, 1806.**
W-AIS (Warship AIS)	**0950.**
Warship Automatic Identification System	See W-AIS.
Warship Electronic Chart Display Information System (Warship ECDIS)	See WECDIS.
Water Track (Course steered)	**0713,** 0806, **1703,** 1722.
Wave of Translation (Canal Effect / Interaction)	**1220.**
Wavelength (Radar)	**1514, 1523**
Waypoint / Waypoints (Great Circle)	**0414, 0440-0441,** 0442, **0521,** 0622.
Waypoint / Waypoints (Track / Route)	0714, **0720, 0951, 1211, 1236, 1827, 1829.**
WECDIS (Warship ECDIS)	*WECDIS* comprises an *IMO*-approved *ECDIS* with additional functionality for military purposes in warships. **0210, 0331,** 0451, 0550, **0551,** 0613, **0631-0632,** 0701, 0712, 0714-0716, **0720-0721,** 0805, 0808, 0811, 0910, 0919, 0950, 1051, 1102, 1110, 1113, 1202, 1210, 1214, 1222, 1230, **1231,** 1232, 1236, **1238,** 1302, **1311,** 1312-1314, 1316, **1317,** 1319-1320, **1321,** 1323-1325, 1327-1328, 1402, 1413, 1415, 1502, **1528,** 1701, 1702, **1806, 1810, 1814-1815, 1827,** 1903, **1911,** 1920, **1921-1925.**
Weekly NMs	See NMs.
Weekly Notices to Mariners	See NMs.
West Australian Current	**1125.**
WGS 72 (Datum / Spheroid)	**0321-0322, 0324.**
WGS 84 (Datum / Spheroid)	0110, 0115, 0310, 0312- 0313, **0321-0322,** 0323, **0324,** 0331-0332, 0423, 0452, 0531, 0551, 0624, 0805, 0910, 0913, 0918, 0931, 1321, 1602, 1803, 1811, **1820,1829,** App 5. See also Datum, Oblate Spheroid and Spheroid.
Wheel-Over	0714, 0715, 0804, 0807, 1234, 1310, 1312, **1314, 1316,** 1319, **1322,** 1323, 1328, 1413, 1923-1924, 1932.
Whistle (Fog signal)	**0934, 0940.**
Wide Area Augmentation System	See WAAS.
Williamson's Turn (Manoeuvring)	**1827.**
Wind Sheer	**1332, 1334.**
World Geodetic System (WGS)	See WGS 84 and GNSS.
Yaw / Yaws / Yawing	0120, **1418,** 1419.
Yaw (Naval Command Systems)	0120.
Zenith	**0116.**
Zenithal Projection	**0413,** 0440.
Zero-PIM (Zero Position / Intended Move't)	1111, **1112.**
Zero-PIM	**1214.**
Zero Pitch (Controllable Pitch Propeller)	**1332**
Zero Position / Intended Movement	See Zero-PIM.
Zones of Longitude (Transverse Mercator)	**0431.**

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